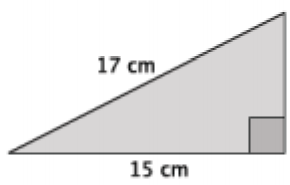
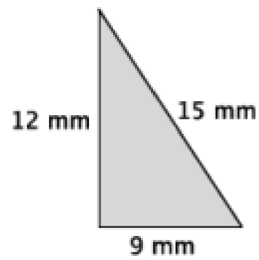
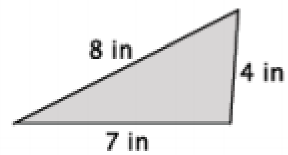
Lesson 1 – The Pythagorean Theorem

Essential Questions:

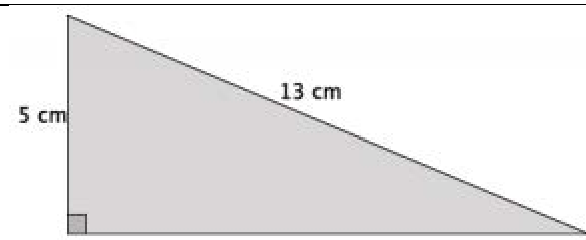
Classwork:



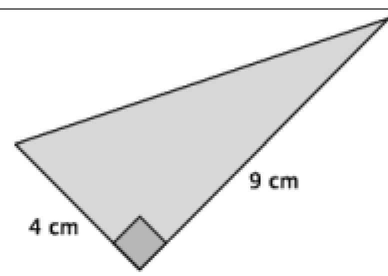


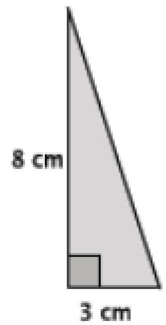
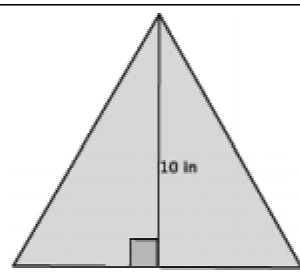


Example 1:
Write an equation that will allow you to determine the length of the unknown side of the right triangle.



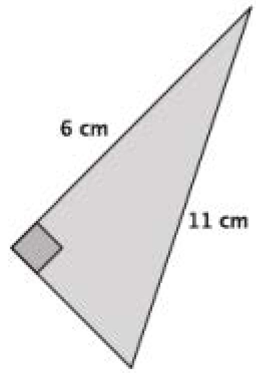
Example 2:
Write an equation that will allow you to determine the length of the unknown side of the right triangle.



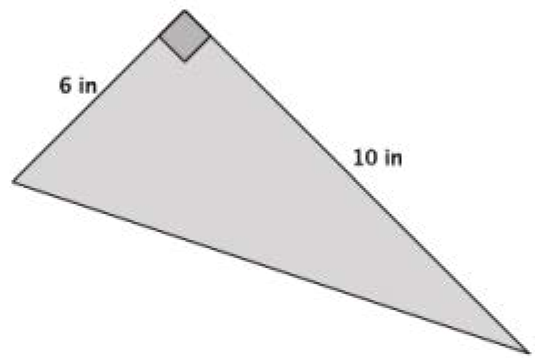
Example 3: Write an equation to determine the length of the unknown side of the right triangle.	
Example 4: In the figure below, we have an equilateral triangle with a height of 10 inches. What do we know about an equilateral triangle?	

On Your Own:

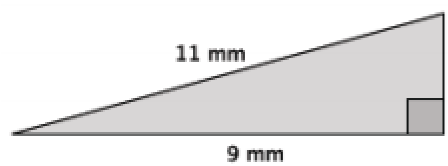
1. Use the Pythagorean theorem to find a whole number estimate of the length of the unknown side of the right triangle. Explain why your estimate makes sense.



2. Use the Pythagorean theorem to find a whole number estimate of the length of the unknown side of the right triangle. Explain why your estimate makes sense.

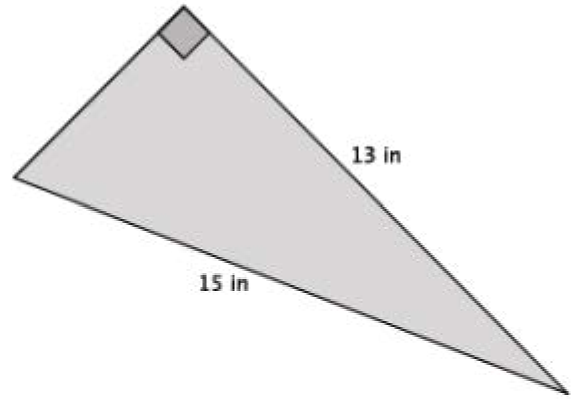
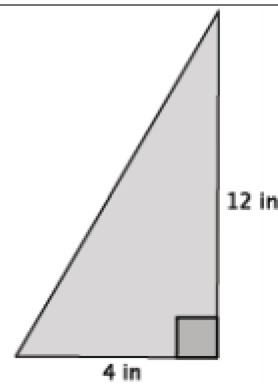


3. Use the Pythagorean theorem to find a whole number estimate of the length of the unknown side of the right triangle. Explain why your estimate makes sense.

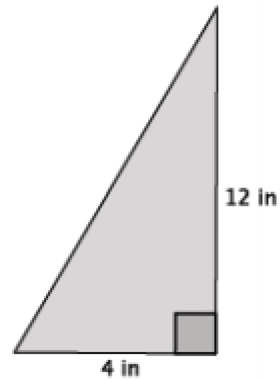


Lesson 1 Summary:

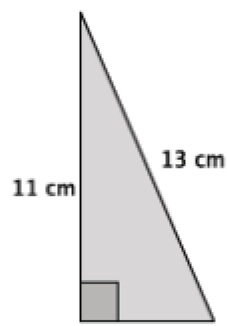
Independent Practice Lesson 1:

1. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.	
2. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.	

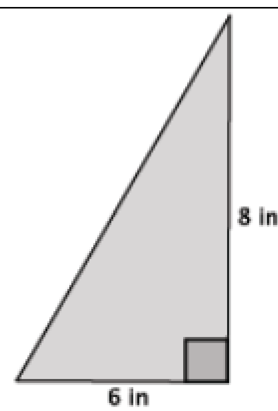
3. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.

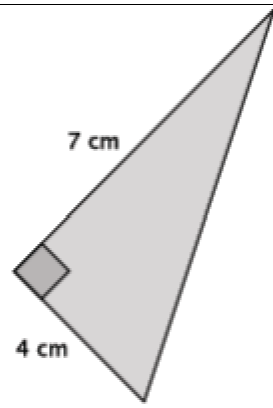
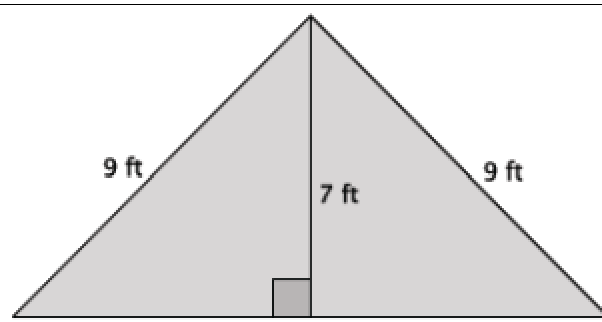


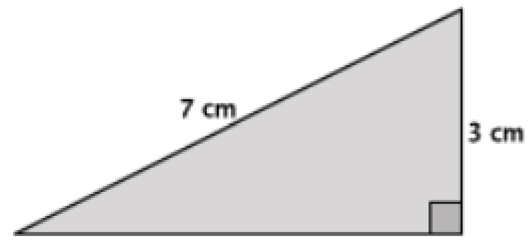
4. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.



5. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.



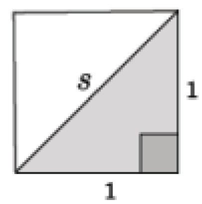
6. Determine the length of the unknown side of the right triangle. Explain how you know your answer is correct.	
7. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.	
8. The triangle below is an isosceles triangle. Use what you know about the Pythagorean theorem to determine the approximate length of the base of the isosceles triangle.	

9. Give an estimate for the area of the triangle shown below. Explain why it is a good estimate.	

Lesson 2 – Square Roots

Essential Questions:

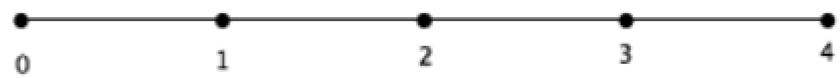
Discussion:

"How can we determine an estimate for the length of the diagonal of the unit square?"	

On Your Own:

1. Determine the positive square root of 81, if it exists. Explain.	
2. Determine the positive square root of 225, if it exists. Explain.	
3. Determine the positive square root of −36, if it exists. Explain.	
4. Determine the positive square root of 49, if it exists. Explain.	

Discussion:	
Can you estimate the value of $\sqrt{2}$?	



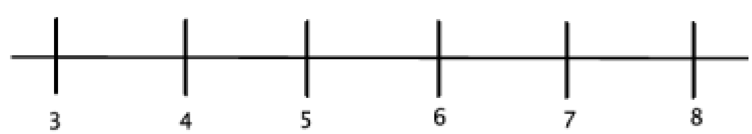
Place the numbers $\sqrt{1}$, $\sqrt{4}$, $\sqrt{9}$, and $\sqrt{16}$ on the number line, and explain how you knew where to place them.	
Place the numbers $\sqrt{2}$ and $\sqrt{3}$ on the number line. Be prepared to explain your reasoning.	
Place the numbers $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, and $\sqrt{8}$ on the number line. Be prepared to explain your reasoning.	
Place the numbers $\sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$, $\sqrt{13}$, $\sqrt{14}$, and $\sqrt{15}$ on the number line. Be prepared to explain your reasoning.	

On Your Own:
Determine the positive square root of the number given. If the number is not a perfect square, determine which whole number the square root would be closest to, and then use guess and check to give an approximate answer to one or two decimal places.

5. $\sqrt{49}$	
6. $\sqrt{62}$	
7. $\sqrt{122}$	
8. $\sqrt{400}$	
9. Which of the numbers in Exercises 5–8 are not perfect squares? Explain.	

Lesson 2 Summary:

Independent Practice Lesson 2
Determine the positive square root of the number given. If the number is not a perfect square, determine the integer to which the square root would be closest.

1. √169	
2. √256	
3. √81	
4. √147	
5. √8	
6. Which of the numbers in Problems 1–5 are not perfect squares? Explain.	
7. Place the following list of numbers in their approximate locations on a number line. √32, √12, √27, √18, √23, and √50	
8. Between which two integers will √45 be located? Explain how you know.	

Lesson 3 – Existence and Uniqueness of Square Roots and Cube Roots

Essential Questions:

Classwork:

The numbers in each column are related. Your goal is to determine how they are related, determine which numbers belong in the blank parts of the columns, and write an explanation for how you know the numbers belong there.

Find the Rule Part 1

1	1
2	
3	9
	81
11	121
15	
	49
10	
12	
	169
m	
	n

.

Find the Rule Part 2

1	1
2	
3	27
	125
6	216
11	
	64
10	
7	
	2,744
p	
	q

On Your Own:
Find the positive value of x that makes each equation true. Check your solution.

1. $x^2 = 169$ a. Explain the first step in solving this equation.	
b. Solve the equation, and check your answer.	
2. A square-shaped park has an area of 324 ft^2. What are the dimensions of the park? Write and solve an equation.	
3. $625 = x^2$	
4. A cube has a volume of 27 in^3. What is the measure of one of its sides? Write and solve an equation.	
5. What positive value of x makes the following equation true: $x^2 = 64$? Explain.	
6. What positive value of x makes the following equation true: $x^3 = 64$? Explain.	

7. Find the positive value of x that makes the equation true: $x^2 = 256^{-1}$.	
8. Find the positive value of x that makes the equation true: $x^3 = 343^{-1}$.	
9. Is 6 a solution to the equation $x^2 - 4 = 5x$? Explain why or why not.	

Lesson 3 Summary:

Independent Practice Lesson 3

Find the positive value of x that makes each equation true. Check your solution.

1. What positive value of x makes the following equation true: $x^2 = 289$? Explain.

2. A square-shaped park has an area of 400 ft^2. What are the dimensions of the park? Write and solve an equation.

3. A cube has a volume of 64 in^3. What is the measure of one of its sides? Write and solve an equation.

4. What positive value of x makes the following equation true: $125 = x^3$? Explain.

5. Find the positive value of x that makes the equation true: $x^2 = 441^{-1}$.

a. Explain the first step in solving this equation.

b. Solve and check your solution.

6. Find the positive value of x that makes the equation true: $x^3 = 125^{-1}$.

7. The area of a square is 196 in^2. What is the length of one side of the square? Write and solve an equation, and then check your solution.

8. The volume of a cube is 729 cm^3. What is the length of one side of the cube? Write and solve an equation, and then check your solution.

9. What positive value of x would make the following equation true: $19 + x^2 = 68$?

Lesson 4- Simplifying Square Roots

Essential Questions:

Opening Exercise:

a. i. What does $\sqrt{16}$ equal? ii. What does 4 x 4 equal? iii. Does $\sqrt{16} = \sqrt{4 \text{ x } 4}$?	
b. i. What does $\sqrt{36}$ equal? ii. What does 6 x 6 equal? iii. Does $\sqrt{36} = \sqrt{6 \text{ x } 6}$?	
c. i. What does $\sqrt{121}$ equal? ii. What does 11 x 11 equal? iii. Does $\sqrt{121} = \sqrt{11 \text{ x } 11}$?	
d. i. What does $\sqrt{81}$ equal? ii. What does 9 x 9 equal? iii. Does $\sqrt{81} = \sqrt{9 \text{ x } 9}$?	
e. What is another way to write $\sqrt{20}$?	
f. What is another way to write $\sqrt{4 \text{ x } 7}$?	

Example 1	
Simplify the square root as much as possible. $\sqrt{50}$	
Example 2	
Simplify the square root as much as possible $\sqrt{28}$	

Exercises 1 – 4 Simplify the square root as much as possible.

$\sqrt{18}$	
$\sqrt{44}$	
$\sqrt{169}$	
$\sqrt{75}$	

Example 3	
Simplify the square root as much as possible $\sqrt{128}$	

Example 4

Simplify the square root as much as possible $\sqrt{288}$	

Exercises 5 – 8

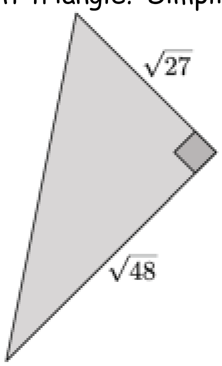
Simplify $\sqrt{108}$	
Simplify $\sqrt{250}$	
Simplify $\sqrt{200}$	
Simplify $\sqrt{504}$	

Independent Practice

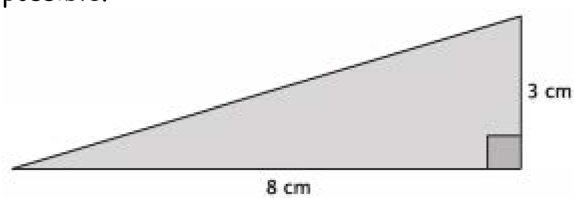
Simplify each of the square roots in Problems 1–5 as much as possible.

1. $\sqrt{20}$	2. $\sqrt{54}$	3. $\sqrt{144}$	4. $\sqrt{512}$	5. $\sqrt{756}$

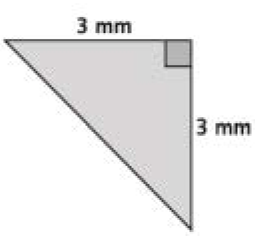
6. What is the length of the unknown side of the right triangle? Simplify your answer, if possible.



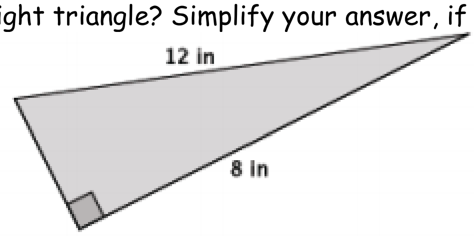
7. What is the length of the unknown side of the right triangle? Simplify your answer, if possible.



8. What is the length of the unknown side of the right triangle? Simplify your answer, if possible



9. What is the length of the unknown side of the right triangle? Simplify your answer, if possible.



10. Josue simplified $\sqrt{450}$ as $15\sqrt{2}$. Is he correct? Explain why or why not.	11. Tiah was absent from school the day that you learned how to simplify a square root. Using $\sqrt{360}$, write Tiah an explanation for simplifying square roots.

Lesson 5 –Solving Radical Equations

Essential Questions:

Example 1:
Transform the equation using the properties of equality until you can determine the positive value of x that makes the equation true.

$x^3 + 9x = \frac{1}{2}(18x + 54)$

Example 2:
Transform the equation using the properties of equality until you can determine the positive value of x that makes the equation true.

$x(x - 3) - 51 = -3x + 13$

On Your Own:

Find the positive value of x that makes each equation true, and then verify your solution is correct.

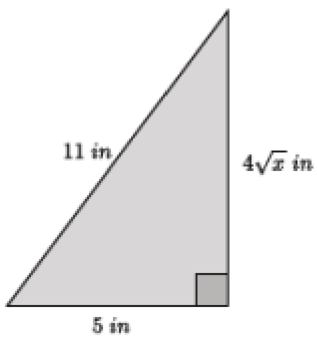
1. a. Solve $x^2 - 14 = 5x + 67 - 5x$	
b. Explain how you solved the equation.	
2. Solve and simplify: $x(x - 1) = 121 - x$	
3. A square has a side length of $3x$ and an area of 324 in^2. What is the value of x?	

4. $-3x^3 + 14 = -67$

5. $x(x + 4) - 3 = 4(x + 19.5)$

6. $216 + x = x(x^2 - 5) + 6x$

7.
a. What are we trying to determine in the diagram?



b. Determine the value of x, and check your answer.

Lesson 5 Summary:

Independent Practice Lesson 5

Find the positive value of x that makes each equation true, and then verify your solution is correct.

1. $x^2(x + 7) = \frac{1}{2}(14x^2 + 16)$

2. $x^3 = 1331^{-1}$

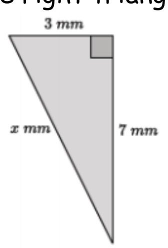
3. Determine the positive value of xx that makes the equation true, and then explain how you solved the equation.

$\frac{x^9}{x^7} - 49 = 0$

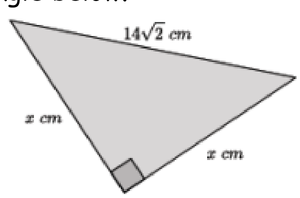
4. Determine the positive value of xx that makes the equation true.
$(8x)^2 = 1$

5. $(9\sqrt{x})^2 - 43x = 76$

6. Determine the length of the hypotenuse of the right triangle below.



7. Determine the length of the legs in the right triangle below.



8. An equilateral triangle has side lengths of 6cm. What is the height of the triangle? What is the area of the triangle?

9. Challenge: Find the positive value of x that makes the equation true.
$\left(\frac{1}{2}x\right)^2 - 3x = 7x + 8 - 10x$

10. Challenge: Find the positive value of x that makes the equation true.

$11x + x(x - 4) = 7(x + 9)$

Lesson 6 – Finite and Infinite Decimals

Essential Questions:

Classwork:

a. Use long division to determine the decimal expansion of $\frac{54}{20}$.

b. Use long division to determine the decimal expansion of $\frac{7}{8}$.

c. Use long division to determine the decimal expansion of $\frac{8}{9}$.

d. Use long division to determine the decimal expansion of $\frac{22}{7}$.

e. What do you notice about the decimal expansions of parts (a) and (b) compared to the decimal expansions of parts (c) and (d)?

Example 1:
Consider the fraction $\frac{5}{8}$. Is it equal to a finite decimal? How do you know?

Example 2:
Consider the fraction $\frac{17}{125}$. Is it equal to a finite or an infinite decimal? How do you know?

On Your Own:
Show your steps, but use a calculator for the multiplication.

1. Convert the fraction $\frac{7}{8}$ to a decimal.

a. Write the denominator as a product of 2's or 5's. Explain why this way of rewriting the denominator helps to find the decimal representation of $\frac{7}{8}$.

b. Find the decimal representation of $\frac{7}{8}$. Explain why your answer is reasonable.

2. Convert the fraction $\frac{43}{64}$ to a decimal.

3. Convert the fraction $\frac{29}{125}$ to a decimal.

4. Convert the fraction $\frac{19}{34}$ to a decimal.	
5. Identify the type of decimal expansion for each of the numbers in Exercises 1–4 as finite or infinite. Explain why their decimal expansion is such.	
Example 3: Write $\frac{7}{80}$ as a decimal. Will it be finite or infinite? Explain.	
Example 4: Write $\frac{3}{160}$ as a decimal. Will it be finite or infinite? Explain.	

On Your Own:

6. Convert the fraction $\frac{37}{40}$ to a decimal. a. Write the denominator as a product of 2's and/or 5's. Explain why this way of rewriting the denominator helps to find the decimal representation of $\frac{37}{40}$.	

b. Find the decimal representation of $\frac{37}{40}$. Explain why your answer is reasonable.

7. Convert the fraction $\frac{3}{250}$ to a decimal.

8. Convert the fraction $\frac{7}{1250}$ to a decimal.

Lesson 6 Summary:

Independent Practice Lesson 6

Convert each fraction to a finite decimal. If the fraction cannot be written as a finite decimal, then state how you know. Show your steps, but use a calculator for the multiplication

1. $\frac{2}{32}$

2. $\frac{99}{125}$

a. Write the denominator as a product of 2's and/or 5's. Explain why this way of rewriting the denominator helps to find the decimal representation of $\frac{99}{125}$.

b. Find the decimal representation of $\frac{99}{125}$. Explain why your answer is reasonable.

3. $\frac{15}{128}$

4. $\frac{8}{15}$

5. $\frac{3}{28}$

6. $\frac{13}{400}$

7. $\frac{5}{64}$

8. $\frac{15}{35}$

9. $\frac{199}{250}$

10. $\frac{219}{625}$

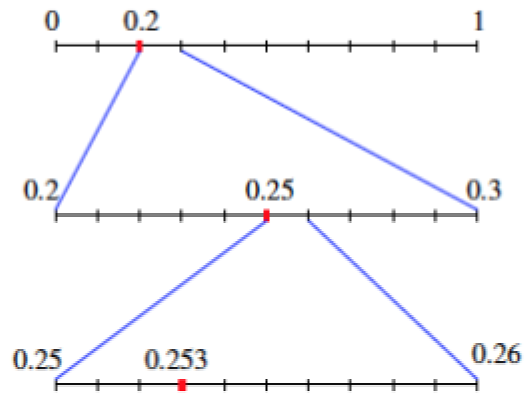
Lesson 7 – Infinite Decimals

Essential Questions:

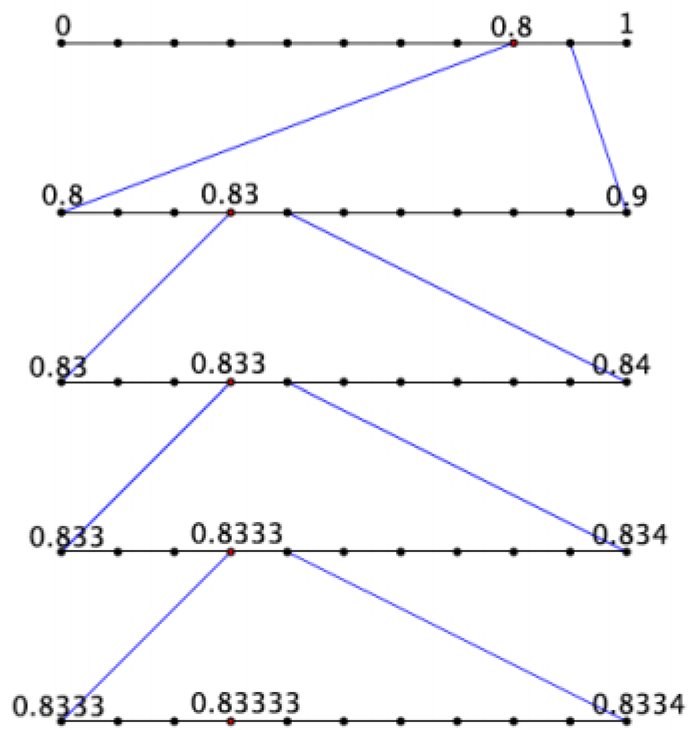
Classwork:

a. Write the expanded form of the decimal 0.3765 using powers of 10.	
b. Write the expanded form of the decimal 0.3333333… using powers of 10.	
c. What is an infinite decimal? Give an example.	
d. Do you think it is acceptable to write that 1 = 0.99999…? Why or why not?	

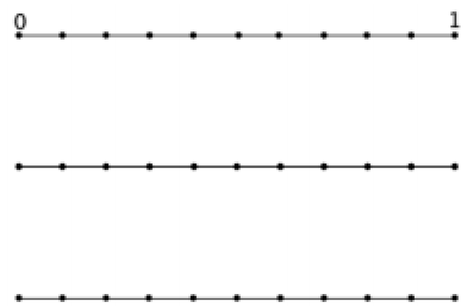
Example 1:
The number 0.253 is represented on the number line.

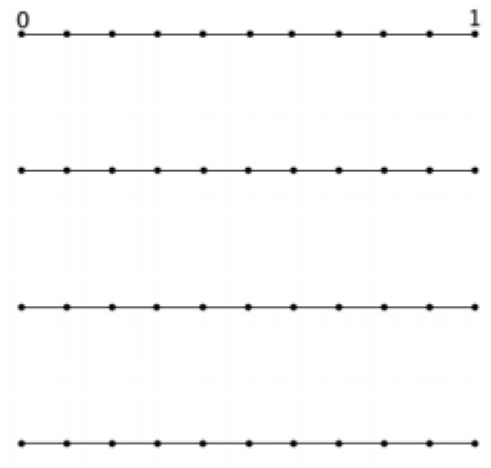


Example 2;
The number $\frac{5}{6}$ = 0.833333… = $0.8\overline{3}$ is represented on the number line.



On Your Own:

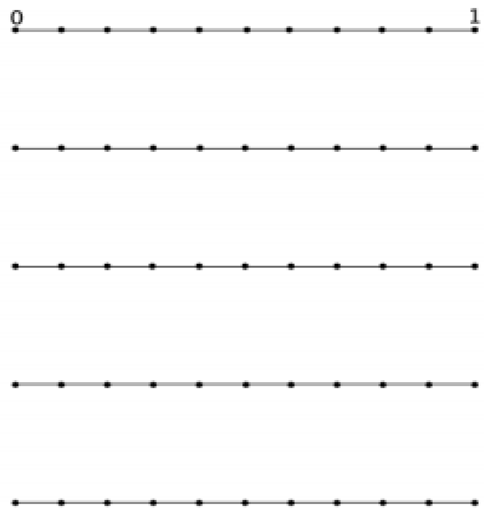
1. a. Write the expanded form of the decimal 0.125 using powers of 10.	
b. Show on the number line the representation of the decimal 0.125.	
c. Is the decimal finite or infinite? How do you know?	

2. a. Write the expanded form of the decimal 0.3875 using powers of 10.	
b. Show on the number line the representation of the decimal 0.3875.	
c. Is the decimal finite or infinite? How do you know?	

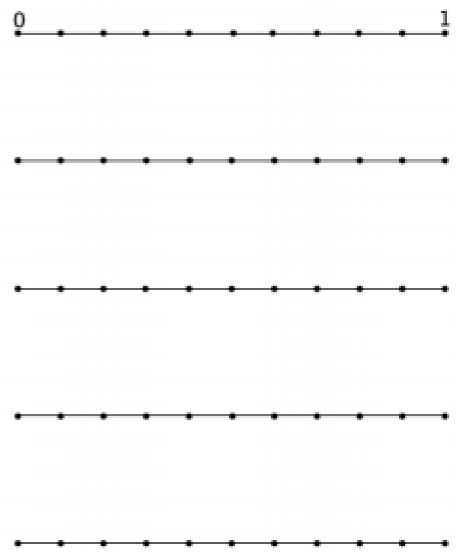
3.

a. Write the expanded form of the decimal 0.777777… using powers of 10.

b. Show on the number line the representation of the decimal 0.777777….



c. Is the decimal finite or infinite? How do you know?

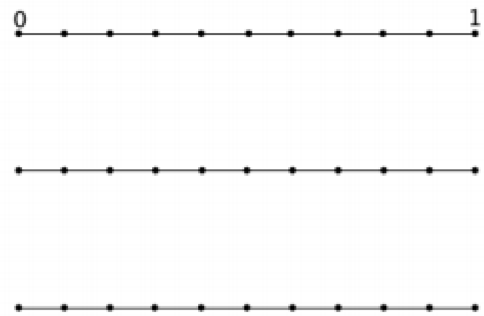
4. a. Write the expanded form of the decimal $0.\overline{45}$ using powers of 10.	
b. Show on the number line the representation of the decimal $0.\overline{45}$.	
c. Is the decimal finite or infinite? How do you know?	

5. Order the following numbers from least to greatest: 2.121212, 2.1, 2.2, and $2.\overline{12}$.	
6. Explain how you knew which order to put the numbers in.	

Lesson 7 Summary:

Independent Practice Lesson 7

1. a. Write the expanded form of the decimal 0.625 using powers of 10.



b. Show on the number line the representation of the decimal 0.625.

c. Is the decimal finite or infinite? How do you know?

2. a. Write the expanded form of the decimal $0.\overline{370}$ using powers of 10.

b. Show on the number line the representation of the decimal 0.370370….

c. Is the decimal finite or infinite? How do you know?

3. Which is a more accurate representation of the number $\frac{2}{3}$: 0.6666 or $0.\overline{6}$? Explain. Which would you prefer to compute with?

4. Explain why we shorten infinite decimals to finite decimals to perform operations. Explain the effect of shortening an infinite decimal on our answers.

5. A classmate missed the discussion about why $0.\overline{9} = 1$. Convince your classmate that this equality is true.

6. Explain why $0.3333 < 0.33333$

Lesson 8 – The Long Division Algorithm

Essential Questions:

Example 1:
Show that the decimal expansion of $\frac{26}{4}$ is 6.5.

On Your Own:

1. Use long division to determine the decimal expansion of $\frac{142}{2}$.

a. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{2}$.

$$142 = \_\_\_\_ \times 2 + \_\_\_\_$$

$$\frac{142}{2} = \frac{\_\_\_\_ \times 2 + \_\_\_\_}{2}$$

$$\frac{142}{2} = \frac{\_\_\_\_ \times 2}{2} + \frac{\_\_\_\_}{2}$$

$$\frac{142}{2} = \_\_\_\_ + \frac{\_\_\_\_}{2}$$

$$\frac{142}{2} = \_\_\_\_\_\_$$

b. Does the number $\frac{142}{2}$ have a finite or an infinite decimal expansion? Explain how you know.	
2. Use long division to determine the decimal expansion of $\frac{142}{4}$.	
a. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{4}$.	$142 = \underline{\qquad} \times 4 + \underline{\qquad}$ $\frac{142}{4} = \frac{\underline{\qquad} \times 4 + \underline{\qquad}}{4}$ $\frac{142}{4} = \frac{\underline{\qquad} \times 4}{4} + \frac{\underline{\qquad}}{4}$ $\frac{142}{4} = \underline{\qquad} + \frac{\underline{\qquad}}{4}$ $\frac{142}{4} = \underline{\qquad\qquad}$
b. Does the number $\frac{142}{4}$ have a finite or an infinite decimal expansion? Explain how you know.	

3. Use long division to determine the decimal expansion of $\frac{142}{6}$.	
a. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{6}$.	$142 = \_\_\_\_ \times 6 + \_\_\_\_$ $\frac{142}{6} = \frac{\_\_\_\_ \times 6 + \_\_\_\_}{6}$ $\frac{142}{6} = \frac{\_\_\_\_ \times 6}{6} + \frac{\_\_\_\_}{6}$ $\frac{142}{6} = \_\_\_\_ + \frac{\_\_\_\_}{6}$ $\frac{142}{6} = \_\_\_\_\_\_\_\_\_\_$
b. Does the number $\frac{142}{6}$ have a finite or an infinite decimal expansion? Explain how you know.	

4. Use long division to determine the decimal expansion of $\frac{142}{11}$.

a. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{11}$.

$$142 = \_\_\_\_ \times 11 + \_\_\_\_$$

$$\frac{142}{11} = \frac{\_\_\_\_ \times 11 + \_\_\_\_}{11}$$

$$\frac{142}{11} = \frac{\_\_\_\_ \times 11}{11} + \frac{\_\_\_\_}{11}$$

$$\frac{142}{11} = \_\_\_\_ + \frac{\_\_\_\_}{11}$$

$$\frac{142}{11} = \_\_\_\_\_\_\_\_\_\_$$

b. Does the number $\frac{142}{11}$ have a finite or an infinite decimal expansion? Explain how you know

5. Which fractions produced an infinite decimal expansion? Why do you think that is?	
Discussion:	
What is the decimal expansion of $\frac{142}{2}$?	
On Your Own:	
6. Does the number $\frac{65}{13}$ have a finite or an infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.	
7. Does the number $\frac{17}{11}$ have a finite or an infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.	

8. Does the number π = 3.1415926535897 … have a finite or an infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.	
9. Does the number $\frac{860}{999}$ = 0.860860860 … have a finite or an infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.	
10. Does the number $\sqrt{2}$ = 1.41421356237 … have a finite or an infinite decimal expansion? Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.	

Lesson 8 Summary:

Independent Practice Lesson 8

1. Write the decimal expansion of $\frac{70000}{9}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

2. Write the decimal expansion of $\frac{6,555,555}{3}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

3. Write the decimal expansion of $\frac{350,000}{11}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

4. Write the decimal expansion of $\frac{12{,}000{,}000}{37}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

5. Someone notices that the long division of 2,222,222 by 6 has a quotient of 370,370 and remainder 2 and wonders why there is a repeating block of digits in the quotient, namely 370. Explain to the person why this happens.

6. Is the number $\frac{9}{11}$ = 0.81818181 … rational? Explain.

7. Is the number $\sqrt{3}$ = 1.73205080 … rational? Explain.

8. Is the number $\frac{41}{333}$ = 0.1231231231 … rational? Explain.

Lesson 9 – Decimal Expansion of Fractions, Part 1

Essential Questions:

Classwork:

a. i. We know that the fraction $\frac{5}{8}$ can be written as a finite decimal because its denominator is a product of 2's. Which power of 10 will allow us to easily write the fraction as a decimal? Explain.	
ii. Write the equivalent fraction using the power of 10.	
b. i. We know that the fraction $\frac{17}{125}$ can be written as a finite decimal because its denominator is a product of 5's. Which power of 10 will allow us to easily write the fraction as a decimal? Explain.	
ii. Write the equivalent fraction using the power of 10.	

Example 1:
Write the decimal expansion of the fraction $\frac{5}{8}$.

Example 2:
Write the decimal expansion of the fraction $\frac{17}{125}$.

Example 3: Write the decimal expansion of the fraction $\frac{35}{11}$.	
Example 4: Write the decimal expansion of the fraction $\frac{6}{7}$.	

On Your Own

1. a. Choose a power of 10 to use to convert this fraction to a decimal: $\frac{4}{13}$. Explain your choice.	
b. Determine the decimal expansion of $\frac{4}{13}$. Verify you are correct using a calculator.	

2. Write the decimal expansion of $\frac{1}{11}$. Verify you are correct using a calculator.	
3. Write the decimal expansion of $\frac{19}{21}$. Verify you are correct using a calculator.	

Lesson 9 Summary:

Independent Practice Lesson 9

1.

a. Choose a power of 10 to convert this fraction to a decimal: $\frac{4}{11}$. Explain your choice.

b. Determine the decimal expansion of $\frac{4}{11}$. Verify you are correct using a calculator.

2. Write the decimal expansion of $\frac{5}{13}$. Verify you are correct using a calculator.

3. Write the decimal expansion of $\frac{23}{39}$. Verify you are correct using a calculator.

4. Tamer wrote the decimal expansion of $\frac{3}{7}$ as 0.418571, but when he checked it on a calculator, it was 0.428571. Identify his error and explain what he did wrong.

$$\frac{3}{7} = \frac{3 \times 10^6}{7} \times \frac{1}{10^6}$$
$$= \frac{3\,000\,000}{7} \times \frac{1}{10^6}$$

$$3\,000\,000 = 418\,571 \times 7 + 3$$

$$\frac{3}{7} = \frac{418\,571 \times 7 + 3}{7} \times \frac{1}{10^6}$$
$$= \left(\frac{418\,571 \times 7}{7} + \frac{3}{7}\right) \times \frac{1}{10^6}$$
$$= \left(418\,571 + \frac{3}{7}\right) \times \frac{1}{10^6}$$
$$= 418\,571 \times \frac{1}{10^6} + \left(\frac{3}{7} \times \frac{1}{10^6}\right)$$
$$= \frac{418\,571}{10^6} + \left(\frac{3}{7} \times \frac{1}{10^6}\right)$$
$$= 0.418\,571 + \left(\frac{3}{7} \times \frac{1}{10^6}\right)$$

5. Given that

($\frac{6}{7}$ = 0.857 142 + ($\frac{6}{7}$ × 1 106), explain why 0.857142 is a good estimate of $\frac{6}{7}$.

Lesson 10 – Converting Repeating Decimals to Fractions

Essential Questions:

Example 1

Find the fraction that is equal to the infinite decimal $0.\overline{81}$.	

Exercises 1 and 2

1. a. Let x = $0.\overline{123}$. Explain why multiplying both sides of this equation by 10^3 will help us determine the fractional representation of x.	
b. After multiplying both sides of the equation by 10^3, rewrite the resulting equation by making a substitution that will help determine the fractional value of x. Explain how you were able to make the substitution.	

c. Solve the equation to determine the value of x.	
d. Is your answer reasonable? Check your answer using a calculator.	
2. Find the fraction equal to $0.\overline{4}$. Check your answer using a calculator.	

Example 2

Find the fraction that is equal to the infinite decimal $2.13\overline{8}$.	

Exercises 3 and 4

3. Find the fraction equal to $1.6\overline{23}$ Check your answer using a calculator.	
4. Find the fraction equal to $2.9\overline{60}$. Check your answer using a calculator.	

Independent Practice

1.

   a. Let x = $0.\overline{631}$. Explain why multiplying both sides of this equation by 10^3 will help us determine the fractional representation of x.

   b. After multiplying both sides of the equation by 10^3, rewrite the resulting equation by making a substitution that will help determine the fractional value of x. Explain how you were able to make the substitution.

   c. Solve the equation to determine the value of x.

   d. Is your answer reasonable? Check your answer using a calculator.

2. Find the fraction equal to $3.40\overline{8}$. Check your answer using a calculator.

3. Find the fraction equal to $0.\overline{5923}$. Check your answer using a calculator.

4. Find the fraction equal to $2.3\overline{82}$. Check your answer using a calculator.

5. Find the fraction equal to $0.\overline{714285}$. Check your answer using a calculator.

6. Explain why an infinite decimal that is not a repeating decimal cannot be rational.

7. In a previous lesson, we were convinced that it is acceptable to write $0.\overline{9} = 1$. Use what you learned today to show that it is true.

8. Examine the following repeating infinite decimals and their fraction equivalents. What do you notice? Why do you think what you observed is true?

$0.\overline{81} = \frac{81}{99}$ $0.\overline{4} = \frac{4}{9}$ $0.\overline{123} = \frac{123}{999}$ $0.\overline{60} = \frac{60}{99}$ $0.\overline{9} = 1.0$

Lesson 11 – The Decimal Expansion of Some Irrational Numbers

Essential Questions:

Classwork:

Place $\sqrt{28}$ on a number line. What decimal do you think $\sqrt{28}$ is equal to? Explain your reasoning.

Example 1:

Recall the basic theorem on inequalities:

Let c and d be two positive numbers, and let n be a fixed positive integer. Then $c < d$ if and only if $cn < dn$.

Write the decimal expansion of $\sqrt{3}$.

First approximation:

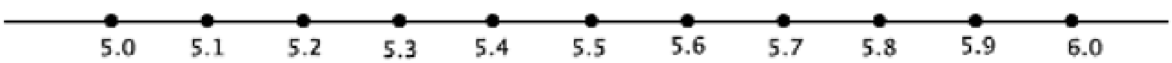
Second approximation:

Third approximation:

Example 2:
Write the decimal expansion of $\sqrt{28}$.

First approximation:

Second approximation:



Third approximation:

Fourth approximation:

On Your Own:

Between which interval of hundredths would $\sqrt{14}$ be located? Show your work.	

Lesson 11 Summary:

Independent Practice Lesson 11

1. Use the method of rational approximation to determine the decimal expansion of $\sqrt{84}$. Determine which interval of hundredths it would lie in.

2. Determine the three-decimal digit approximation of the number $\sqrt{34}$.

3. Write the decimal expansion of $\sqrt{47}$ to at least two-decimal digits.

4. Write the decimal expansion of $\sqrt{46}$ to at least two-decimal digits.

5. Explain how to improve the accuracy of the decimal expansion of an irrational number.

6. Is the number $\sqrt{125}$ rational or irrational? Explain.

7. Is the number $0.646464646\ldots$ rational or irrational? Explain.

8. Is the number $3.741657387\ldots$ rational or irrational? Explain.

9. Is the number $\sqrt{99}$ rational or irrational? Explain.

10. Challenge: Determine the two-decimal digit approximation of the number $\sqrt[3]{9}$.

Lesson 12 – Decimal Expansion of Fractions, Part 2

Essential Questions:

Classwork:

Write the decimal expansion of $\frac{35}{11}$.

Locate $\frac{35}{11}$ on the number line.

On Your Own:

1. Use rational approximation to determine the decimal expansion of $\frac{5}{3}$.

2. Use rational approximation to determine the decimal expansion of $\frac{5}{11}$.

3.

a. Determine the decimal expansion of the number $\frac{23}{99}$ using rational approximation and long division.

Chapter 12 Summary:

Independent Practice Lesson 12

1. Explain why the tenths digit of $\frac{3}{11}$ is 2, using rational approximation.

2. Use rational approximation to determine the decimal expansion of $\frac{25}{9}$.

3. Use rational approximation to determine the decimal expansion of $\frac{11}{41}$ to at least 5 digits.

4. Use rational approximation to determine which number is larger, $\sqrt{10}$ or $\frac{28}{9}$.

5. Sam says that $\frac{7}{11}$ = 0.63, and Jaylen says that $\frac{7}{11}$ = 0.636. Who is correct? Why?

Lesson 13 – Comparing Irrational Numbers

Essential Questions:

On Your Own:

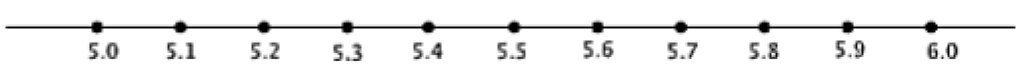
1. Rodney thinks that $\sqrt[3]{64}$ is greater than $\frac{17}{4}$. Sam thinks that $\frac{17}{4}$ is greater. Who is right and why?	
2. Which number is smaller, $\sqrt[3]{27}$ or 2.89? Explain.	
3. Which number is smaller, $\sqrt{121}$ or $\sqrt[3]{125}$? Explain.	

4. Which number is smaller, $\sqrt{49}$ or $\sqrt[3]{216}$? Explain.	
5. Which number is greater, $\sqrt{50}$ or $\frac{319}{45}$? Explain.	
6. Which number is greater, $\frac{5}{11}$ or $0.\overline{4}$? Explain.	

7. Which number is greater, $\sqrt{38}$ or $\frac{154}{25}$? Explain.

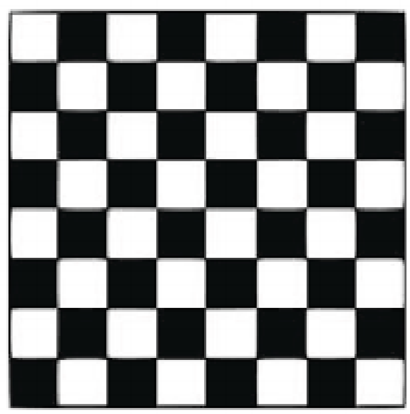
8. Which number is greater, , $\sqrt{2}$ or $\frac{15}{9}$? Explain.

9. Place each of the following numbers at its approximate location on the number line: $\sqrt{25}$, $\sqrt{28}$, $\sqrt{30}$, $\sqrt{32}$, $\sqrt{35}$, and $\sqrt{36}$.



10. Challenge: Which number is larger $\sqrt{5}$ or $\sqrt[3]{11}$?

11. A certain chessboard is being designed so that each square has an area of 3 in^2. What is the length of one edge of the board rounded to the tenths place? (A chessboard is composed of 64 squares as shown.)



Definitions:

Rational Numbers:	
Irrational Numbers:	

Lesson 13 Summary:

Independent Practice Lesson 13

1. Which number is smaller, $\sqrt[3]{343}$ or $\sqrt{48}$ Explain.

2. Which number is smaller, $\sqrt{100}$ or $\sqrt[3]{1000}$? Explain.

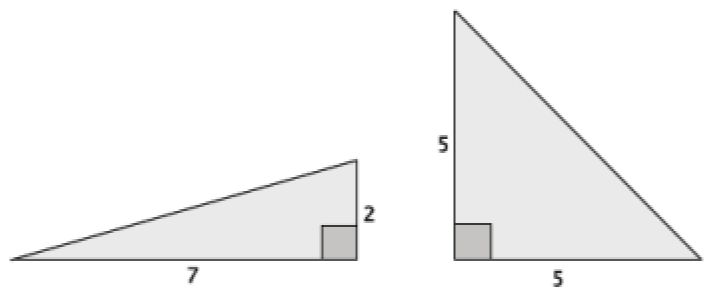
3. Which number is larger, $\sqrt{87}$ or $\frac{929}{99}$? Explain.

4. Which number is larger, $\frac{9}{13}$ or $0.\overline{692}$? Explain.

5. Which number is larger, 9.1 or, $\sqrt{82}$? Explain.

6. Place each of the following numbers at its approximate location on the number line: $\sqrt{144}$, $\sqrt[3]{1000}$, $\sqrt{130}$, $\sqrt{110}$, $\sqrt{120}$, $\sqrt{115}$, and $\sqrt{133}$. Explain how you knew where to place the numbers.

7. Which of the two right triangles shown below, measured in units, has the longer hypotenuse? Approximately how much longer is it?

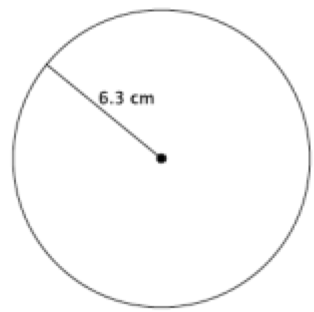


Lesson 14 – Decimal Expansion of π

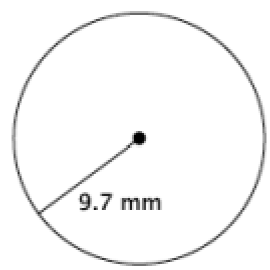
Essential Questions:

Classwork:

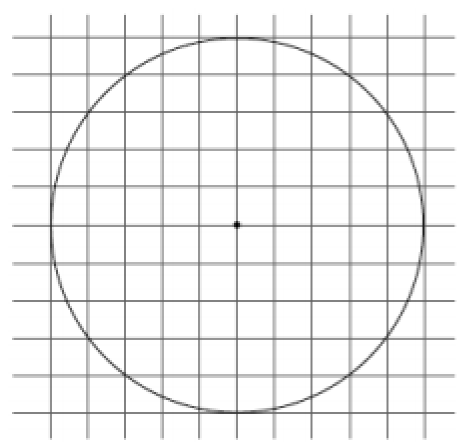
a. Write an equation for the area, A, of the circle shown.



b. Write an equation for the circumference, C, of the circle shown.

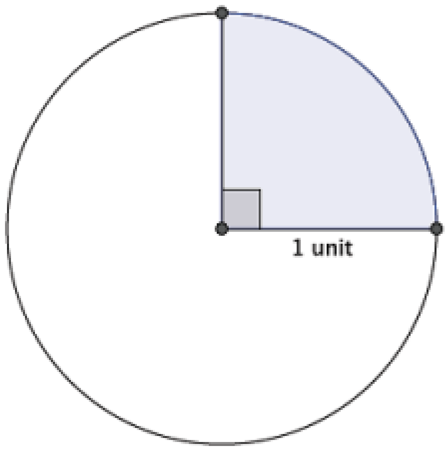


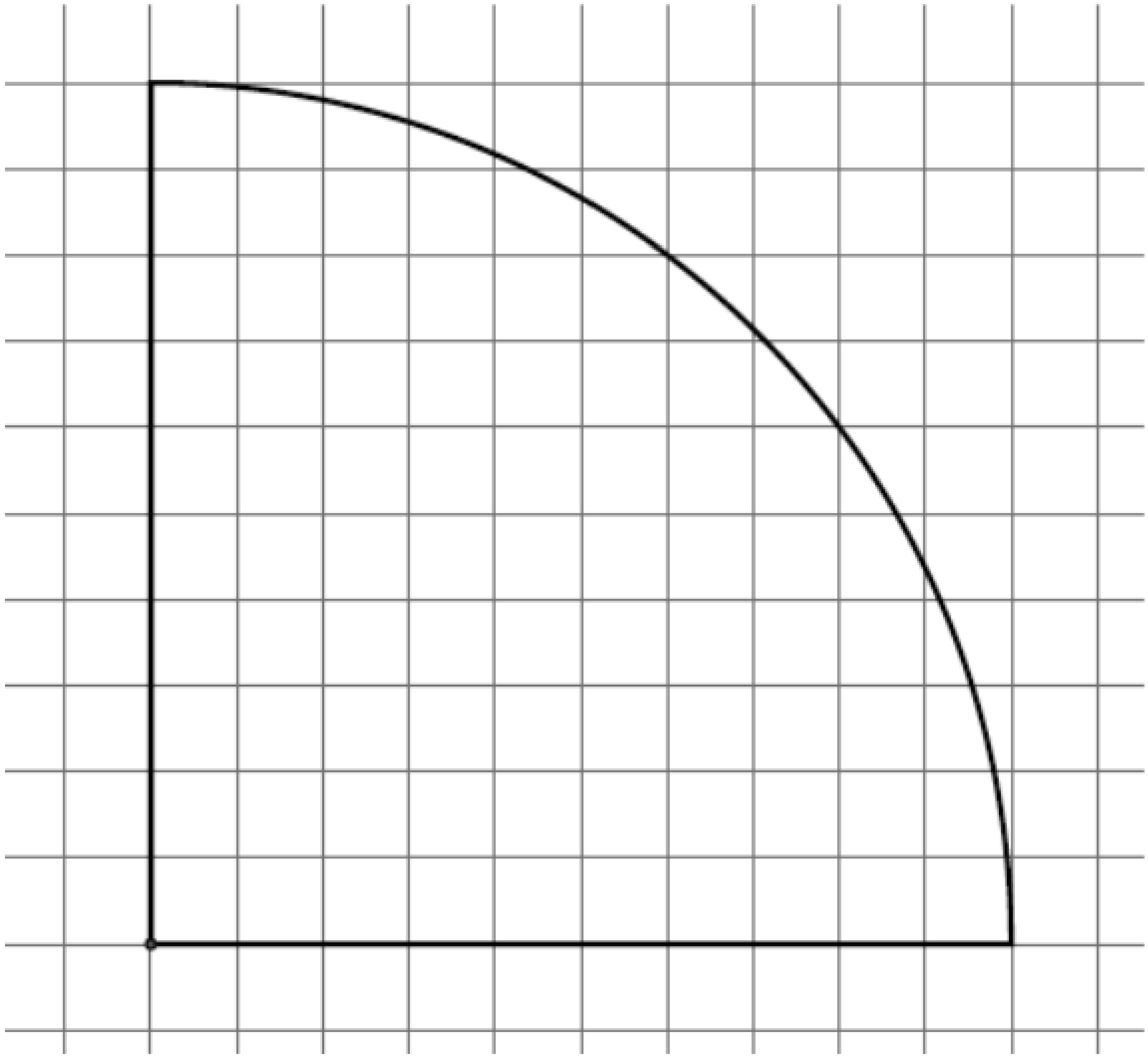
c. Each of the squares in the grid below has an area of 1 unit^2.
i. Estimate the area of the circle shown by counting squares.



ii. Calculate the area of the circle using a radius of 5 units. Use 3.14 as an approximation for π.

How do we determine the decimal expansion for π?





On Your Own:

1. Gerald and Sarah are building a wheel with a radius of 6.5 cm and are trying to determine the circumference. Gerald says, "Because $6.5 \times 2 \times 3.14 = 40.82$, the circumference is 40.82 cm." Sarah says, "Because $6.5 \times 2 \times 3.10 = 40.3$ and $6.5 \times 2 \times 3.21 = 41.73$, the circumference is somewhere between 40.3 and 41.73." Explain the thinking of each student.	
2. Estimate the value of the irrational number $(6.12486\ldots)^2$.	
3. Estimate the value of the irrational number $(9.204107\ldots)^2$.	
4. Estimate the value of the irrational number $(4.014325\ldots)^2$.	

Lesson 14 Summary:

Lesson 14 Independent Practice

1. Caitlin estimated π to be $3.10 < \pi < 3.21$. If she uses this approximation of π to determine the area of a circle with a radius of 5 cm, what could the area be?

2. Myka estimated the circumference of a circle with a radius of 4.5 in. to be 28.44 in. What approximate value of π did she use? Is it an acceptable approximation of π? Explain.

3. A length of ribbon is being cut to decorate a cylindrical cookie jar. The ribbon must be cut to a length that stretches the length of the circumference of the jar. There is only enough ribbon to make one cut. When approximating π to calculate the circumference of the jar, which number in the interval $3.10 < \pi < 3.21$ should be used? Explain.

4. Estimate the value of the irrational number $(1.86211\ldots)^2$.

5. Estimate the value of the irrational number $(5.9035687\ldots)^2$.

6. Estimate the value of the irrational number $(12.30791\ldots)^2$.

7. Estimate the value of the irrational number $(0.6289731\ldots)^2$.

8. Estimate the value of the irrational number $(1.112223333\ldots)^2$.

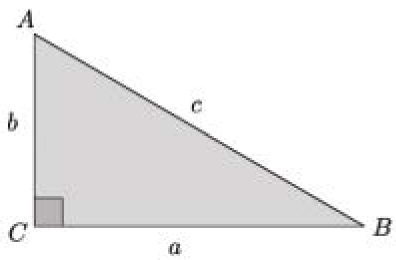
9. Which number is a better estimate for π, $\frac{22}{7}$ or 3.14? Explain.

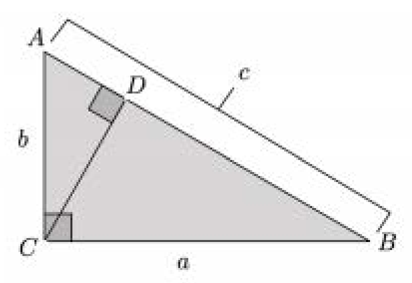
10. To how many decimal digits can you correctly estimate the value of the irrational number $(4.56789012\ldots)^2$?

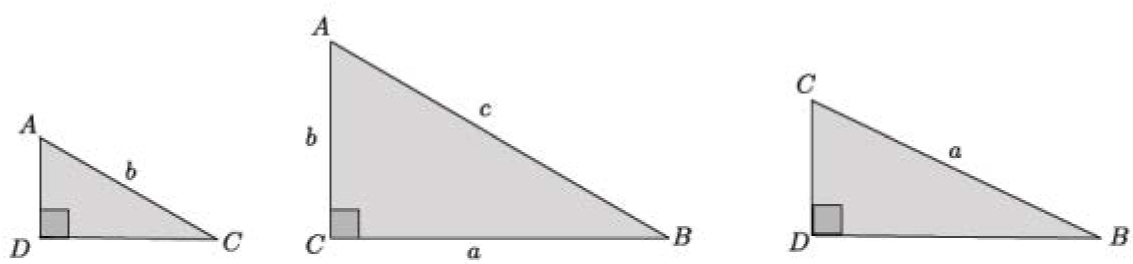
Lesson 15 – Pythagorean Theorem, Revisited

Essential Questions:

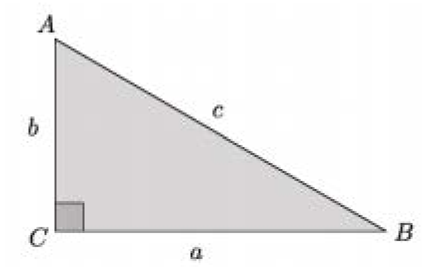
Discussion

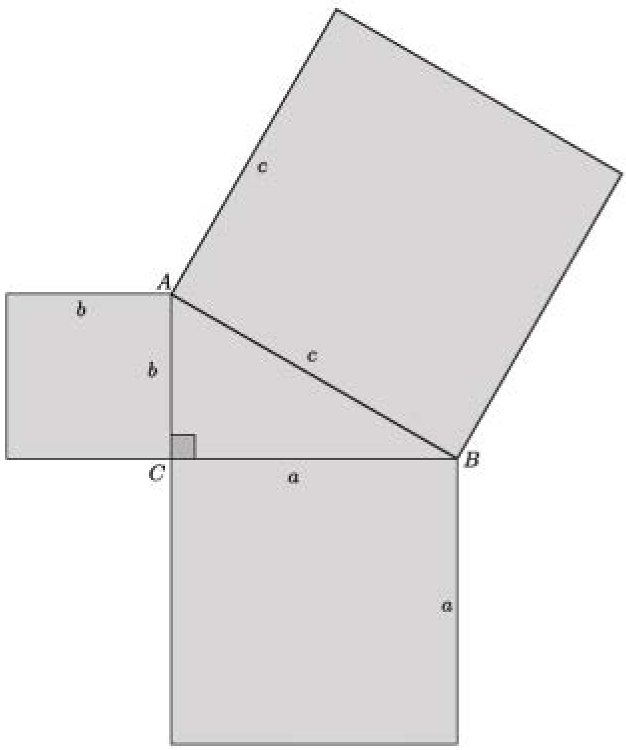






Discussion

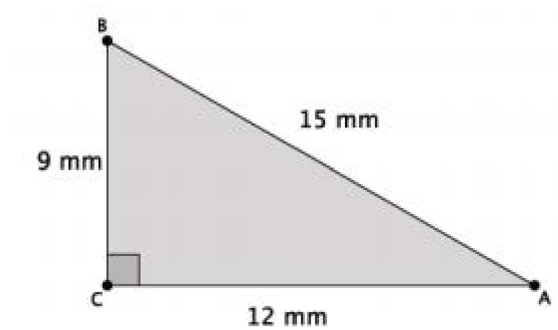




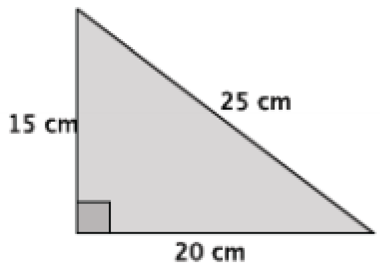
Lesson 15 Summary:

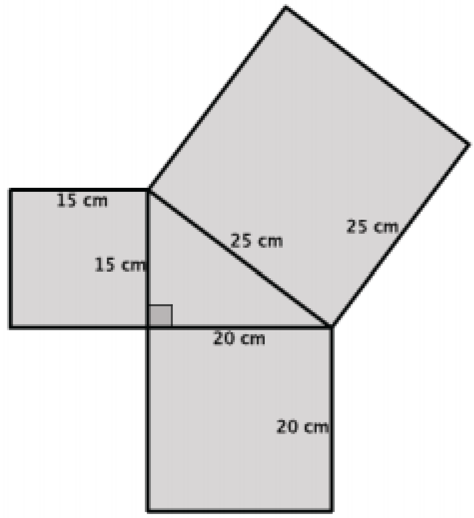
Independent Practice Lesson 15

1. For the right triangle shown below, identify and use similar triangles to illustrate the Pythagorean theorem.

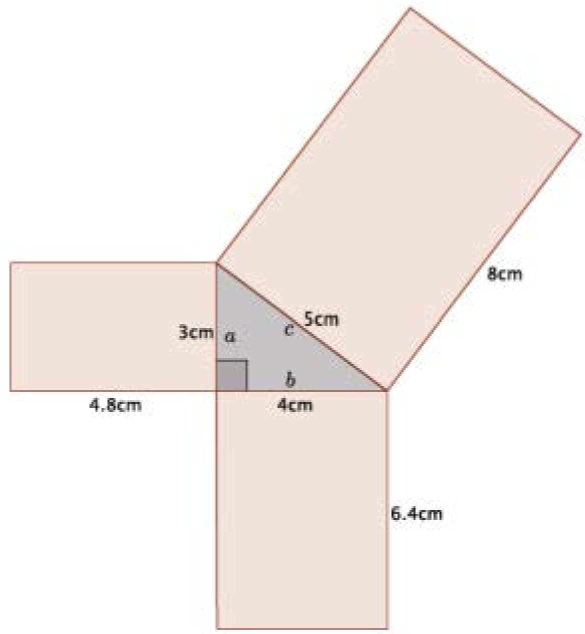


2. For the right triangle shown below, identify and use squares formed by the sides of the triangle to illustrate the Pythagorean theorem.

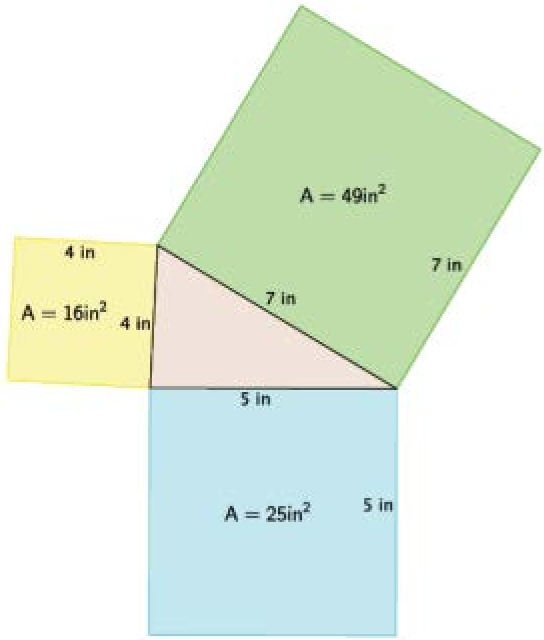




3. Reese claimed that any figure can be drawn off the sides of a right triangle and that as long as they are similar figures, then the sum of the areas off of the legs will equal the area off of the hypotenuse. She drew the diagram by constructing rectangles off of each side of a known right triangle. Is Reese's claim correct for this example? In order to prove or disprove Reese's claim, you must first show that the rectangles are similar. If they are, then you can use computations to show that the sum of the areas of the figures off of the sides a and b equals the area of the figure off of side c.



4. After learning the proof of the Pythagorean theorem using areas of squares, Joseph got really excited and tried explaining it to his younger brother. He realized during his explanation that he had done something wrong. Help Joseph find his error. Explain what he did wrong.



5. Draw a right triangle with squares constructed off of each side that Joseph can use the next time he wants to show his younger brother the proof of the Pythagorean theorem.

6. Explain the meaning of the Pythagorean theorem in your own words.

7. Draw a diagram that shows an example illustrating the Pythagorean theorem.

Lesson 16 – Converse of the Pythagorean Theorem

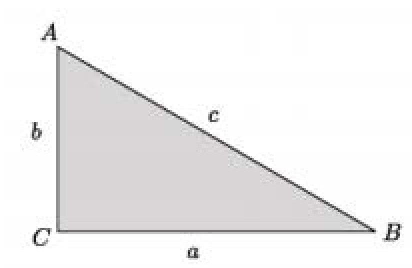
Essential Questions:

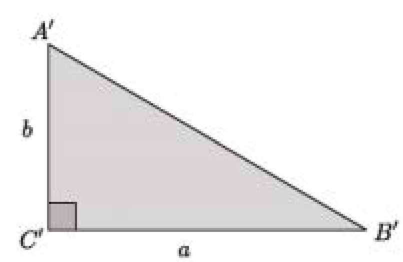
Discussion:

THEOREM: If the lengths of the legs of a right triangle are a and b, and the length of the hypotenuse is c, then $a^2 + b^2 = c^2$.

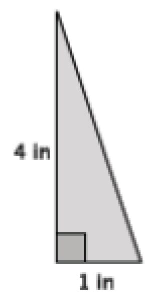
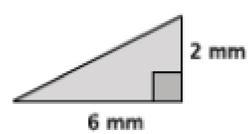
Vocabulary:

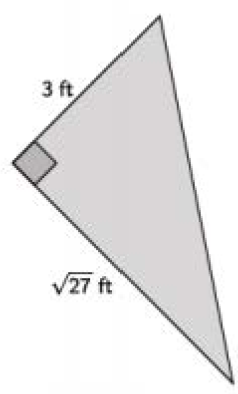
Converse:	





On Your Own:

1. Is the triangle with leg lengths of 3 mi., 8 mi., and hypotenuse of length $\sqrt{73}$ mi. a right triangle? Show your work, and answer in a complete sentence.	
2. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.	
3. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.	

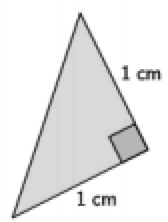
4. Is the triangle with leg lengths of 9 in., 9 in., and hypotenuse of length $\sqrt{175}$ in. a right triangle? Show your work, and answer in a complete sentence.	
5. Is the triangle with leg lengths of $\sqrt{28}$ cm, 6 cm, and hypotenuse of length 8 cm a right triangle? Show your work, and answer in a complete sentence.	
6. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence.	

7. The triangle shown below is an isosceles right triangle. Determine the length of the legs of the triangle. Show your work, and answer in a complete sentence.	

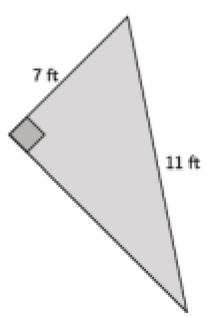
Lesson 16 Summary:

Lesson 16 Independent Practice

1. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



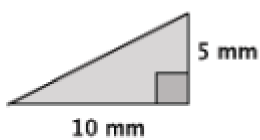
2. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



3. Is the triangle with leg lengths of $\sqrt{3}$ cm, 9 cm, and hypotenuse of length $\sqrt{84}$ cm a right triangle? Show your work, and answer in a complete sentence.

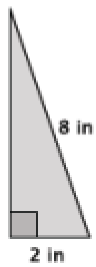
4. Is the triangle with leg lengths of $\sqrt{7}$ km, 5 km, and hypotenuse of length $\sqrt{48}$ km a right triangle? Show your work, and answer in a complete sentence.

5. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



6. Is the triangle with leg lengths of 3, 6, and hypotenuse of length $\sqrt{45}$ a right triangle? Show your work, and answer in a complete sentence.

7. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



8. Is the triangle with leg lengths of 1 and $\sqrt{3}$ and hypotenuse of length 2 a right triangle? Show your work, and answer in a complete sentence.

9. Corey found the hypotenuse of a right triangle with leg lengths of 2 and 3 to be $\sqrt{13}$. Corey claims that since $\sqrt{13}$ = 3.61 when estimating to two decimal digits, that a triangle with leg lengths of 2 and 3 and a hypotenuse of 3.61 is a right triangle. Is he correct? Explain.

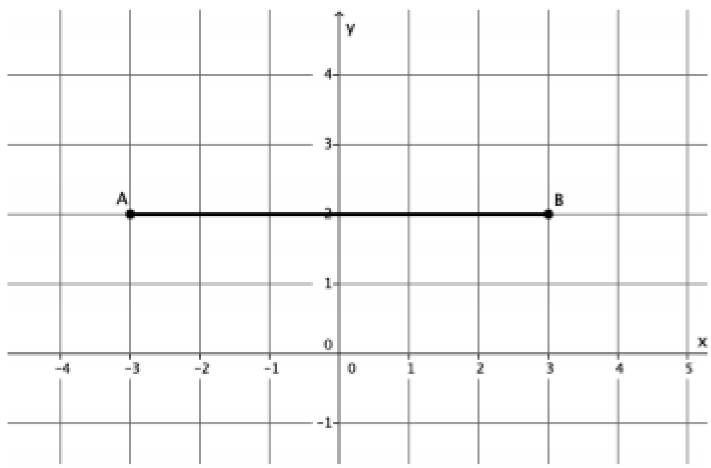
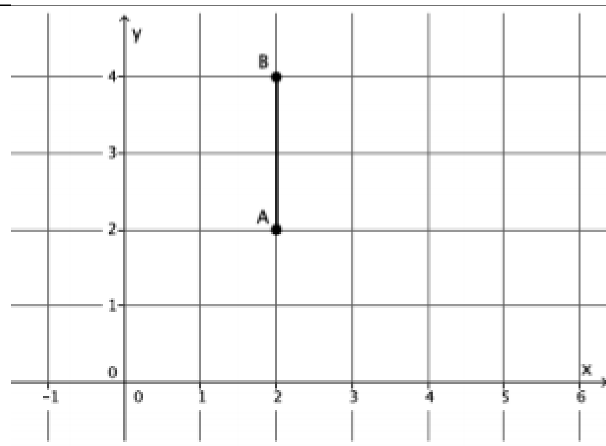
10. Explain a proof of the Pythagorean theorem.

11. Explain a proof of the converse of the Pythagorean theorem.

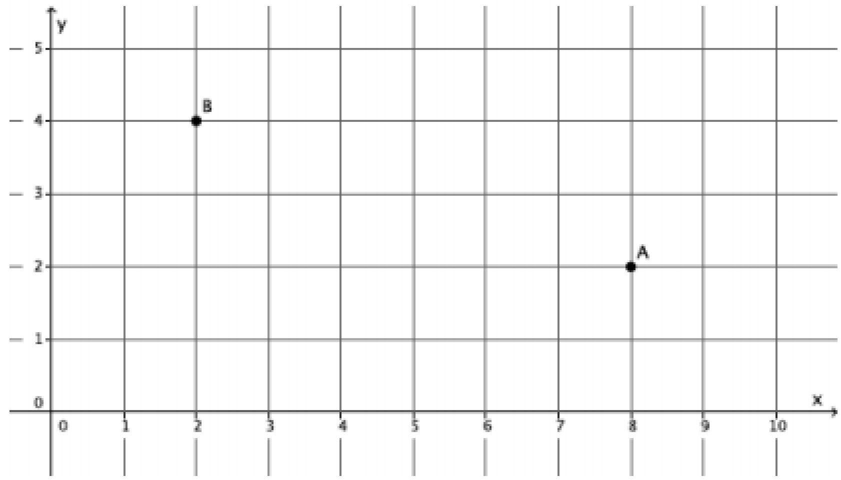
Lesson 17 – Distance on the Coordinate Plane

Essential Questions:

Classwork:

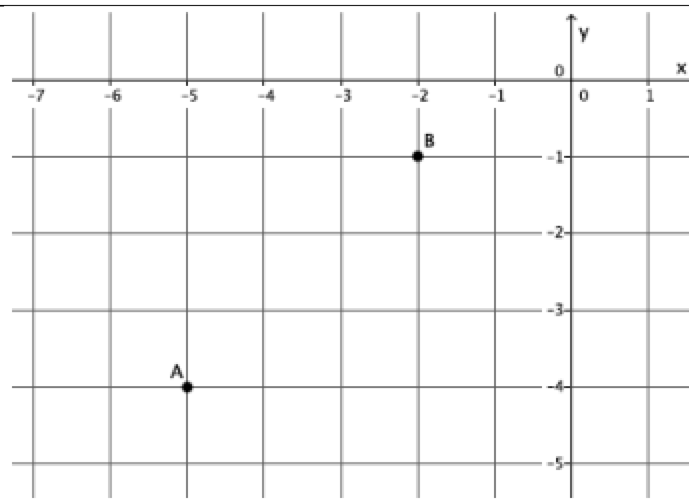
Example 1 i. What is the distance between the two points A and B on the coordinate plane?	
ii. What is the distance between the two points A and B on the coordinate plane?	

iii. What is the distance between the two points A and B on the coordinate plane? Round your answer to the tenths place.



Example 2

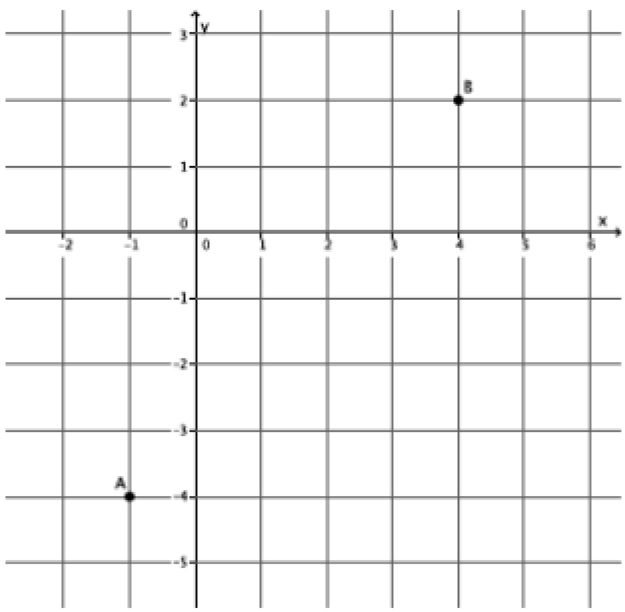
Given two points A and B on the coordinate plane, determine the distance between them. First, make an estimate; then, try to find a more precise answer. Round your answer to the tenths place.



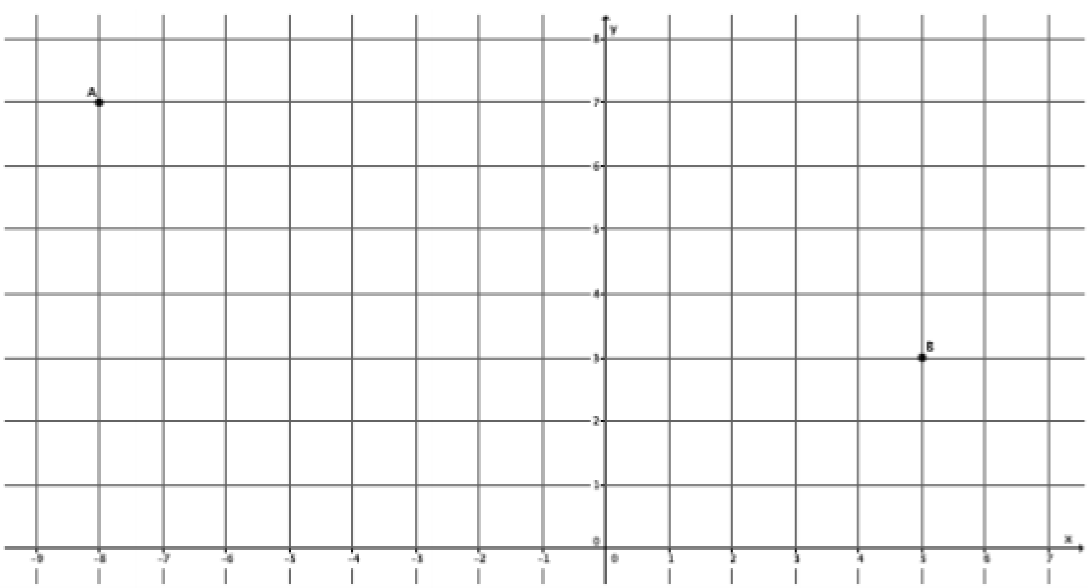
On Your Own:

For each of the Exercises 1–4, determine the distance between points A and B on the coordinate plane. Round your answer to the tenths place.

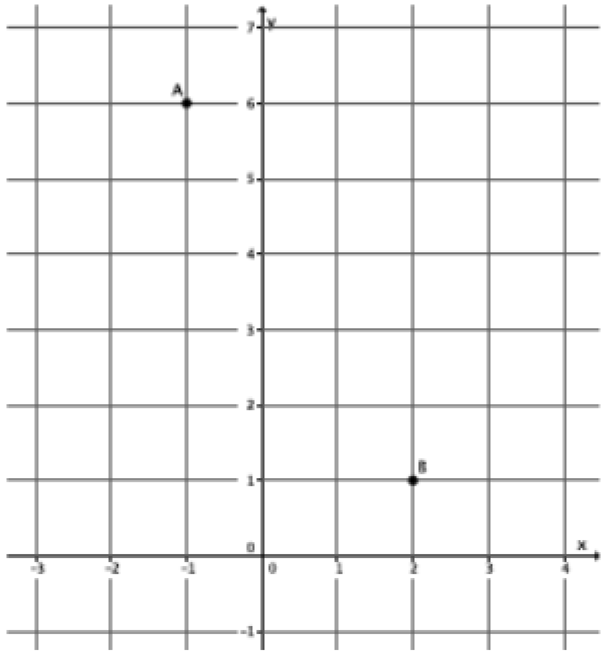
1.



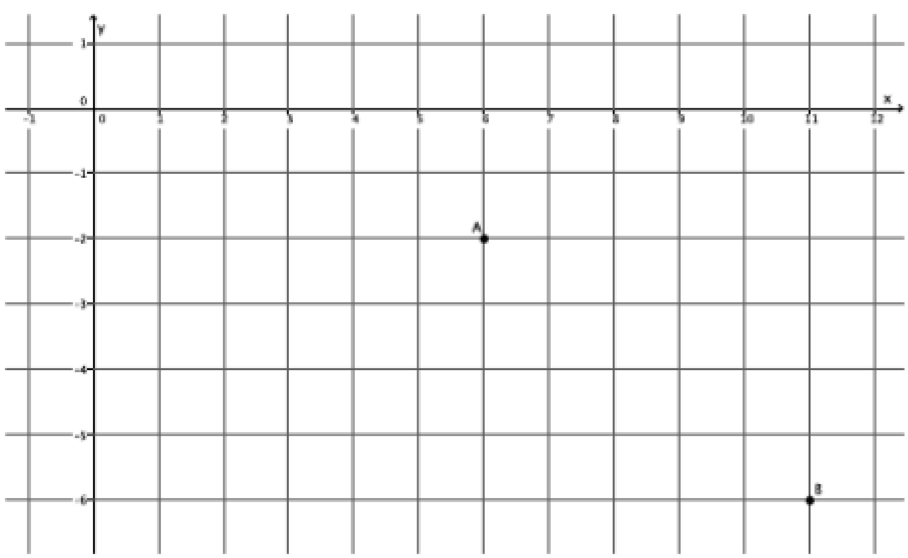
2.



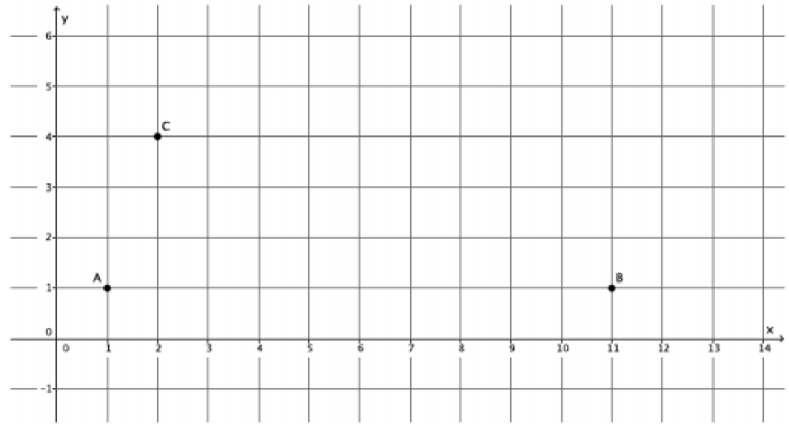
3.



4.



Example 3
Is the triangle formed by the points A,B,C a right triangle?

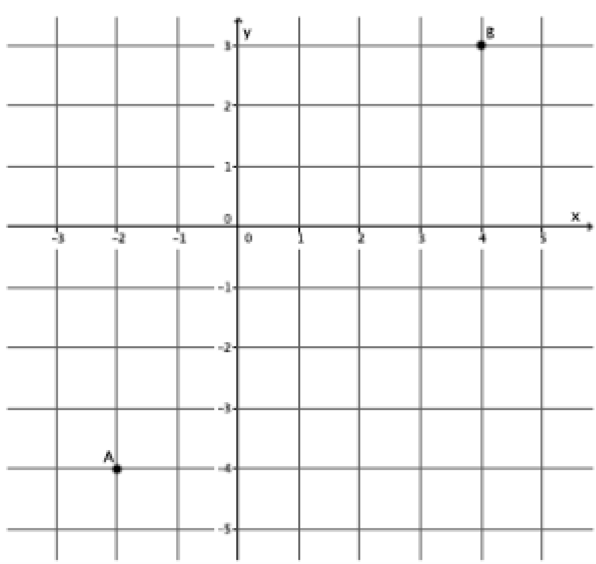


Lesson 17 Summary:

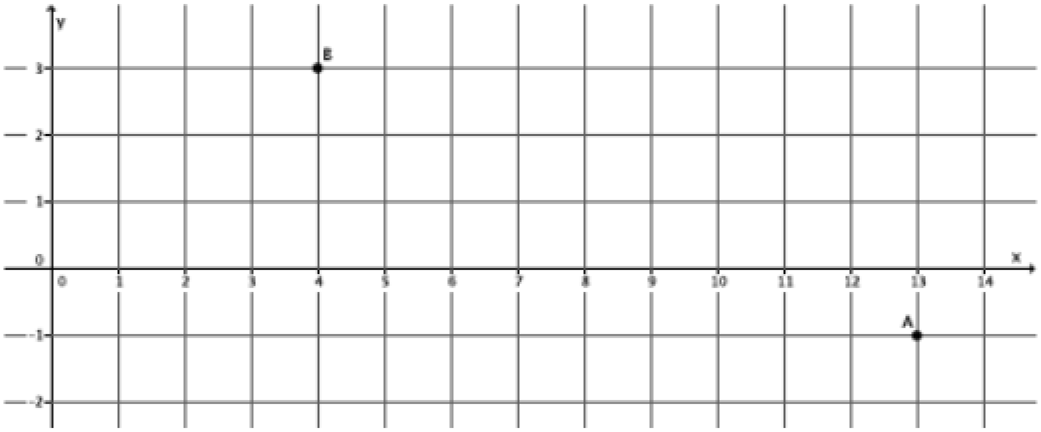
Independent Practice Lesson 17

For each of the Problems 1–4, determine the distance between points A and B on the coordinate plane. Round your answer to the tenths place.

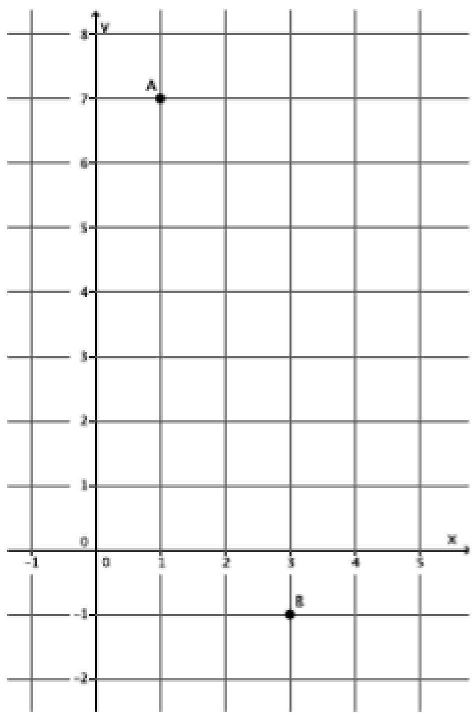
1.



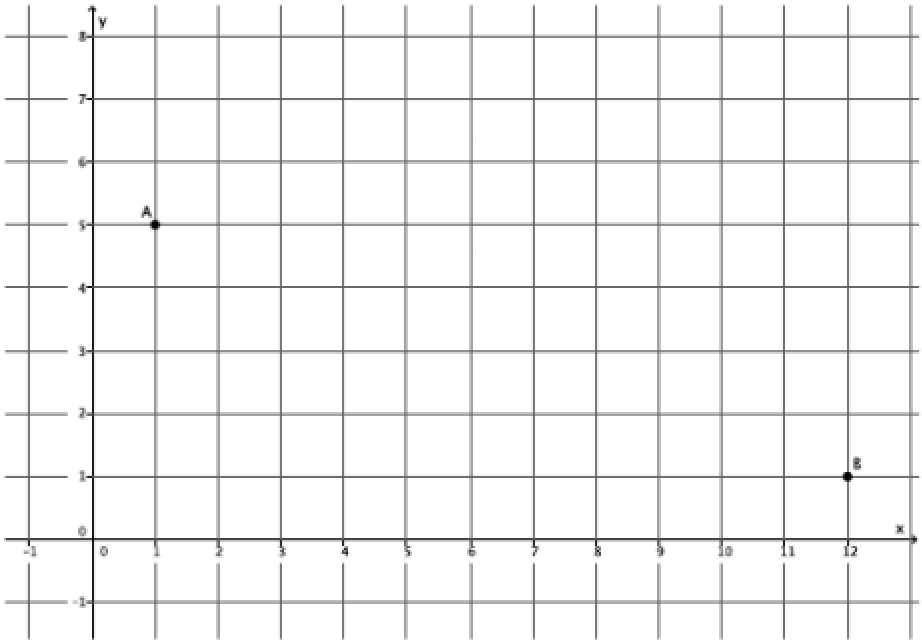
2.



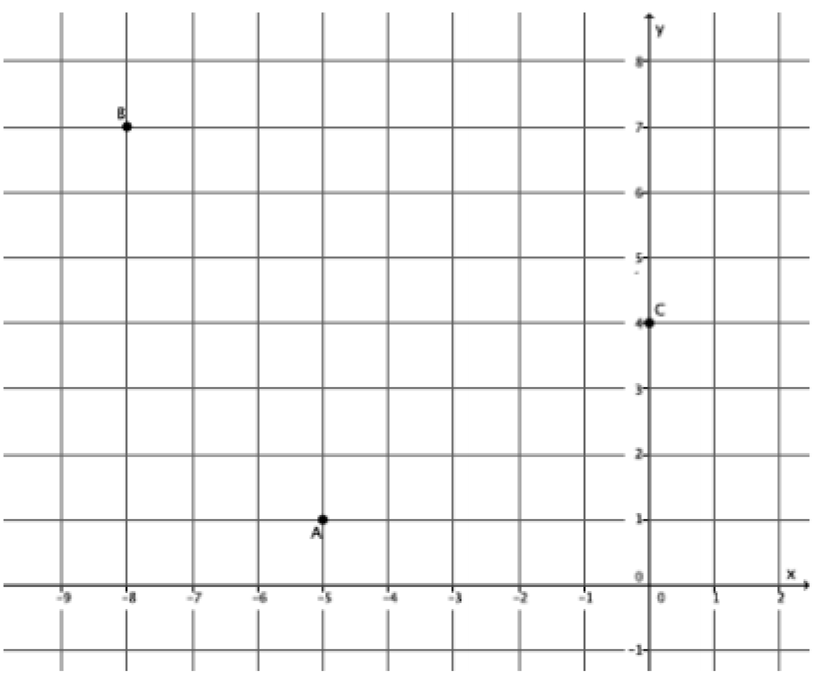
3.



4.



5. Is the triangle formed by points A, B, C a right triangle?

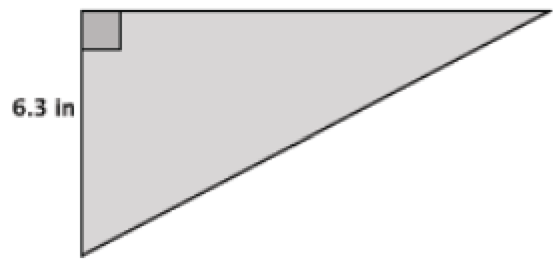


Lesson 18 – Applications of the Pythagorean Theorem

Essential Questions:

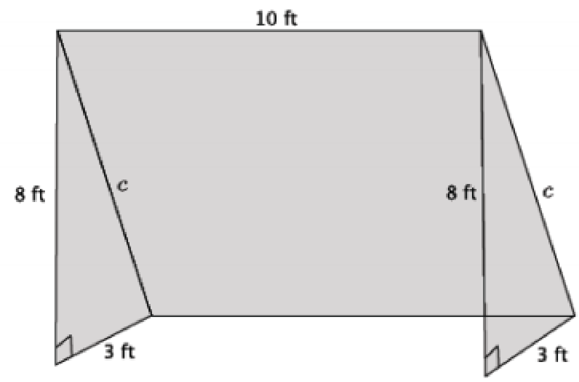
On Your Own:

1. The area of the right triangle shown is 26.46 in^2. What is the perimeter of the right triangle? Round your answer to the tenths place.

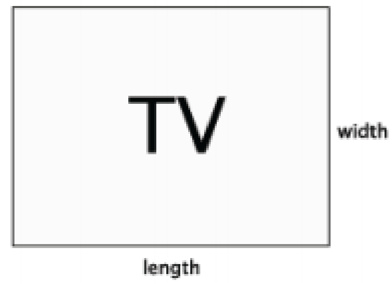


2. The diagram is a representation of a soccer goal.

a. Determine the length of the bar, , that would be needed to provide structure to the goal. Round your answer to the tenths place.



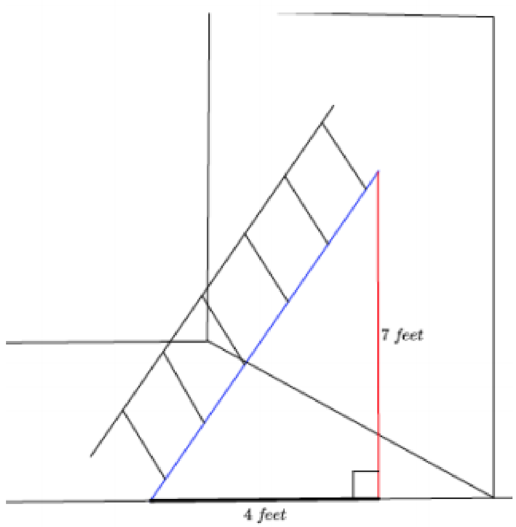
b. How much netting (in square feet) is needed to cover the entire goal?

3. The typical ratio of length to width that is used to produce televisions is 4: 3.	
a. A TV with those exact measurements would be quite small, so generally the size of the television is enlarged by multiplying each number in the ratio by some factor of xx. For example, a reasonably sized television might have dimensions of 4 × 5: 3 × 5, where the original ratio 4: 3 was enlarged by a scale factor of 5. The size of a television is described in inches, such as a 60" TV, for example. That measurement actually refers to the diagonal length of the TV (distance from an upper corner to the opposite lower corner). What measurement would be applied to a television that was produced using the ratio of 4 × 5: 3 × 5?	
b. A 42" TV was just given to your family. What are the length and width measurements of the TV?	
c. Check that the dimensions you got in part (b) are correct using the Pythagorean theorem.	

d. The table that your TV currently rests on is 30" in length. Will the new TV fit on the table? Explain.	
4. Determine the distance between the following pairs of points. Round your answer to the tenths place. Use graph paper if necessary.	
a. (7, 4) and (–3, –2)	
b. (–5, 2) and (3, 6)	

c. Challenge: (x_1, y_1) and (x_2, y_2). Explain your answer.

5. What length of ladder will be needed to reach a height of 7 feet along the wall when the base of the ladder is 4 feet from the wall? Round your answer to the tenths place.



Discussion

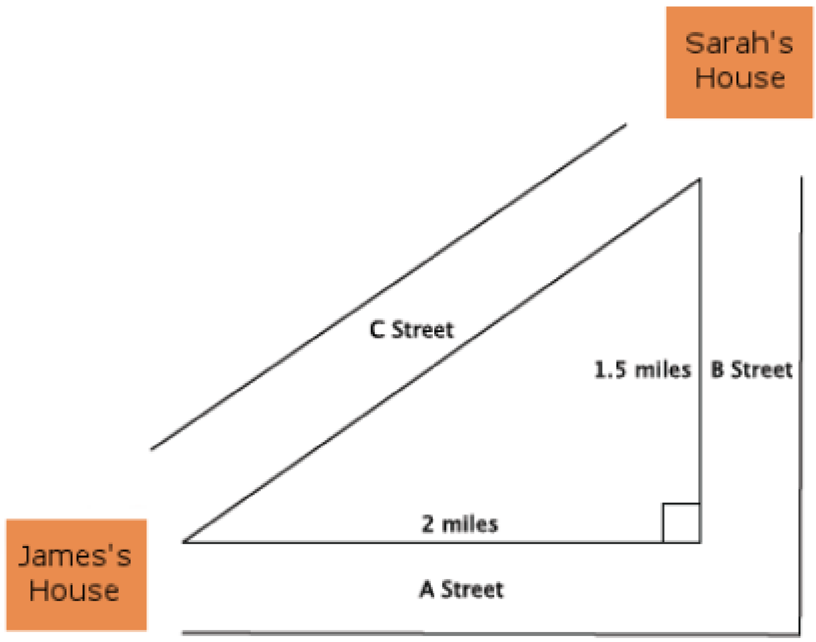
Can we use Pythagorean Theorem in 3-D? How?

Lesson 18 Summary:

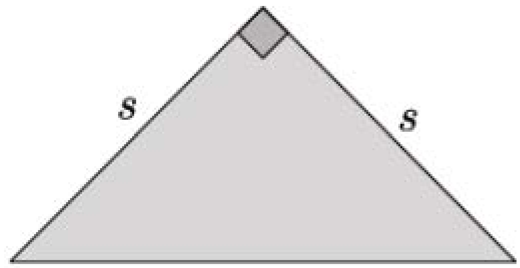
Independent Practice Lesson 18

1. A 70" TV is advertised on sale at a local store. What are the length and width of the television?

2. There are two paths that one can use to go from Sarah's house to James's house. One way is to take C Street, and the other way requires you to use A Street and B Street. How much shorter is the direct path along C Street?



3. An isosceles right triangle refers to a right triangle with equal leg lengths, s, as shown below.

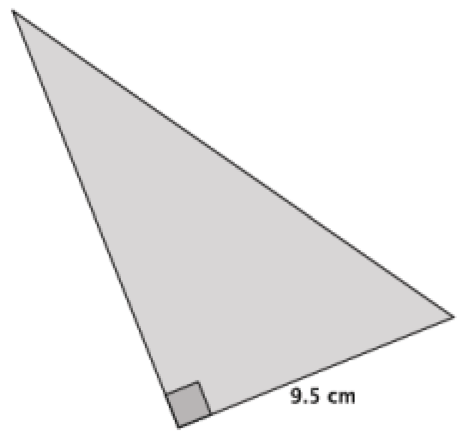


What is the length of the hypotenuse of an isosceles right triangle with a leg length of 9 cm? Write an exact answer using a square root and an approximate answer rounded to the tenths place.

4. The area of the right triangle shown is 66.5 cm^2.

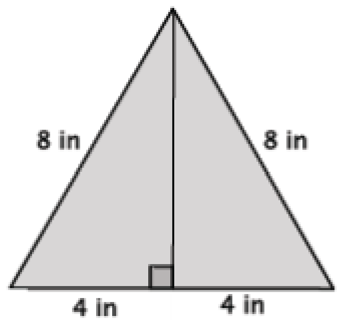
a. What is the height of the triangle?

b. What is the perimeter of the right triangle? Round your answer to the tenths place.



5. What is the distance between points (1, 9) and (−4, −1)? Round your answer to the tenths place.

6. An equilateral triangle is shown below. Determine the area of the triangle. Round your answer to the tenths place.

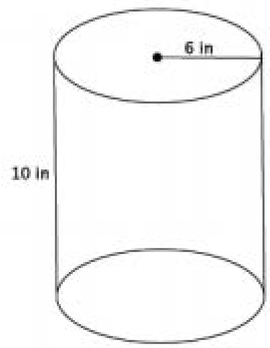
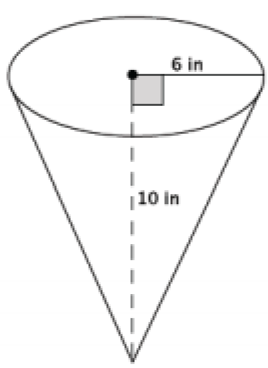


Lesson 19 – Cones and Spheres

Essential Questions:

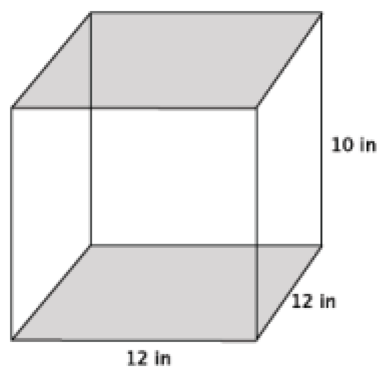
On Your Own:

Note: Figures not drawn to scale.

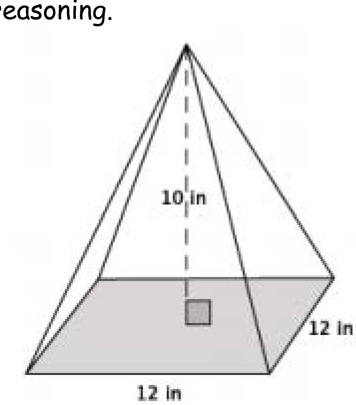
1. Determine the volume for each figure. a. Write an expression that shows volume in terms of the area of the base, B, and the height of the figure. Explain the meaning of the expression, and then use it to determine the volume of the figure.  	
b. Write an expression that shows volume in terms of the area of the base, B, and the height of the figure. Explain the meaning of the expression, and then use it to determine the volume of the figure.  	

2.

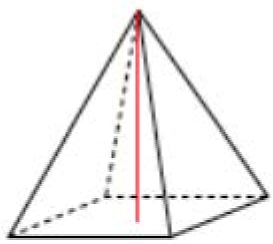
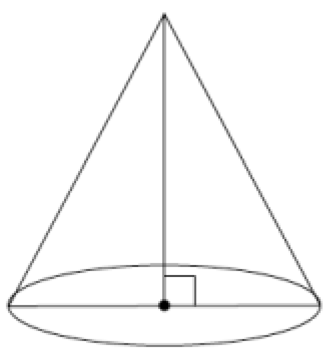
a. Write an expression that shows volume in terms of the area of the base, B, and the height of the figure. Explain the meaning of the expression, and then use it to determine the volume of the figure.



b. The volume of the pyramid shown below is 480 in^3. What do you think the formula to find the volume of a pyramid is? Explain your reasoning.

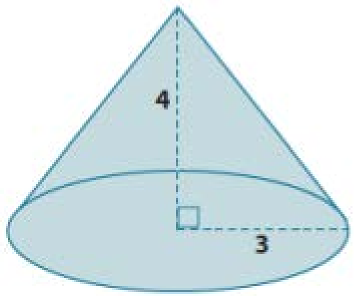


Discussion:

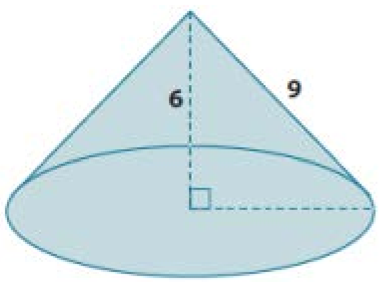
What do you think the formula to find the volume of a pyramid is? Explain. 	
Example 1 State as many facts as you can about a cone. 	

On Your Own:

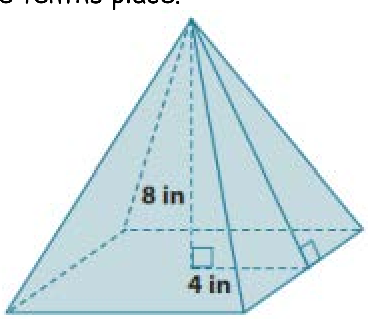
3. What is the lateral length of the cone shown?



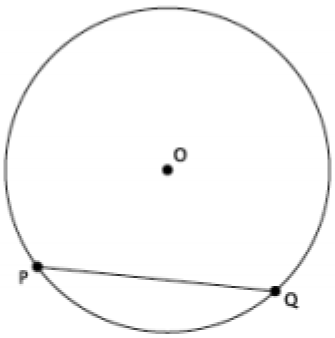
4. Determine the exact volume of the cone shown.



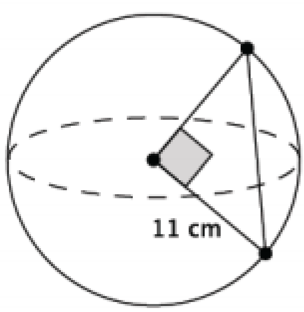
5. What is the lateral length (slant height) of the pyramid shown? Give an exact square root answer and an approximate answer rounded to the tenths place.



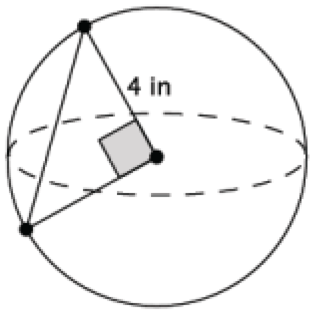
Discussion:

Let O be the center of a circle, and let P and be two points on the circle as shown. Then PQ is called a chord of the circle. 	

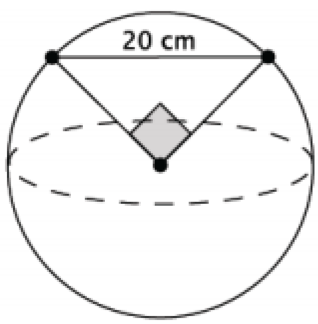
On Your Own:

7. What is the length of the chord of the sphere shown below? Give an exact answer using a square root. 	

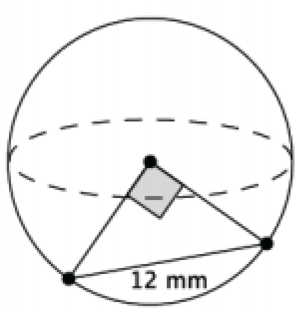
8. What is the length of the chord of the sphere shown below? Give an exact answer using a square root.



9. What is the volume of the sphere shown below? Give an exact answer using a square root.



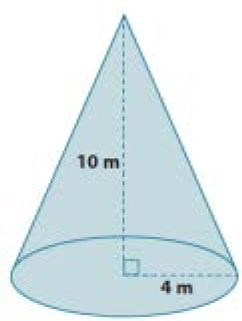
10. What is the volume of the sphere shown below? Give an exact answer using a square root.



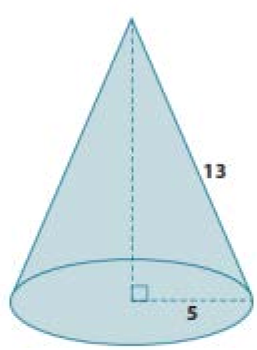
Lesson 19 Summary:

Independent Practice Lesson 19

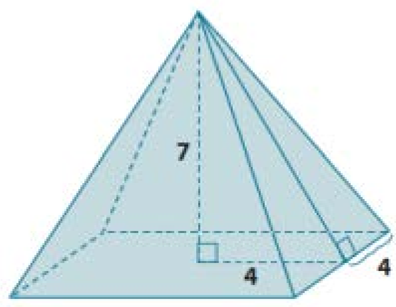
1. What is the lateral length of the cone shown below? Give an approximate answer rounded to the tenths place.



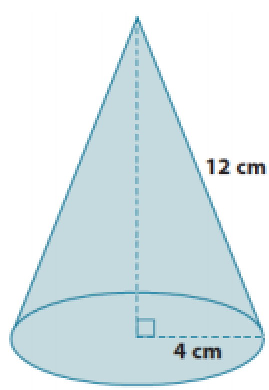
2. What is the volume of the cone shown below? Give an exact answer.



3. Determine the volume and surface area of the pyramid shown below. Give exact answers.

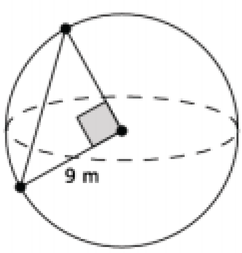


4. Alejandra computed the volume of the cone shown below as 64π cm^2. Her work is shown below. Is she correct? If not, explain what she did wrong, and calculate the correct volume of the cone. Give an exact answer.

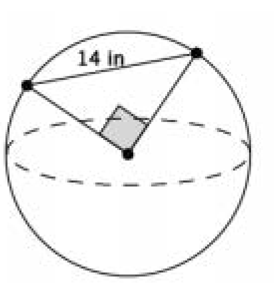


$$V = \frac{1}{3}\pi(4^2)(12)$$
$$= \frac{16(12)\pi}{3}$$
$$= 64\pi$$
$$= 64\pi \text{ cm}^3$$

5. What is the length of the chord of the sphere shown below? Give an exact answer using a square root.



6. What is the volume of the sphere shown below? Give an exact answer using a square root.



Lesson 20 – Truncated Cones

Essential Questions:

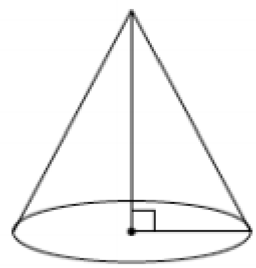
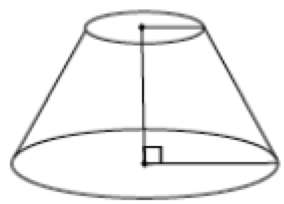
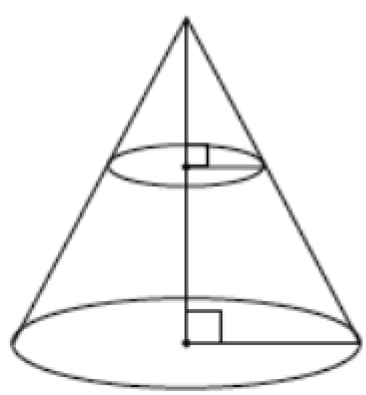
On Your Own:

1. Examine the bucket below. It has a height of 9 inches and a radius at the top of the bucket of 4 inches.

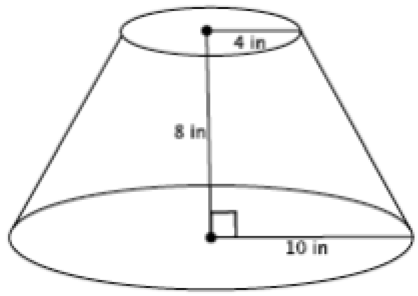
a. Describe the shape of the bucket. What is it similar to?

b. Estimate the volume of the bucket.

Discussion:

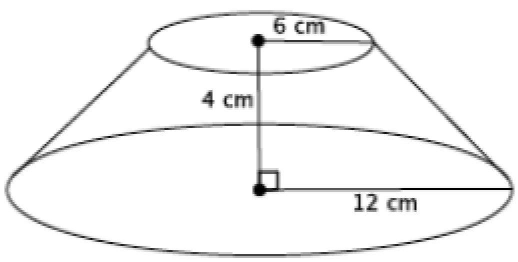
Cone:	
Truncated Cone:	
	

Example 1
Determine the volume of the truncated cone shown below.



On Your Own:

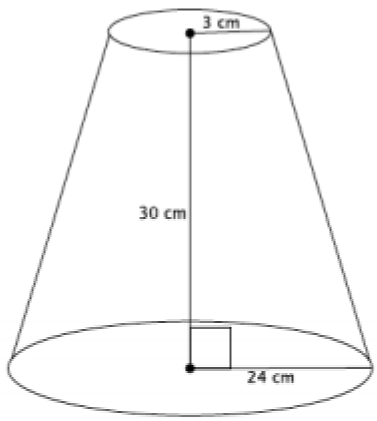
2. Find the volume of the truncated cone.



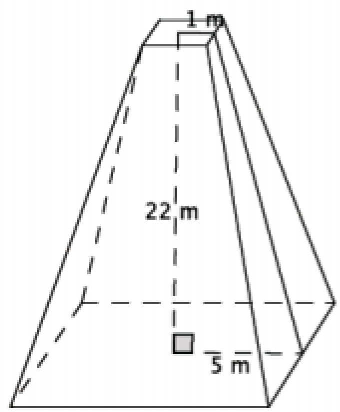
a. Write a proportion that will allow you to determine the height of the cone that has been removed. Explain what all parts of the proportion represent.

b. Solve your proportion to determine the height of the cone that has been removed.	
c. Write an expression that can be used to determine the volume of the truncated cone. Explain what each part of the expression represents.	
d. Calculate the volume of the truncated cone.	

3. Find the volume of the truncated cone.



4. Find the volume of the truncated pyramid with a square base.



a. Write a proportion that will allow you to determine the height of the cone that has been removed. Explain what all parts of the proportion represent.

b. Solve your proportion to determine the height of the pyramid that has been removed.

c. Write an expression that can be used to determine the volume of the truncated pyramid. Explain what each part of the expression represents.

d. Calculate the volume of the truncated pyramid.

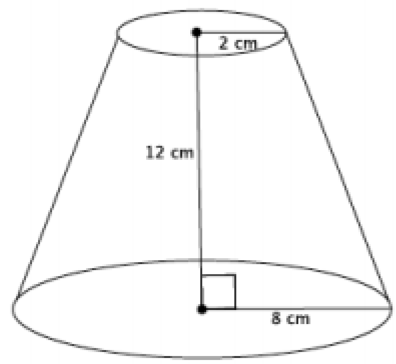
5. A pastry bag is a tool used to decorate cakes and cupcakes. Pastry bags take the form of a truncated cone when filled with icing. What is the volume of a pastry bag with a height of 6 inches, large radius of 2 inches, and small radius of 0.5 inches?

6. Explain in your own words what a truncated cone is and how to determine its volume.

Lesson 20 Summary:

Independent Practice Lesson 20

1. Find the volume of the truncated cone.



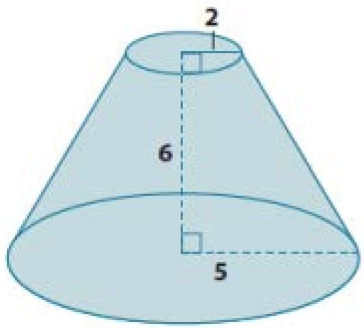
a. Write a proportion that will allow you to determine the height of the cone that has been removed. Explain what each part of the proportion represents.

b. Solve your proportion to determine the height of the cone that has been removed.

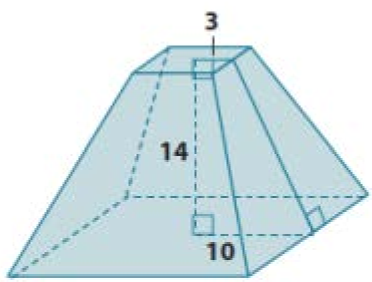
c. Show a fact about the volume of the truncated cone using an expression. Explain what each part of the expression represents.

d. Calculate the volume of the truncated cone.

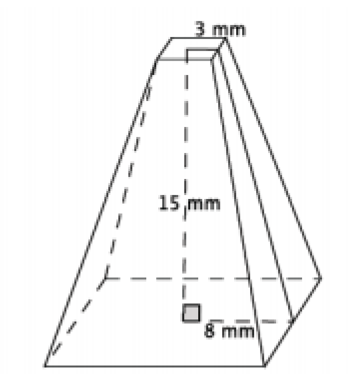
2. Find the volume of the truncated cone.



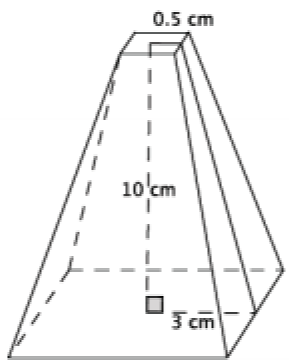
3. Find the volume of the truncated pyramid with a square base.



4. Find the volume of the truncated pyramid with a square base. Note: 3 mm is the distance from the center to the edge of the square at the top of the figure.

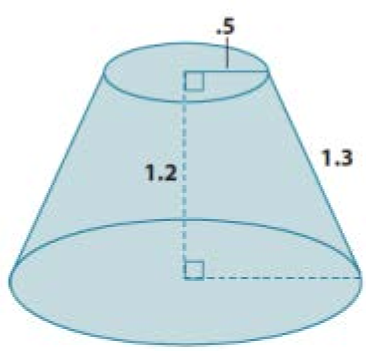


5. Find the volume of the truncated pyramid with a square base. Note: 0.5 cm is the distance from the center to the edge of the square at the top of the figure.



6. Explain how to find the volume of a truncated cone.

7. Challenge: Find the volume of the truncated cone.



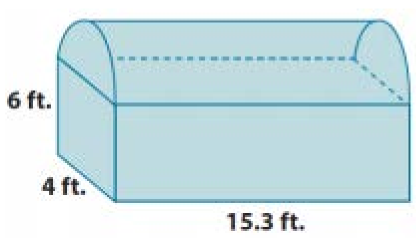
Lesson 21 – Volume of Composite Solids

Essential Questions:

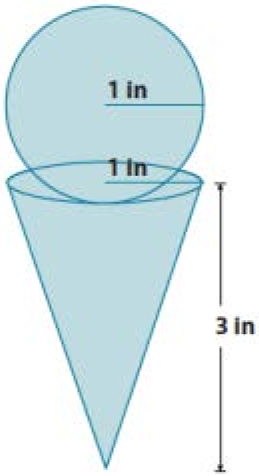
On Your Own:

1.

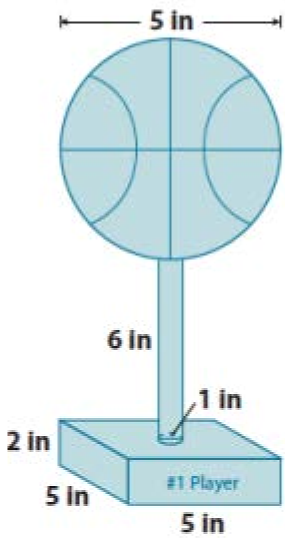
a. Write an expression that can be used to find the volume of the chest shown. Explain what each part of your expression represents.



b. What is the approximate volume of the chest shown above? Use 3.14 for an approximation of π. Round your final answer to the tenths place.

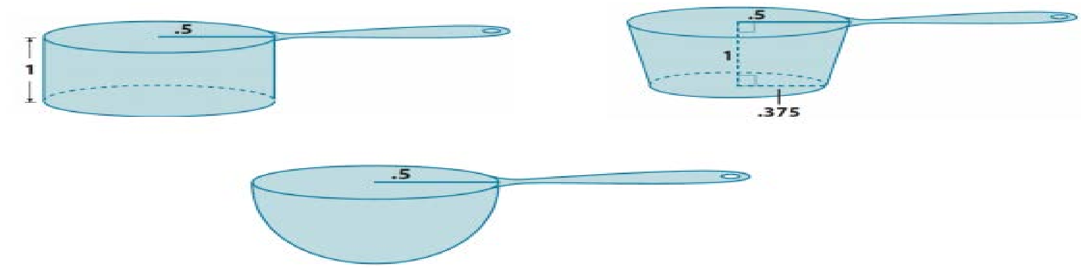
2. a. Write an expression for finding the volume of the figure shown. Explain what each part of your expression represents.  	
b. Assuming every part of the cone can be filled with ice cream, what is the exact and approximate volume of the cone and scoop? (Recall that exact answers are left in terms of $\pi\pi$ and approximate answers use 3.14 for π). Round your approximate answer to the hundredths place.	

3. a. Write an expression for finding the volume of the figure shown. Explain what each part of your expression represents.



b. Every part of the trophy shown is made out of silver. How much silver is used to produce one trophy? Give an exact and approximate answer rounded to the hundredths place.

4. Use the diagram of scoops to answer parts (a) and (b).



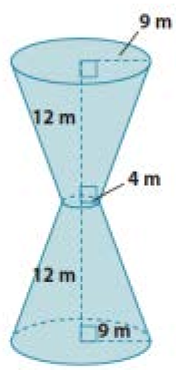
a. Order the scoops from least to greatest in terms of their volumes. Each scoop is measured in inches.

b. How many of each scoop would be needed to add a half-cup of sugar to a cupcake mixture? (One-half cup is approximately 7 in^3.) Round your answer to a whole number of scoops.

Lesson 21 Summary:

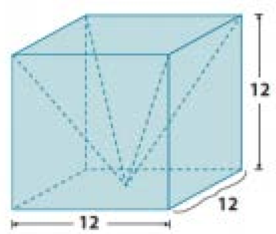
Independent Practice Lesson 21

1. What volume of sand would be required to completely fill up the hourglass shown below? Note: 12 m is the height of the truncated cone, not the lateral length of the cone.



2.

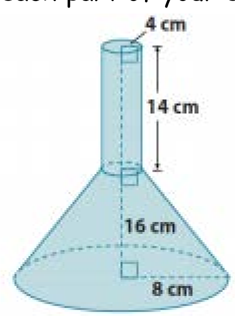
a. Write an expression that can be used to find the volume of the prism with the pyramid portion removed. Explain what each part of your expression represents.



b. What is the volume of the prism shown above with the pyramid portion removed?

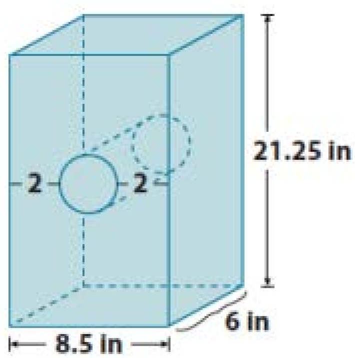
3.

a. Write an expression that can be used to find the volume of the funnel shown. Explain what each part of your expression represents.

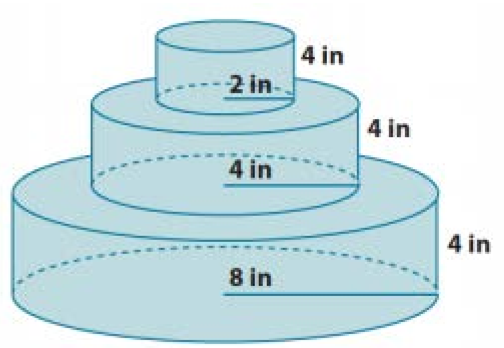


b. Determine the exact volume of the funnel.

4. What is the approximate volume of the rectangular prism with a cylindrical hole shown? Use 3.14 for π. Round your answer to the tenths place.



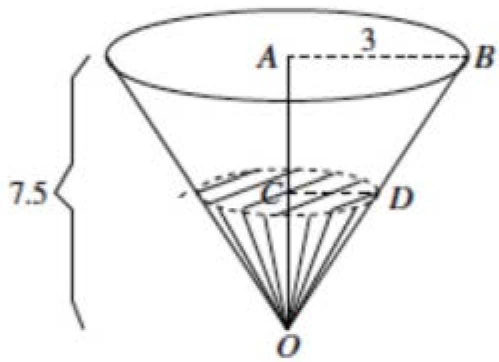
5. A layered cake is being made to celebrate the end of the school year. What is the exact total volume of the cake shown below?

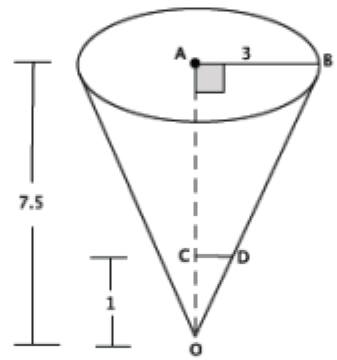


Lesson 22 – Average Rate of Change

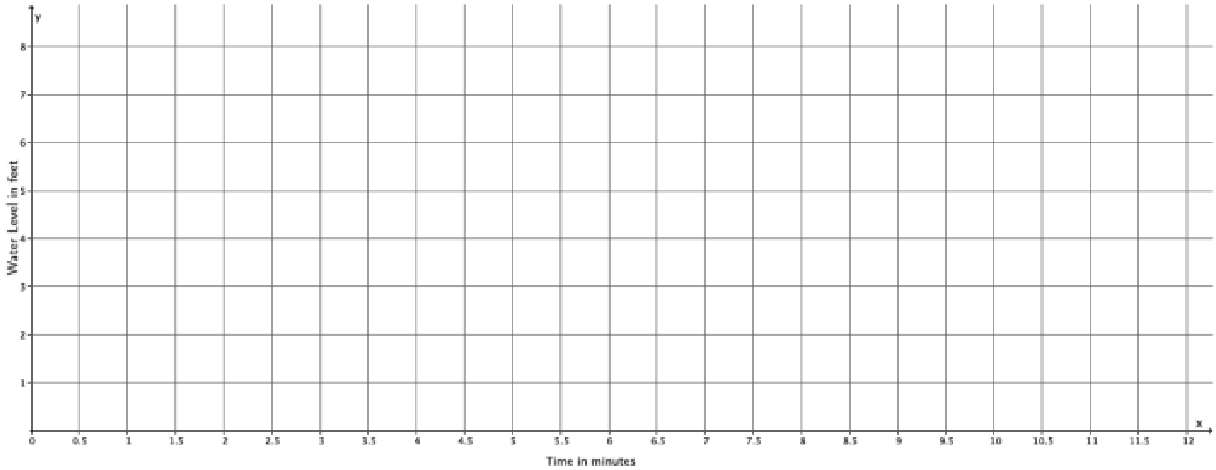
Essential Questions:

On Your Own:

The height of a container in the shape of a circular cone is 7.5 ft., and the radius of its base is 3 ft., as shown. What is the total volume of the cone? 	



Time (in minutes)	Water Level (in feet)
	1
	2
	3
	4
	5
	6
	7
	7.5

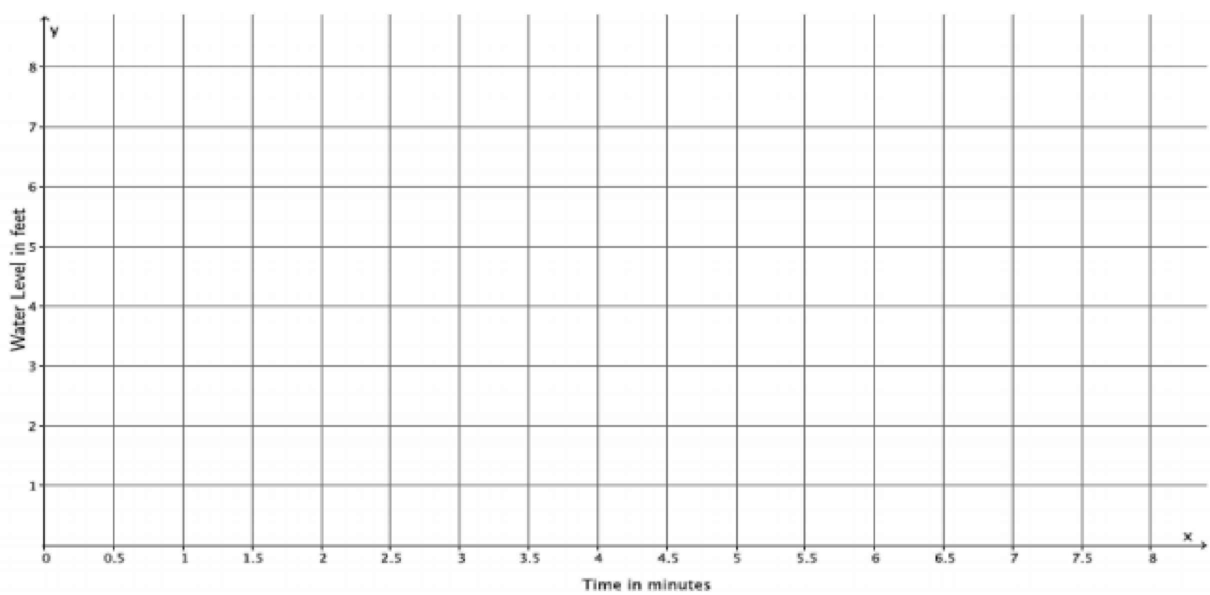


Lesson 22 Summary:

Independent Practice Lesson 22

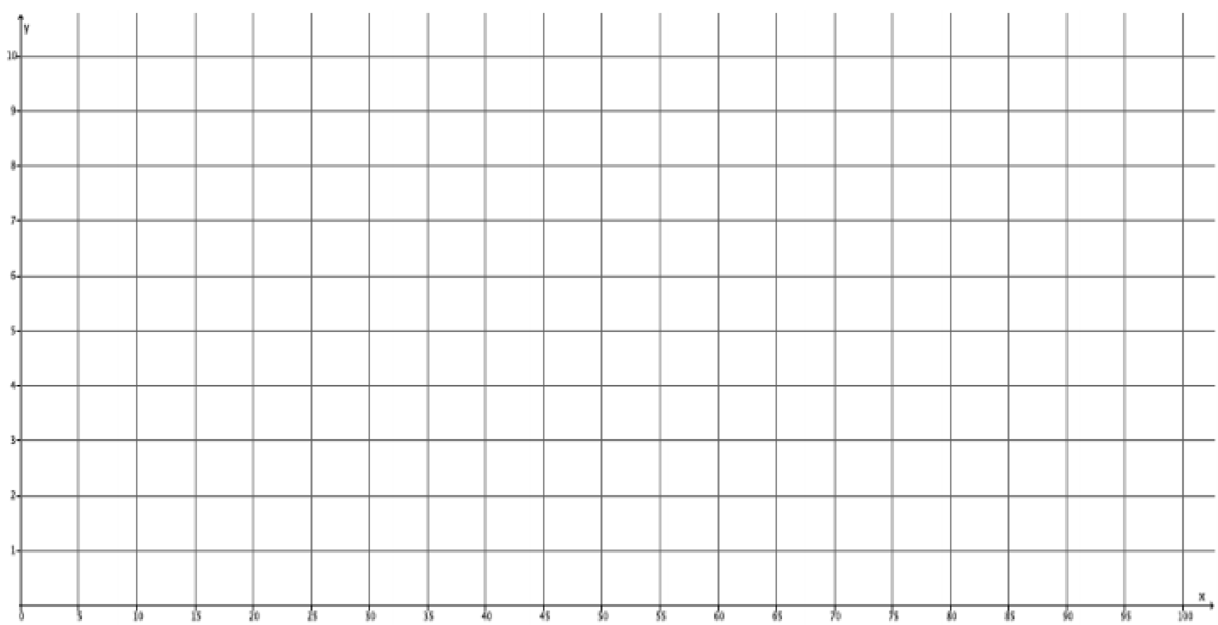
1. Complete the table below for more intervals of water levels of the cone discussed in class. Then, graph the data on a coordinate plane.

Time (in minutes)	Water Level (in feet)
	1
	1.5
	2
	2.5
	3
	3.5
	4
	4.5
	5
	5.5
	6
	6.5
	7
	7.5

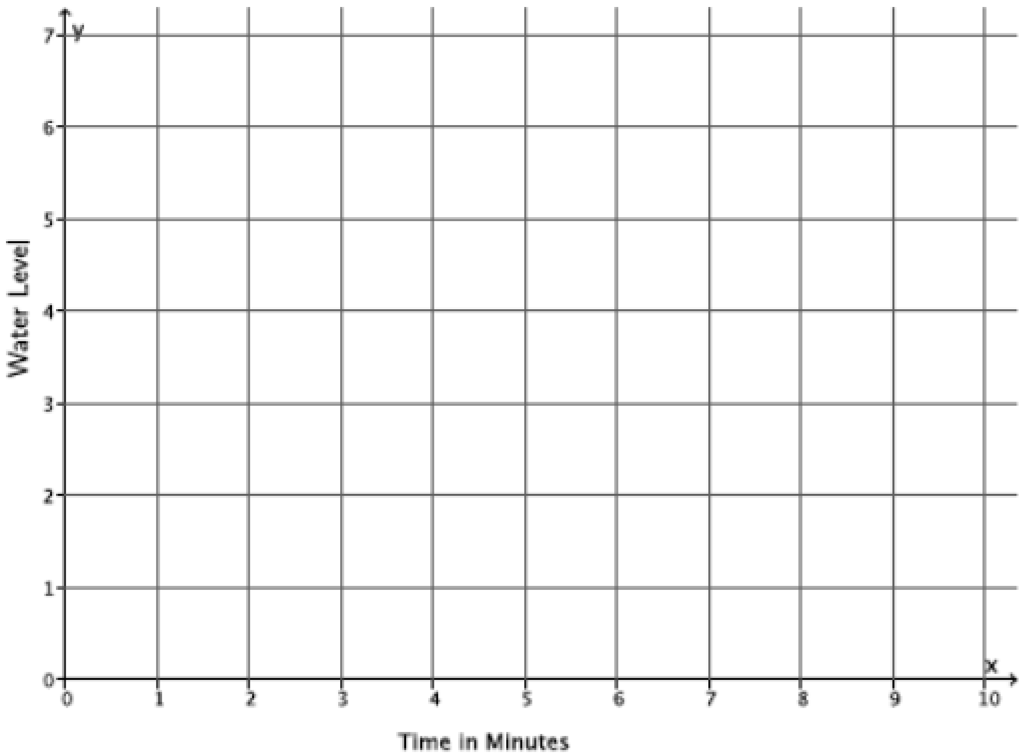


2. Complete the table below, and graph the data on a coordinate plane. Compare the graphs from Problems 1 and 2. What do you notice? If you could write a rule to describe the function of the rate of change of the water level of the cone, what might the rule include?

x	$\sqrt{x}$
1	
4	
9	
16	
25	
36	
49	
64	
81	
100	



3. Describe, intuitively, the rate of change of the water level if the container being filled were a cylinder. Would we get the same results as with the cone? Why or why not? Sketch a graph of what filling the cylinder might look like, and explain how the graph relates to your answer.



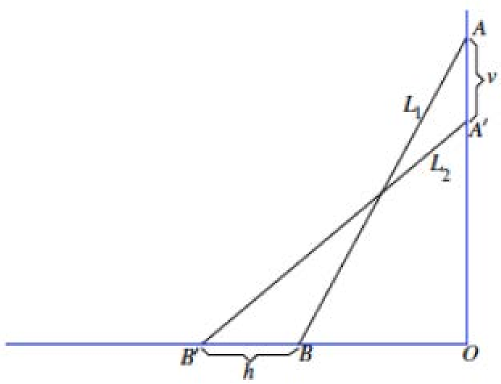
4. Describe, intuitively, the rate of change if the container being filled were a sphere. Would we get the same results as with the cone? Why or why not?

Lesson 23 – Nonlinear Motion

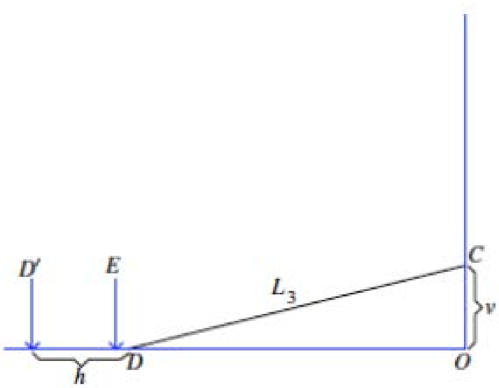
Essential Questions:

Classwork:

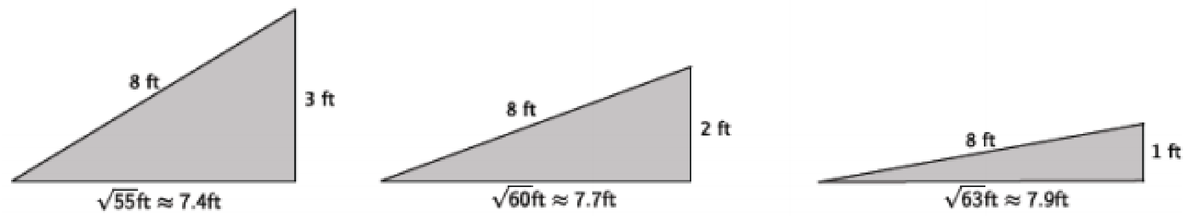
A ladder of length L ft. leaning against a wall is sliding down. The ladder starts off being flush with (right up against) the wall. The top of the ladder slides down the vertical wall at a constant speed of v ft. per second. Let the ladder in the position L_1 slide down to position L_2 after 1 second, as shown below.

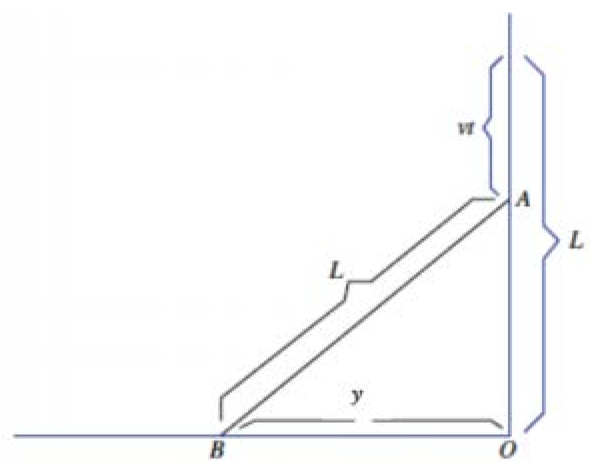


Will the bottom of the ladder move at a constant rate away from point O?

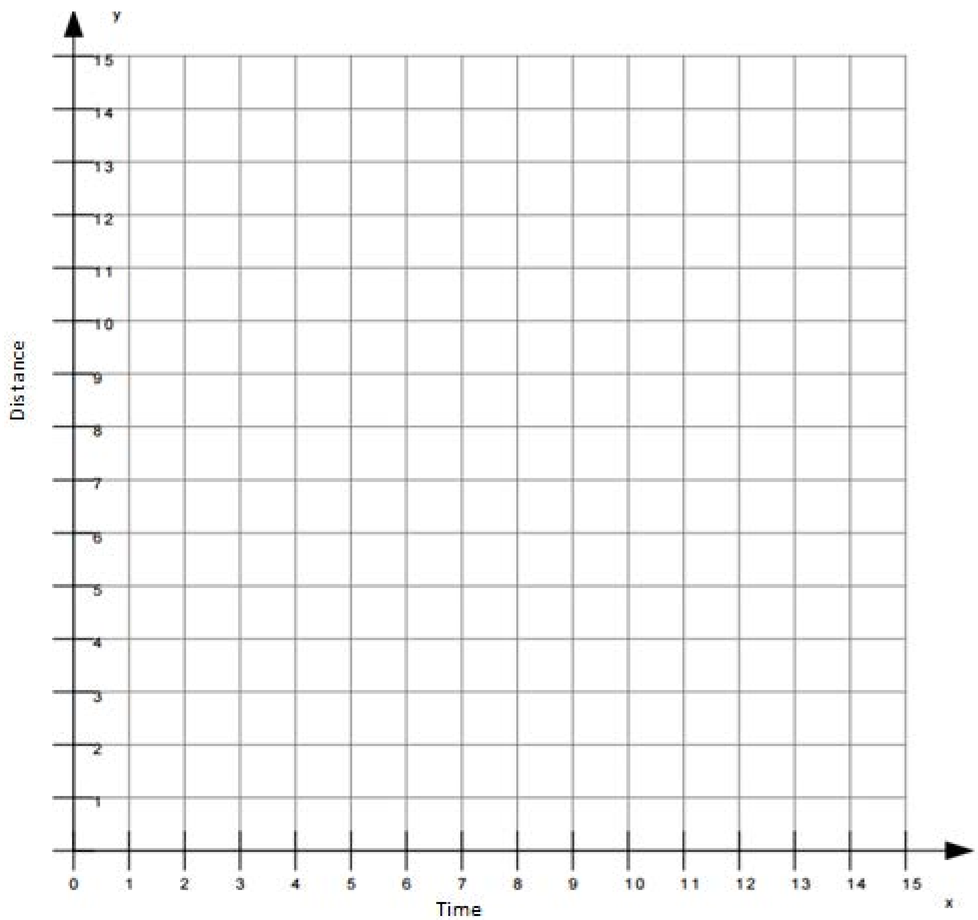


Consider the three right triangles shown below, specifically the change in the length of the base as the height decreases in increments of 1 ft.





Input t	Output $y = \sqrt{t(30 - t)}$
0	
1	
3	
4	
7	
8	
14	
15	



Lesson 23 Summary:

Independent Practice Lesson 23

1. Suppose the ladder is 10 feet long, and the top of the ladder is sliding down the wall at a rate of 0.8 ft. per second. Compute the average rate of change in the position of the bottom of the ladder over the intervals of time from 0 to 0.5 seconds, 3 to 3.5 seconds, 7 to 7.5 seconds, 9.5 to 10 seconds, and 12 to 12.5 seconds. How do you interpret these numbers?

Input t	Output $y = \sqrt{0.8t(20 - 0.8t)}$
0	
0.5	
3	
3.5	
7	
7.5	
9.5	
10	
12	
12.5	

2. Will any length of ladder, L, and any constant speed of sliding of the top of the ladder v ft. per second, ever produce a constant rate of change in the position of the bottom of the ladder? Explain.