Lesson 1 – Writing Equations Using Symbols

Essential Questions:

Discussion:

The number 1,157 is the sum of the squares of two consecutive odd integers divided by the difference between the two consecutive odd integers. Which do you prefer, the word description or the numerical description? Why?	$1{,}157 = \frac{x^2+(x+2)^2}{(x+2)-x}$

Example 1:

We want to express the following statement using symbolic language: A whole number has the property that when the square of half the number is subtracted from five times the number, we get the number itself.

Example 2:

We want to express the following statement using symbolic language:

Paulo has a certain amount of money. If he spends $6.00, then he has $\frac{1}{4}$ of the original amount left.

Example 3:

We want to write the following statement using symbolic language:

When a fraction of 57 is taken away from 57, what remains exceeds $\frac{2}{3}$ of 57 by 4.

Example 4:

We want to express the following statement using symbolic language:

The sum of three consecutive integers is 372.

Example 5:

We want to express the following statement using symbolic language:

The sum of three consecutive odd integers is 93.

On Your Own:

1. The sum of four consecutive even integers is –28.	
2. A number is four times larger than the square of half the number.	
3. Steven has some money. If he spends \$9.00, then he will have $\frac{3}{5}$ of the amount he started with.	
4. The sum of a number squared and three less than twice the number is 129.	
5. Miriam read a book with an unknown number of pages. The first week, she read five less than $\frac{1}{3}$ of the pages. The second week, she read 171 more pages and finished the book. Write an equation that represents the total number of pages in the book.	

Lesson 1 Summary:

Lesson 1 – Independent Practice

Write each of the following statements using symbolic language.

1. Bruce bought two books. One book costs \$4.00 more than three times the other. Together, the two books cost him \$72.

2. Janet is three years older than her sister Julie. Janet's brother is eight years younger than their sister Julie. The sum of all of their ages is 55 years.

3. The sum of three consecutive integers is 1,623.

4. One number is six more than another number. The sum of their squares is 90.

5. When you add 18 to $\frac{1}{4}$ of a number, you get the number itself.

6. When a fraction of 12 is taken away from 17, what remains exceeds one-third of seventeen by six.

7. The sum of two consecutive even integers divided by four is 189.5.

8. Subtract seven more than twice a number from the square of one-third of the number to get zero.

9. The sum of three consecutive integers is 42. Let x be the middle of the three integers. Transcribe the statement accordingly.

Lesson 2 – Linear and Nonlinear Expressions in x

Essential Questions:

Discussion:

What is an expression?	
What is an equation?	

The following chart contains both linear and nonlinear expressions in x. Sort them into two groups and be prepared to explain what is different about the two groups.

Identify which equations you placed in each group. Explain your reasoning for grouping the expressions.	

How can we distinguish between linear and nonlinear equations?	Linear: Nonlinear:

Example 1:

A linear expression in x is an expression where each term is either a constant or a product of a constant and x. For example, the expression ($57-x$) is a linear expression. However, the expression $2x^2 + 9x + 5$ is not a linear expression.

Why is $2x^2 + 9x + 5$ not a linear expression in x?	

Example 2:

Let's examine the expression $4 + 3x^5$ more deeply. To begin, we want to identify the terms of the expression.

Is $4 + 3x^5$ a linear or non linear expression in x? Why or why not?	
Example 3: How many terms does the expression $7x + 9 + 6 + 3x$ have? What are they?	
Is $10x + 15$ a linear or nonlinear expression in x? Why or why not?	

Example 4: How many terms does the expression $5 + 9x \cdot 7 + 2x^9$ have? What are they?	
Is $5 + 9x \cdot 7 + 2x^9$ a linear or nonlinear expression in x? Why or why not?	
Example 5: Is $94 + x + 4x^{-6} - 2$ a linear or nonlinear expression in x? Why or why not?	
Example 6: Is the expression $x^1 + 9x - 4$ a linear expression in x ?	
What powers of x are acceptable in the definition of a linear expression in x?	

On Your Own:

Write each of the following statements in Exercises 1–12 as a mathematical expression. State whether or not the expression is linear or nonlinear. If it is nonlinear, then explain why.

1. The sum of a number and four times the number.	
2. The product of five and a number	
3. Multiply six and the reciprocal of the quotient of a number and seven.	
4. Twice a number subtracted from four times a number, added to 15.	
5. The square of the sum of six and a number	
6. The cube of a positive number divided by the square of the same positive number	
7. The sum of four consecutive numbers	

8. Four subtracted from the reciprocal of a number.	
9. Half of the product of a number multiplied by itself three times.	
10. The sum that shows how many pages Maria read if she read 45 pages of a book yesterday and $\frac{2}{3}$ of the remaining pages today.	
11. An admission fee of $10 plus an additional $2 per game.	
12. Five more than four times a number and then twice that sum.	

Lesson 2 Summary:

Lesson 2 – Independent Practice

Write each of the following statements as a mathematic expression. State whether the expression is linear or nonlinear. If it is nonlinear, then explain why.

1. A number decreased by three squared.

2. The quotient of two and a number, subtracted from seventeen.

3. The sum of thirteen and twice a number.

4. 5.2 more than the product of seven and a number.

5. The sum that represents the number of tickets sold if 35 tickets were sold Monday, half of the remaining tickets were sold on Tuesday, and 14 tickets were sold on Wednesday.

6. The product of 19 and a number, subtracted from the reciprocal of the number cubed.

7. The product of 15 and a number, and then the product multiplied by itself four times.

8. A number increased by five and then divided by two.

9. Eight times the result of subtracting three from a number.

10. The sum of twice a number and four times a number subtracted from the number squared.

11. One-third of the result of three times a number that is increased by 12.

12. Five times the sum of one-half and a number.

13. Three-fourths of a number multiplied by seven.

14. The sum of a number and negative three, multiplied by the number.

15. The square of the difference between a number and 10.

Lesson 3 – Linear Equations in x

Essential Questions:

Concept Development: We want to define a linear equation in x

Using what you know about the words **linear** and **equation** to develop a mathematical definition of a "linear equation in x"	

$x + 11 = 15$	$5 + 3 = 8$	$-\frac{1}{2}x = 22$
$15 - 4x = x + \frac{4}{5}$	$3 - (x + 2) = -12x$	$\frac{3}{4}x + 6(x - 1) = 9(2 - x)$

Consider the following equations. Which are true, and how do you know? $4 + 1 = 5$ $6 + 5 = 16$ $21 - 6 = 15$ $6 - 2 = 1$	
Is $4 + 15x = 49$ true? How do you know?	

Define linear equation in x:	
What points can we make about linear equations in x?	
Example 1: $4 + 15x = 49$ Is there a number x that makes the linear expression $4 + 15x$ equal to the linear expression 49?	
Example 2: $8x - 19 = -4 - 7x$. Is 5 a solution to the equation?	

Example 3:

$3(x + 9) = 4x - 7 + 7x.$

Is $\frac{5}{4}$ a solution to the equation?

Example 4:

$-2x + 11 - 5x = 5 - 6x.$

Is 6 a solution to the equation?

On Your Own:

1. Is the equation a true statement when $x = -3$; in other words, is -3 a solution to the equation

$6x + 5 = 5x + 8 + 2x$?

Explain.

2. Does $x = 12$ satisfy the equation

$16 - \frac{1}{2}x = \frac{3}{4}x + 1$?

Explain.

3. Chad solved the equation $24x + 4 + 2x = 3(10x - 1)$ and is claiming that x = 2 makes the equation true. Is Chad correct? Explain.	
4. Lisa solved the equation $x + 6 = 8 + 7x$ and claimed that the solution is $x = -\frac{1}{3}$. Is she correct? Explain.	
5. Angel transformed the following equation from $6x + 4 - x = 2(x + 1)$ *to* $10 = 2(x + 1)$. He then stated that the solution to the equation is x = 4. Is he correct? Explain.	

6. Claire was able to verify that $x = 3$ was a solution to her teacher's linear equation, but the equation got erased from the board. What might the equation have been? Identify as many equations as you can with a solution of $x = 3$.	
7. Does an equation always have a solution? Could you come up with an equation that does not have a solution?	

Lesson 3 Summary:

Lesson 3 Independent Practice

1. Given that $2x + 7 = 27$ and $3x + 1 = 28$, does $2x + 7 = 3x + 1$? Explain.

2. Is –5 a solution to the equation $6x + 5 = 5x + 8 + 2x$? Explain.

3. Does x = 1.6 satisfy the equation $6 - 4x = -\frac{x}{4}$? Explain.

4. Use the linear equation $3(x + 1) = 3x + 3$ to answer parts (a)–(d).

a. Does x = 5 satisfy the equation above? Explain.

b. Is x = –8 a solution of the equation above? Explain.

c. Is $x = \frac{1}{2}$ a solution of the equation above? Explain.

d. What interesting fact about the equation $3(x + 1) = 3x + 3$ is illuminated by the answers to parts (a), (b), and (c)? Why do you think this is true?

Lesson 4: Solving a Linear Equation

EQ:

Concept Development

What does it mean to *solve* an equation?	
In some cases, some simple guess work can lead us to a solution. For example, consider the following equation: $$4x + 1 = 13$$ What number x would make this equation true?	
$3(4x - 9) + 10 = 15x + 2 + 7x$ Can you guess a number for x that would make this equation true?	
Is guessing an efficient strategy for problem solving?	

$4 + 1 = 7 - 2$ Is this equation true?	
Perform each of the following operations and state whether or not the equation is still true.	
Add three to both sides of the equal sign.	
Add three to the left side of the equal sign, and add two to the right side of the equal sign.	
Subtract six from both sides of the equal sign	
Subtract three from one side of the equal sign, and subtract three from the other side of the equal sign.	
Multiply both sides of the equal sign by ten.	
Multiply the left side of the equation by ten and the right side of the equation by four.	

Divide both sides of the equation by two.	
Divide the left side of the equation by two and the right side of the equation by five.	
What do you notice? Describe any patterns you see.	• • • •

Example	List the properties that you use
Example 1: Solve the linear equation. $2x - 3 = 4x$	List the properties that you use
Example 2: Solve the linear equation. $\frac{3}{5}x - 21 = 15$	List the properties that you use

Example 3: What are other properties that make solving an equation more efficient?	• •
Solve the linear equation. $\frac{1}{5}x + 13 + x = 1 - 9x + 22$	Explain your work

On Your Own:

1. Solve the linear equation $x + x + 2 + x + 4 + x + 6 = -28$ State the property that justifies your first step and why you chose it.	
2. Solve the linear equation $2(3x + 2) == 2x - 1 + x$ State the property that justifies your first step and why you chose it.	

3. Solve the linear equation

$$x - 9 = \frac{3}{5}x$$

State the property that justifies your first step and why you chose it.

4. Solve the linear equation

$$29 - 3x = 5x + 5.$$

State the property that justifies your first step and why you chose it.

5. Solve the linear equation $\frac{1}{3}x - 5 + 171 = x$.

State the property that justifies your first step and why you chose it.

Lesson 4 Summary:

Lesson 4 Independent Practice

For each problem, show your work and check that your solution is correct.

1. Solve the linear equation $x + 4 + 3x = 72$. State the property that justifies your first step and why you chose it.

2. Solve the linear equation $x + 3 + x - 8 + x = 55$. State the property that justifies your first step and why you chose it.

3. Solve the linear equation $\frac{1}{2}x + 10 = \frac{1}{4}x + 54$. State the property that justifies your first step and why you chose it.

4. Solve the linear equation $\frac{1}{4}x + 18 = x$. State the property that justifies your first step and why you chose it.

5. Solve the linear equation $17 - x = \frac{1}{3} \cdot 15 + 6$. State the property that justifies your first step and why you chose it.

6. Solve the linear equation $\frac{x+x+2}{4} = 189.5$. State the property that justifies your first step and why you chose it.

7. Alysha solved the linear equation $2x - 3 - 9x = 14 + x - 1$. Her work is shown below. When she checked her answer, the left side of the equation did not equal the right side. Find and explain Alysha's error, and then solve the equation correctly.

$$2x - 3 - 9x = 14 + x - 1$$

$$-6x - 3 = 13 + 2x$$

$$-6x - 3 + 3 = 13 + 3 + 2x$$

$$-6x = 16 + 2x$$

$$-6x + 2x = 16$$

$$-4x = 16$$

$$\frac{-4}{-4}x = \frac{16}{-4}$$

$$x = -4$$

Lesson 5: Writing and Solving Linear Equations

Essential Questions:

Example 1

Solve:

One angle is five less than three times the size of another angle. Together they have a sum of $\mathbf{143}^\circ$. What is the measure each angle?

Using either method, solve the equation. Verify that the measures of the angles are the same as before.	

Example 2:

Given a right triangle, find the measure of the angles if one angle is ten more than four times the other angle, and the third angle is the right angle.

Draw a triangle and solve:

Find another way to solve:

Example 3:

A pair of alternate interior angles are described as follows. One angle measure is fourteen more than half a number. The other angle measure is six less than half that number. Are the angles congruent?

On Your Own For each of the following problems, write an equation and solve.	
1. A pair of congruent angles are described as follows: The measure of one angle is three more than twice a number, and the other angle's measure is 54.5 less than three times the number. Determine the size of the angles.	

2. The measure of one angle is described as twelve more than four times a number. Its supplement is twice as large.

Find the measure of each angle.

3. A triangle has angles described as follows: The measure of the first angle is four more than seven times a number, the measure of the second angle is four less than the first, and the measure of the third angle is twice as large as the first.

What is the measure of each angle?

4. One angle measures nine more than six times a number. A sequence of rigid motions maps the angle onto another angle that is described as being thirty less than nine times the number.

What is the measure of the angle?

5. A right triangle is described as having an angle of measure "six less than negative two times a number," another angle measure that is "three less than negative one-fourth the number," and a right angle.

What are the measures of the angles?

6. One angle is one less than six times the measure of another. The two angles are complementary angles. Find the measure of each angle.	

Lesson 5 Summary:

Lesson 5 Independent Practice

For each of the following problems, write an equation and solve.

1. The measure of one angle is thirteen less than five times the measure of another angle. The sum of the measures of the two angles is 140°. Determine the measure of each angle.

2. An angle measures seventeen more than three times a number. Its supplement is three more than seven times the number. What is the measure of each angle?

3. The angles of a triangle are described as follows: $\angle A$ is the largest angle; its measure is twice the measure of $\angle B$. The measure of $\angle C$ is 2 less than half the measure of $\angle B$. Find the measures of the three angles.

4. A pair of corresponding angles are described as follows: The measure of one angle is five less than seven times a number, and the measure of the other angle is eight more than seven times the number. Are the angles congruent? Why or why not?

5. The measure of one angle is eleven more than four times a number. Another angle is twice the first angle's measure. The sum of the measures of the angles is 195°. What is the measure of each angle?

6. Three angles are described as follows: $\angle B$ is half the size of $\angle A$. The measure of $\angle C$ is equal to one less than two times the measure of $\angle B$. The sum of $\angle A$ and $\angle B$ is 114°. Can the three angles form a triangle? Why or why not?

Lesson 6: Solutions of a Linear Equation

Essential Questions:

Example 1: What value of x would make the linear equation true? $4x + 3(4x + 7) = 4(7x + 3) - 3$	
What are the steps to solve the following equation: $4x + 3(4x + 7) = 4(7x + 3) - 3$	List the properties that you use.

Example 2:

What value of x would make the following linear equation true:

$$20 - (3x - 9) - 2 = -(-11x + 1)?$$

$20 - (3x - 9) - 2 = -(-11x + 1)$	Record the steps to solving the equation here:

Example 3:

What value of x would make the following linear equation true:

$\frac{1}{2}(4x + 6) - 2 = -(5x + 9)$?

Solve.

Example 4:

Consider the following equation:

$$2(x + 1) = 2x - 3.$$

What value of x would make the following linear equation true: $9(4 - 2x) - 3 = 4 - 6(3x - 5)$?	
Example 5: Solve: $3x + 15 = -6$.	

On Your Own

Find the value of x that makes the equation true.

1. $17 - 5(2x - 9) = -(-6x + 10) + 8$

2. $-(x - 7) + 5 = 2(x + 9)$

3. $4 + 4(x - 1) = 28 - (x - 7x) + 1$

4. $5(3x + 4) - 2x = 7x - 3(-2x + 11)$

5. $7x - (3x + 5) - 8 = \frac{1}{2}(8x + 20) - 7x + 5$

6. Write at least three equations that have no solution.

Lesson 6 Summary:

Lesson 6 Independent Practice

Transform the equation if necessary, and then solve it to find the value of x that makes the equation true.

1. $x - (9x - 10) + 11 = 12x + 3\left(-2x + \frac{1}{3}\right)$

2. $7x + 8\left(x + \frac{1}{4}\right) = 3(6x - 9) - 8$

3. $-4x - 2(8x + 1) = -(-2x - 10)$

4. $11(x + 10) = 132$

5. $37x + \frac{1}{2} - (x + \frac{1}{4}) = 9(4x - 7) + 5$

6. $3(2x - 14) + x = 15 - (-9x - 5)$

7. $8(2x + 9) = 56$

Lesson 7: Classification of Solutions

Essential Questions:

On Your Own

Solve each of the following equations for x.

1. $7x - 3 = 5x + 5$

2. $7x - 3 = 7x + 5$

3. $7x - 3 = -3 + 7x$

$7x - 3 = 5x + 5$	$7x - 3 = 7x + 5$	$7x - 3 = -3 + 7x$

Activity

Generalize what you know about each of each of these equations.

On Your Own

Give a brief explanation as to what kind of solution(s) you expect the following linear equations to have. Transform the equation into a simpler form if necessary.

4. $11x - 2x + 15 = 8 + 7 + 9x$

5. $3(x - 14) + 1 = -4x + 5$

6. $-3x + 32 - 7x = -2(5x + 10)$

7. $\frac{1}{2}(8x + 26) = 13 + 4x$

8. Write two equations that have no solutions.

9. Write two equations that have one unique solution each.

10. Write two equations that have infinitely many solutions.

Lesson 7 Summary:

Lesson 7 Independent Practice

1. Give a brief explanation as to what kind of solution(s) you expect for the linear equation $18x + \frac{1}{2} = 6(3x + 25)$. Transform the equation into a simpler form if necessary.

2. Give a brief explanation as to what kind of solution(s) you expect for the linear equation $8 - 9x = 15x + 7 + 3x$. Transform the equation into a simpler form if necessary.

3. Give a brief explanation as to what kind of solution(s) you expect for the linear equation $5(x + 9) = 5x + 45$. Transform the equation into a simpler form if necessary.

4. Give three examples of equations where the solution will be unique, that is, only one solution is possible.

5. Solve one of the equations you wrote in Problem 4, and explain why it is the only solution.

6. Give three examples of equations where there will be no solution.

7. Attempt to solve one of the equations you wrote in Problem 6, and explain why it has no solution.

8. Give three examples of equations where there will be infinitely many solutions.

9. Attempt to solve one of the equations you wrote in Problem 8, and explain why it has infinitely many solutions

Lesson 8: Linear Equations in Disguise

Essential Questions:

Concept Development

Given:

$$\frac{x}{5} = \frac{6}{12}$$

What do we call this kind of problem, and how do we solve it?	
THEOREM. Given rational numbers A, B, C, and D, so that $B \neq 0$ and $D \neq 0$…	
Solve.	

Example 1:
Given a linear equation in disguise, we will try to solve it. To do so, we must first assume that the following equation is true for some number x.

$$\frac{x-1}{2}=\frac{x+\frac{1}{3}}{4}$$

Solve for x.	
Example 2: Can we solve the following equation? Explain.	$$\frac{\frac{1}{5}-x}{7}=\frac{2x+9}{3}$$

Is this a linear equation?
How do you know?

Example 3:
Can this equation be solved?

$$\frac{6+x}{7x+\frac{2}{3}}=\frac{3}{8}$$

$$\frac{6+x}{7x+\frac{2}{3}}=\frac{3}{8}$$

Example 4:
Can this equation be solved?

$$\frac{7}{3x+9}=\frac{1}{8}$$

$$\frac{7}{3x+9}=\frac{1}{8}$$

Example 5:

In the diagram below, $\triangle ABC \sim \triangle A'B'C'$. Using what we know about similar triangles, we can determine the value of x.

Use x to determine the length of AB and AC

On Your Own

Solve the following equations of rational expressions, if possible.

1. $\frac{2x+1}{9} = \frac{1-x}{6}$

2. $\frac{5+2x}{3x-1} = \frac{6}{7}$

3. $\frac{x+9}{12} = \frac{-2x-\frac{1}{2}}{3}$

4. $\frac{8}{3-4x} = \frac{5}{2x+\frac{1}{4}}$

Lesson 8 Summary:

Lesson 8 Independent Practice

1. $\frac{5}{6x-2} = \frac{-1}{x+1}$

2. $\frac{4-x}{8} = \frac{7x-1}{3}$

3. $\frac{3x}{x+2} = \frac{5}{9}$

4. $\frac{\frac{1}{2}x+6}{3} = \frac{x-3}{2}$

5. $\frac{7-2x}{6} = \frac{x-5}{1}$

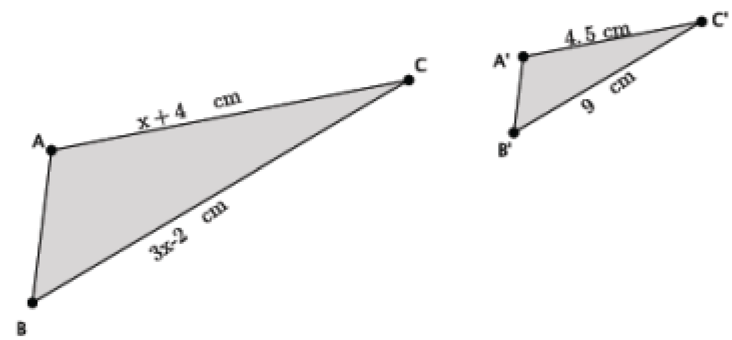
6. $\frac{2x+5}{2} = \frac{3x-2}{6}$

7. $\frac{6x+1}{3} = \frac{9-x}{7}$

8. $\frac{\frac{1}{3}x-8}{12} = \frac{-2-x}{15}$

9. $\frac{3-x}{1-x} = \frac{3}{2}$

10. In the diagram below, $\triangle ABC \sim \triangle A'B'C$. Determine the lengths of AC and BC.



Lesson 9: An Application of Linear Equations

Essential Question:

Discussion

You sent a photo of you and your family on vacation to seven Facebook friends. If each of them sends it to five of their friends, and each of those friends sends it to five of their friends, and those friends send it to five more, how many people (not counting yourself) will see your photo? Assume that no friend received the photo twice.

In Module 1, you were asked to express your answer in exponential notation. The solution is given here:

(1) The number of friends you sent a photo to =

(2) The number of friends 7 people sent the photo to =

(3) The number of friends $7 \cdot 5$ people sent the photo to =

(4) The number of friends $(7 \cdot 5) \cdot 5$ people sent the photo to $=$

Therefore, the total number of people who received the photo is

Since we are talking about steps, we will refer to the sum after step one as S_1, the sum after step two as S_2, the sum after step three as S_3, and so on. Thus:	S_1 = (1) S_2 = (2) S_3 = (3) S_4 = (4)
What patterns do you notice within each of the equation (1)-(4)?	

Generalize with Equation 2:	
Generalize with Equation 3:	
Generalize with Equation 4:	
What did we do next?	

On Your Own

1. Write the equation for the 10th step.	
2. How many people would see the photo after 10 steps? Use a calculator if needed.	

Exercises 3-11

3. Marvin paid an entrance fee of \$5 plus an additional \$1.25 per game at a local arcade. Altogether, he spent \$26.25. Write and solve an equation to determine how many games Marvin played.	

4. The sum of four consecutive integers is –26. What are the integers?

5. A book has x pages. How many pages are in the book if Maria read 45 pages of a book on Monday, $\frac{1}{2}$ the book on Tuesday, and the remaining 72 pages on Wednesday?

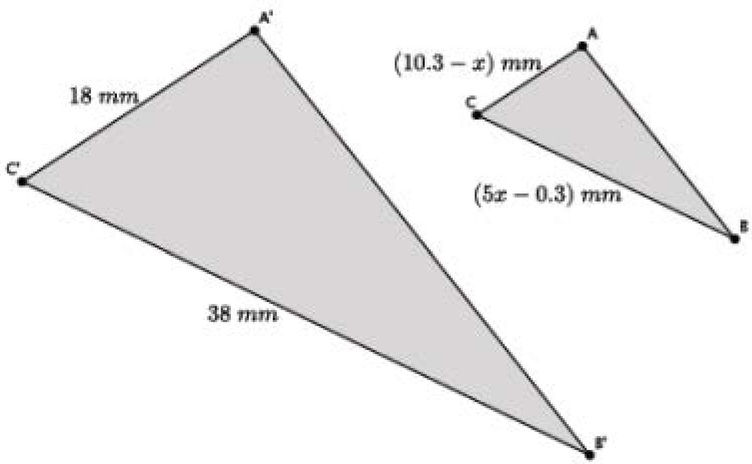
6. A number increased by 5 and divided by 2 is equal to 75. What is the number?	
7. The sum of thirteen and twice a number is seven less than six times a number. What is the number?	

8. The width of a rectangle is 7 less than twice the length. If the perimeter of the rectangle is 43.6 inches, what is the area of the rectangle?

9. Two hundred and fifty tickets are available for sale for a school dance. On Monday, 35 tickets were sold. An equal number of tickets were sold each day for the next five days. How many tickets were sold on one of those days?

10. Shonna skateboarded for some number of minutes on Monday. On Tuesday, she skateboarded for twice as many minutes as she did on Monday, and on Wednesday, she skateboarded for half the sum of minutes from Monday and Tuesday. Altogether, she skateboarded for a total of three hours. How many minutes did she skateboard each day?	

11. In the diagram below, $\triangle\ ABC \sim \triangle\ A'B'C'$. Determine the length of AC and BC.



Lesson 9 Summary:

Lesson 9 Independent Practice

1. You forward an e-card that you found online to three of your friends. They liked it so much that they forwarded it on to four of their friends, who then forwarded it on to four of their friends, and so on. The number of people who saw the e-card is shown below. Let S_1represent the number of people who saw the e-card after one step, let S_2represent the number of people who saw the e-card after two steps, and so on.

$$S_1 = 3$$

$$S_2 = 3 + 3 \cdot 4$$

$$S_3 = 3 + 3 \cdot 4 + 3 \cdot 4^2$$

$$S_4 = 3 + 3 \cdot 4 + 3 \cdot 4^2 + 3 \cdot 4^3$$

a. Find the pattern in the equations.

b. Assuming the trend continues, how many people will have seen the e-card after 10 steps?

c. How many people will have seen the e-card after nn steps?

For each of the following questions, write an equation and solve to find each answer.

2. Lisa has a certain amount of money. She spent \$39 and has $\frac{3}{4}$ of the original amount left. How much money did she have originally?

3. The length of a rectangle is 4 more than 3 times the width. If the perimeter of the rectangle is 18.4 cm, what is the area of the rectangle?

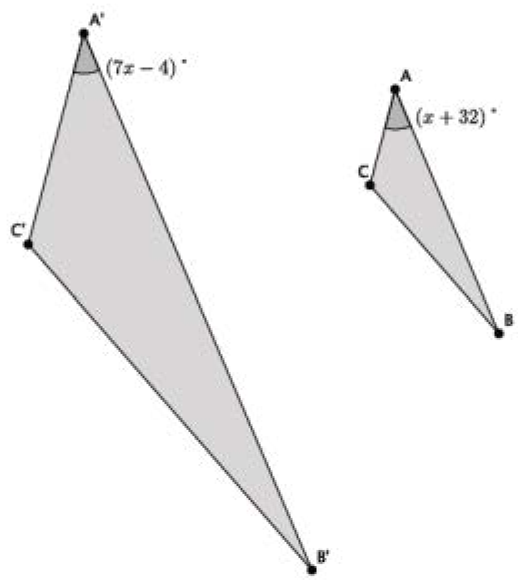
4. Eight times the result of subtracting 3 from a number is equal to the number increased by 25. What is the number?

5. Three consecutive odd integers have a sum of 3. What are the numbers?

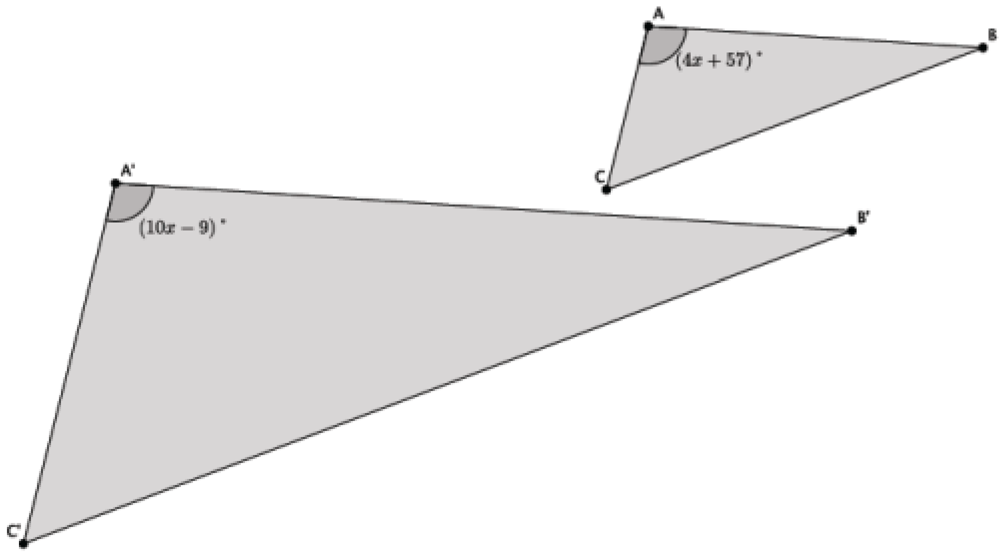
6. Each month, Liz pays \$35 to her phone company just to use the phone. Each text she sends costs her an additional \$0.05. In March, her phone bill was \$72.60. In April, her phone bill was \$65.85. How many texts did she send each month?

7. Claudia is reading a book that has 360 pages. She read some of the book last week. She plans to read 46 pages today. When she does, she will be of $\frac{4}{5}$ the way through the book. How many pages did she read last week?

8. In the diagram below, $\triangle\ ABC \sim \triangle\ A'B'C'$. Determine the measure of $\angle A$.



9. In the diagram below, $\triangle\ ABC \sim \triangle\ A'B'C'$. Determine the measure of $\angle A$.



Lesson 10-A Critical Look at Proportional Relationships

Essential Questions:

Discussion:

Consider the word problem below.

Paul walks 2 miles in 25 minutes. How many miles can Paul walk in 137.5 minutes?

Time (in minutes)	Distance (in miles)
25	2

How many miles, y, can Paul walk in x minutes?

Example 1:

Let's look at another problem where only a table is provided.

Time (in hours)	Distance (in miles)
3	123
6	246
9	369
12	492
	y

We want to know how many miles, y, can be traveled in any number of hours x.

Example 2: Alexxa walked from Grand Central Station on 42nd Street to Penn Station on 7th Avenue. The total distance traveled was 1.1 miles. It took Alexxa 25 minutes to make the walk. How many miles did she walk in the first 10 minutes?	
How can we calculate Alexxa's average speed? If Alexxa walks y miles in x minutes, then write and solve an equation for minutes, then write and solve an equation for y.	

On your own:

Exercise 1:

Wesley walks at a constant speed from his house to school 1.5 miles away. It took him 25 minutes to get to school.

a. What fraction represents his constant speed, C?

b. You want to know how many miles he has walked after 15 minutes. Let y represent the distance he traveled after 15 minutes of walking at the given constant speed. Write a fraction that represents the constant speed, C, in terms of y.	

c. Write the fractions from parts a. and b. as a proportion and solve to find how many miles Wesley walked after 15 minutes.	
d. Let y be the distance in miles that Wesley traveled after x minutes. Write a linear equation in two variables that represents how many miles Wesley walked after x minutes.	

Exercise 2:

Stefanie drove at a constant speed from her apartment to her friend's house 20 miles away. It took her 45 minutes to reach her destination.

a. What fraction represents her constant speed, C?	
b. What fraction represents constant speed, C, if it takes her x number of minutes to get halfway to her friend's house?	
c. Write a proportion using the fractions from parts a. and b. to determine how many minutes it takes her to get to the halfway point.	
d. Write a two-variable equation to represent how many miles Stefanie can drive over any time interval.	

Discussion:

Consider the problem: Dave lives 15 miles from town *A*. He is driving at a constant speed of 50 miles per hour from his home away from the city. How far away is Dave from the town after x hours of driving?	

On Your Own:

3. The equation that represents how many miles, y, Dave travels after x hours is $y = 50x + 15$. Us the equation to complete the table below.

x (hours)	Linear equation in y: $y = 50x + 15$	y (miles)
1	$y = 50(1) + 15$	65
2		
3		
3.5		
4.1		

Summary:

Lesson 10 – Independent Practice

1. Eman walks from the store to her friend's house 2 miles away. It takes her 35 minutes.
    a. What fraction represents her constant speed, C?

    b. Write the fraction that represents her constant speed, C, if she walks y miles in 10 minutes.

    c. Write a proportion using the fractions from parts a. and b. to determine how many miles she walks after 10 minutes. Round your answer to the hundredths place.

    d. Write a two-variable equation to represent how many miles Eman can walk over any time interval.

2. Erika drives from school to soccer practice 1.3 miles away. It takes her 7 minutes.
    a. What fraction represents her constant speed, C?

    b. What fraction represents her constant speed, C, if it takes her x minutes to drive exactly 1 mile?

c. Write a proportion using the fractions from parts a. and b. to determine how much time it takes her to drive exactly 1 mile. Round your answer to the tenths place.

d. Write a two-variable equation to represent how many miles Erika can drive over any time interval.

3. Darla drives at a constant speed of 45 miles per hour.
   a. If she drives for your y miles and it takes her x hours, write the two-variable equation to represent the number of miles Darla can drive in x hours.

   b. Darla plans to drive to the market 14 miles from her house, then to the post office 3 miles from the market, and then return home, which is 15 miles from the post office. Assuming she drives at a constant speed the entire time, how long will it take her to get back home after running her errands? Round your answer to the hundredths place.

4. Aaron walks from his sister's house to his cousin's house, a distance of 4 miles, in 80 minutes. How far does he walk in 30 minutes?

5. Carlos walks 4 miles every night for exercise. It takes him exactly 63 minutes to finish his walk.
   a. Assuming he walks at a constant rate, write an equation that represents how many miles, y, Carlos can walk in x minutes.

   b. Use your equation from part a. to complete the table below. Use a calculator and round all values to the hundredths place.

x (minutes)	Linear equation in y:	y (miles)
15		
30		
40		
60		
75		

Lesson 11-Constant Rate

Essential Questions:

Discussion:

Example 1: Pauline mows a lawn at a constant rate. Suppose she mows a 35 square foot lawn in 2.5 minutes. What area, in square feet, can she mow in 10 minutes? t minutes?	
Write a two-variable linear equation that represents the area of lawn, y, Pauline can mow in ,x, minutes.	

t (time in minutes)	Linear Equation	y (area in square feet)
0		
1		
2		
3		
4		

Vocabulary/Examples:	
Average Rate:	
Constant Rate:	
What is the difference between constant rate and average rate?	
Example 2: Water flows at a constant rate out of a faucet. Suppose the volume of water that comes out in three minutes is 10.5 gallons. How many gallons of water comes out of the faucet in t minutes?	

t (time in minutes)	**Linear Equation**	**V (in gallons)**
0		
1		
2		
3		
4		

On your own:

Example 1:

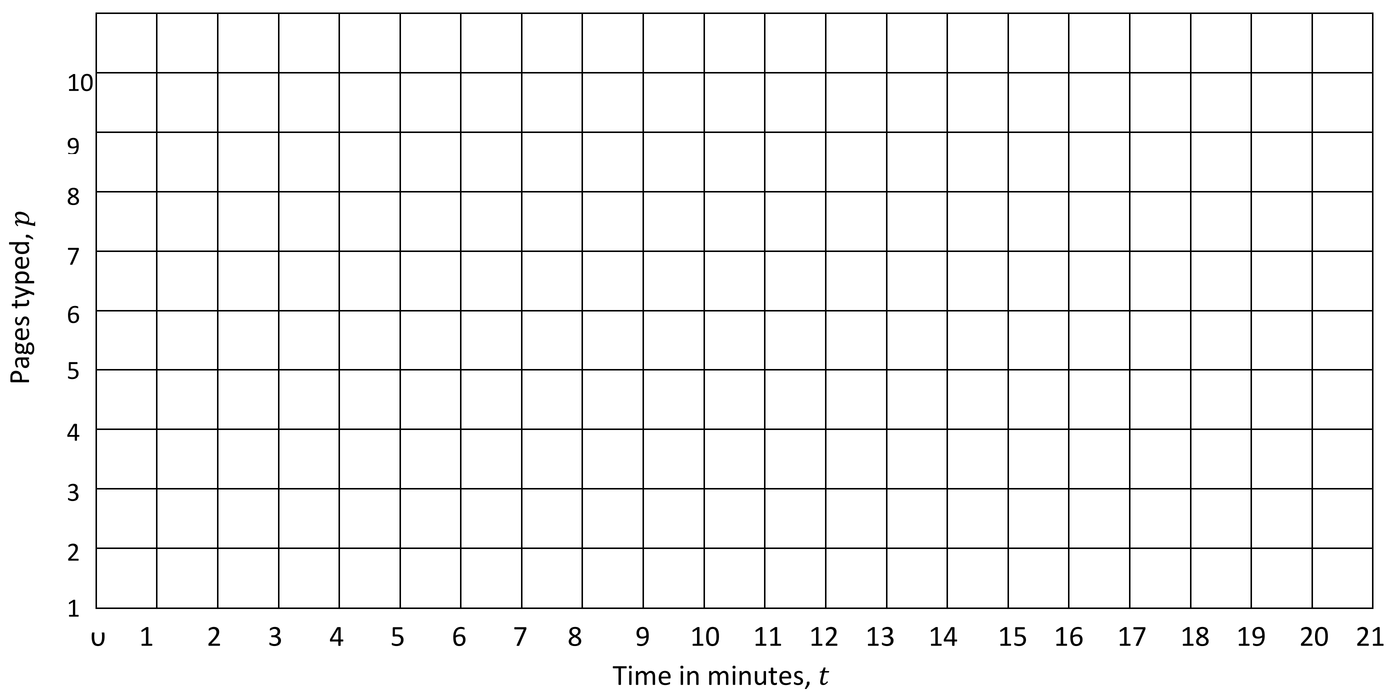
Juan types at a constant rate. He can type a full page of text in $3\frac{1}{2}$ minutes. We want to know how many pages, p, Juan can type after t minutes.

a. Write the linear equation in two variables that represents the number of pages Juan types in any given time interval.

b. Complete the table below. Use a calculator and round your answer to the tenths place.

t (time in minutes)	Linear equation:	p (pages typed)
0		
5		
10		
15		
20		

c. Graph the data on a coordinate plane.



d. About how long would it take Juan to type a 5-page paper? Explain.

Example 2:

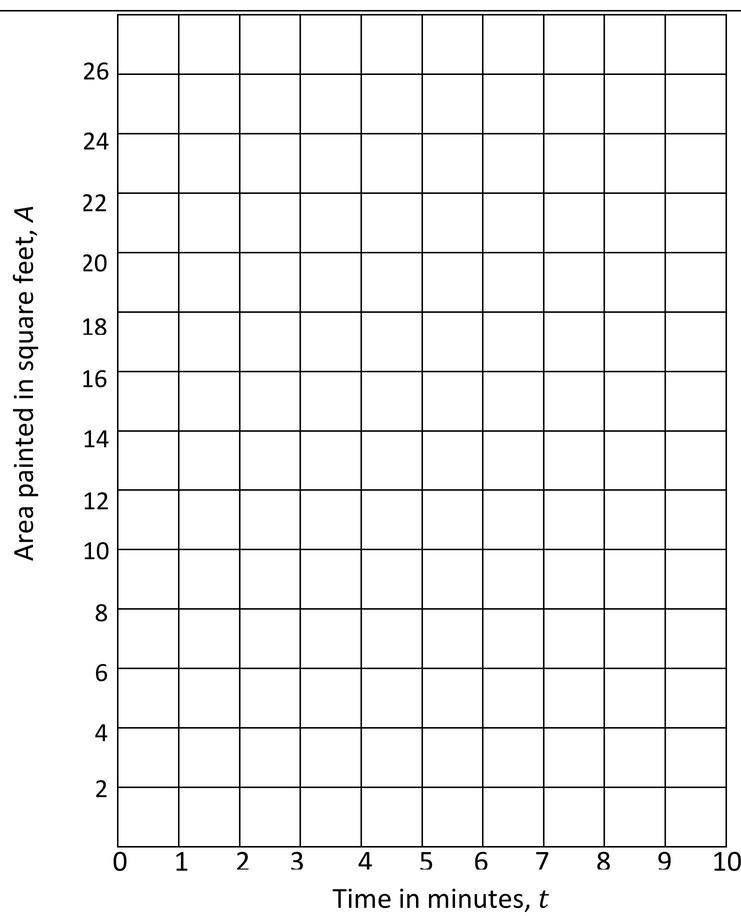
Emily paints at a constant rate. She can paint 32 square feet in 5 minutes. What area, A, in square feet, can she paint in t minutes?

a. Write the linear equation in two variables that represents the number of square feet Emily can paint in any given time interval.

b. Complete the table below. Use a calculator and round your answer to the tenths place.

t (time in minutes)	Linear equation:	A (area painted in square feet)
0		
1		
2		
3		
4		

c. Graph the data on a coordinate plane.



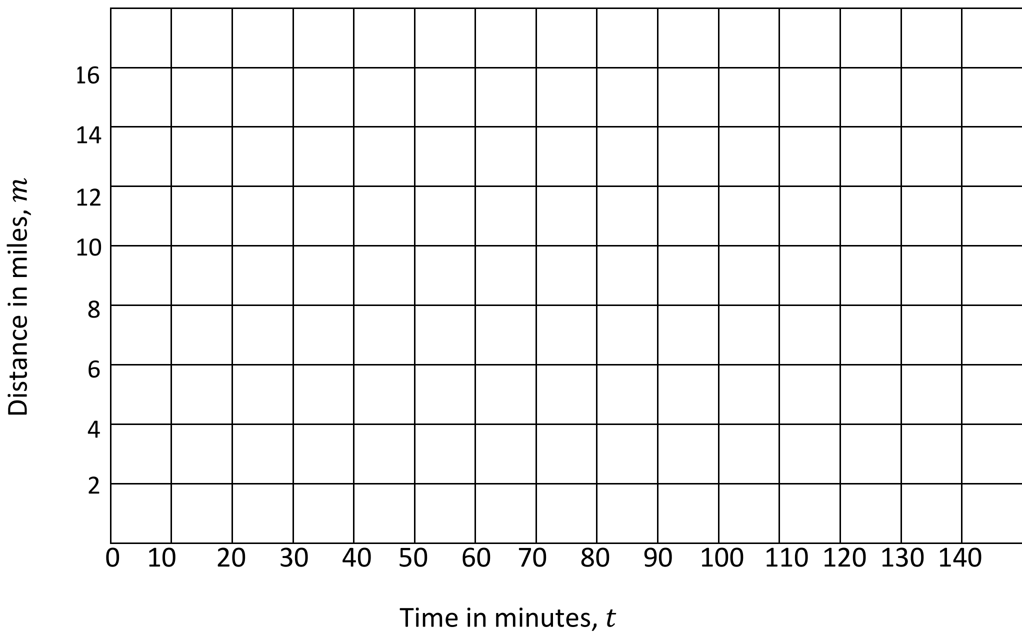
d. About how many square feet can Emily paint in $2\frac{1}{2}$ minutes? Explain.	

Example 3: Joseph walks at a constant speed. He walked to a store that is one-half mile away in 6 minutes. How many miles, m, can he walk in t minutes?	

b. Complete the table below. Use a calculator and round your answer to the tenths place.

t (time in minutes)	Linear equation:	m (distance in miles)
0		
30		
60		
90		
120		

c. Graph the data on a coordinate plane.



d. Joseph's friend lives 4 miles away from him. About how long would it take Joseph to walk to his friend's house? Explain.

Summary:

Lesson 11 – Independent Practice

1. A train travels at a constant rate of 45 miles per hour.
   a. What is the distance, d, in miles, that the train travels in t hours?

   b. How many miles will it travel in 2.5 hours?

2. Water is leaking from a faucet at a constant rate of $\frac{1}{3}$ gallons per minute.
   a. What is the amount of water, w, in gallons per minute, that is leaked from the faucet after t minutes?

   b. How much water is leaked after an hour?

3. A car can be assembled on an assembly line in 6 hours. Assume that the cars are assembled at a constant rate.
   a. How many cars, y, can be assembled in t hours?

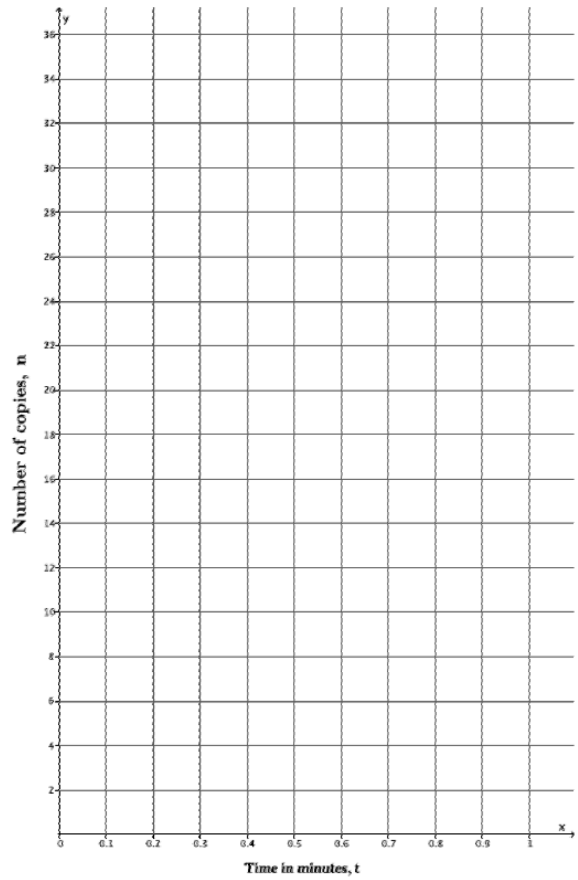
b. How many cars can be assembled in a week?

4. A copy machine makes copies at a constant rate. The machine can make 80 copies in 2 $\frac{1}{2}$ minutes.
   a. Write an equation to represent the number of copies, n, that can be made over an time interval, t.

   b. Complete the table below.

t (time in minutes)	Linear equation:	n (number of copies)
0		
0.25		
0.5		
0.75		
1		

Graph the data on a coordinate plane.

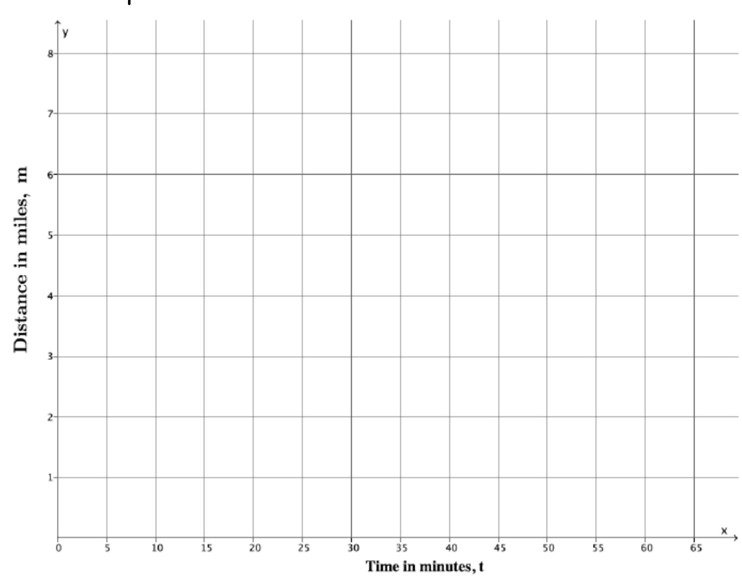


d. The copy machine runs for 20 seconds, then jams. About how many copies were made before the jam occurred? Explain.

5. Connor runs at a constant rate. It takes him 34 minutes to run 4 miles.
   a. Write the linear equation in two variables that represents the number of miles Connor can run in any given time interval, t.

   b. Complete the table below. Use a calculator and round answers to the tenths place.

   c. Graph the data on a coordinate plane.



d. Connor ran for 40 minutes before tripping and spraining his ankle. About how many miles did he run before he had to stop? Explain.

Lesson 12-Linear Equations in Two Variables

Essential Questions:

Opening

Emily tells you that she scored 32 points in a basketball game with only two- and three-point baskets (no free throws). How many of each type of basket did she score? Use the table below to organize your work.

Number of Two-Pointers	Number of Three-Pointers

Let x be the number of two-pointers and y be the number of three-pointers that Emily scored. Write an equation to represent the situation.

Vocabulary:

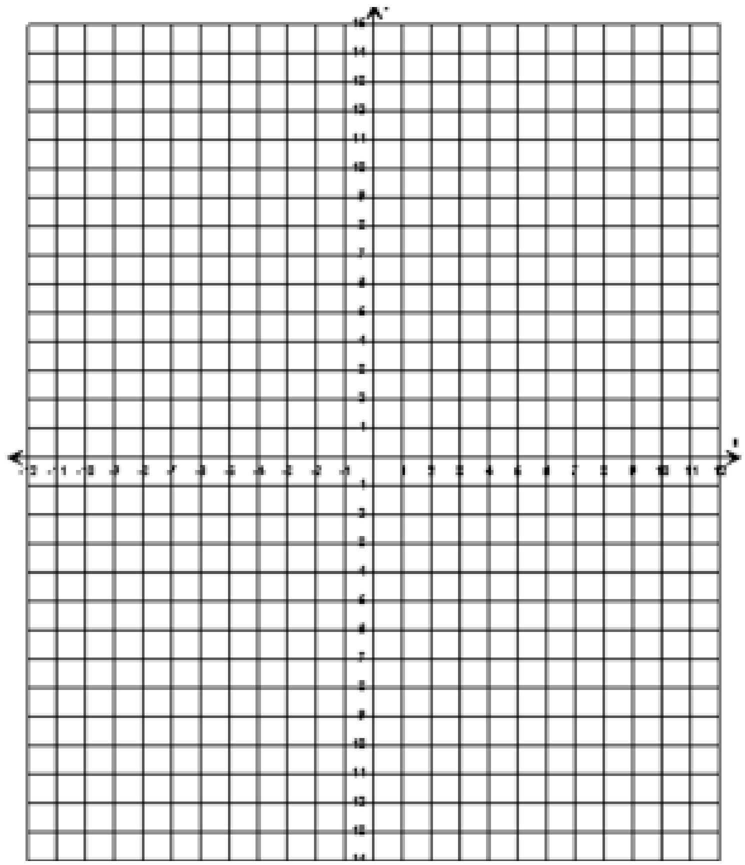
Linear equation in 2 variables	
Standard Form	

Constants	
Variables	
What is a solution to a linear equation in two variables?	

Exploratory Challenge

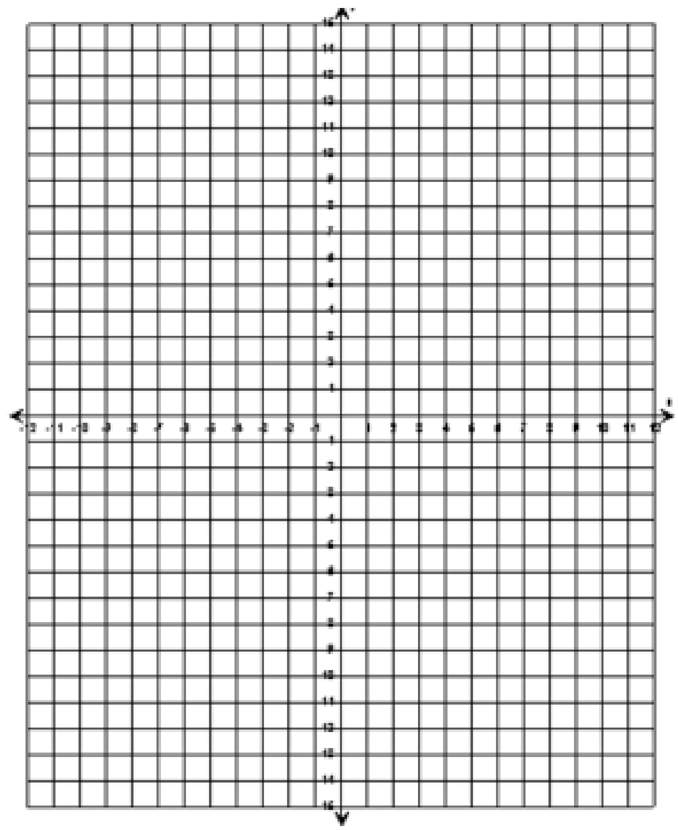
1. Find five solutions for the linear equation $x + y = 3$, and plot the solutions as points on a coordinate plane.

x	Linear Equation	y



2. Find five solutions for the linear equation $2x - y = 10$, and plot the solutions as points on a coordinate plane.

x	Linear Equation	y



3. Find five solutions for the linear equation $x + 5y = 21$, and plot the solutions as points on a coordinate plane.

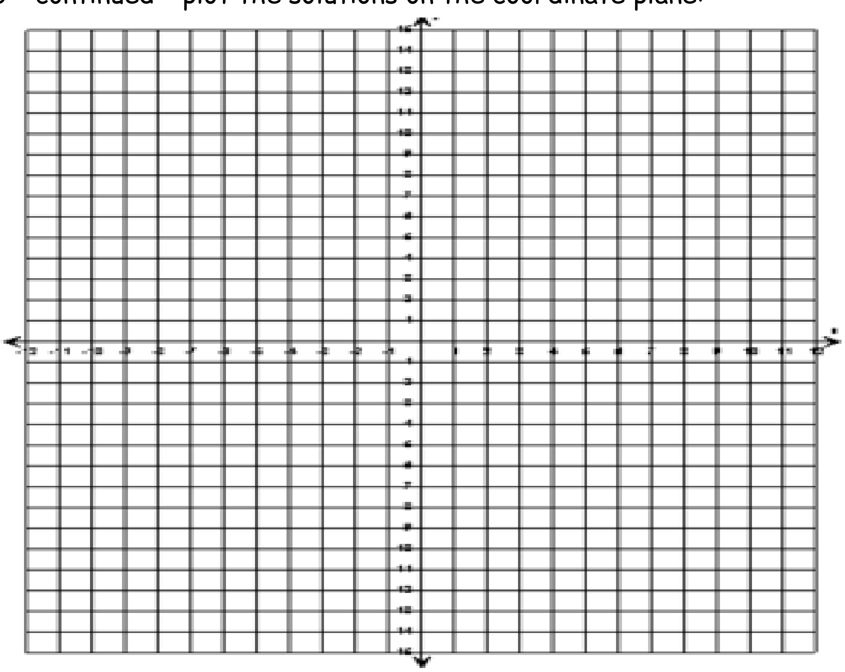
Begin by fixing x and solving for y

x	Linear equation: $x + 5y = 21$	y

In this table, fix y and solve for x

x	Linear equation: $x + 5y = 21$	y

3 – continued – plot the solutions on the coordinate plane:



4. Consider the linear equation $\frac{2}{5}x + y = 11$.

a. Will you choose to fix values for x or y? Explain.

b. Are there specific numbers that would make your computational work easier? Explain.

c. Find five solutions to the linear equation $\frac{2}{5}x + y = 11$, and plot the solutions as points on a coordinate plane.

x	Linear equation: $\frac{2}{5}x + y = 11$	y

5. At the store, you see that you can buy a bag of candy for \$2 and a drink for \$1. Assume you have a total of \$35 to spend. You are feeling generous and want to buy some snacks for you and your friends.

a. Write an equation in standard form to represent the number of bags of candy, x, and the number of drinks, y, that you can buy with \$35.

b. Find five solutions to the linear equation from part (a), and plot the solutions as points on a coordinate plane.

x	Linear Equations 2x + y = 35	y

Summary Lesson 12:

Lesson 12 Independent Practice

1. Consider the linear equation $x - \frac{3}{2}y = -2$.

a. Will you choose to fix values for x or y? Explain.

b. Are there specific numbers that would make your computational work easier? Explain.

c. Find five solutions to the linear equation $x - \frac{3}{2}y = -2$, and plot the solutions as points on a coordinate plane.

X	Linear Equation $x - \frac{3}{2}y = -2$	Y

2. Find five solutions for the linear equation $\frac{1}{3}x + y = 12$, and plot the solutions as points on a coordinate plane.

X	Linear Equation $\frac{1}{3}x + y = 12$	Y

3. Find five solutions for the linear equation $-x + \frac{3}{4}y = -6$, and plot the solutions as points on a coordinate plane.

X	Linear Equation $-x + \frac{3}{4}y = -6$	y

4. Find five solutions for the linear equation $2x + y = 5$, and plot the solutions as points on a coordinate plane.

x	Linear Equation: $2x + y = 5$	y

5. Find five solutions for the linear equation $3x - 5y = 15$, and plot the solutions as points on a coordinate plane.

x	Linear Equation: $3x - 5y = 15$	y

Lesson 13-The Graph of a Linear Equation in Two Variables

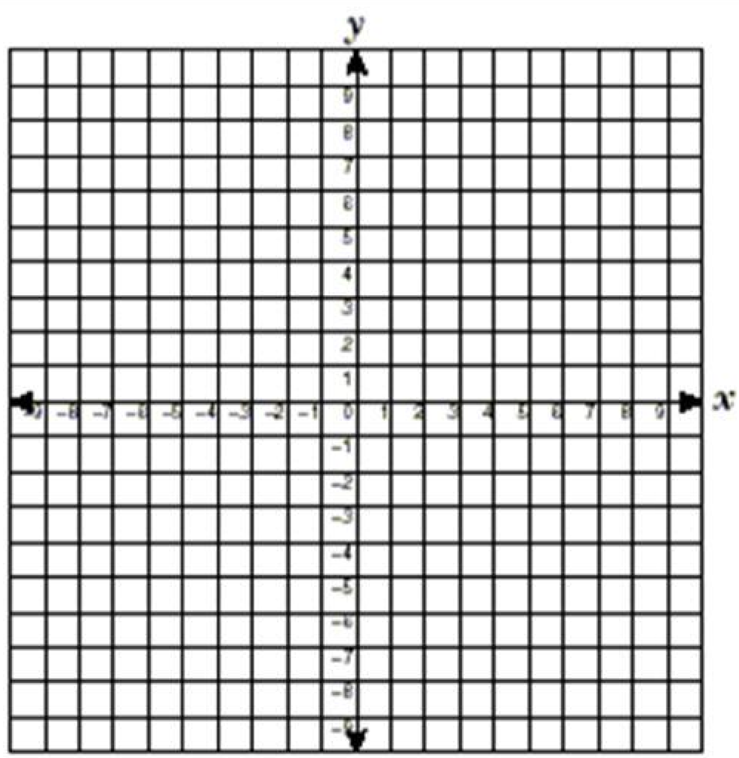
Essential Questions:

Discussion:

Do you think it's possible to plot ALL of the solutions of a linear equation on a coordinate plane?

Find solutions for $x + y = 6$

x	y



On Your Own

1. Find at least ten solutions to the linear equation $3x + y = -8$, and plot the points on a coordinate plane.

X	Linear Equation: 3x + y = -8	Y

What shape is the graph of the linear equation taking?

2. Find at least ten solutions to the linear equation $x - 5y = 11$, and plot the points on a coordinate plane.

x	Linear Equation: $X - 5y = 11$	y

What shape is the graph of the linear equation taking?

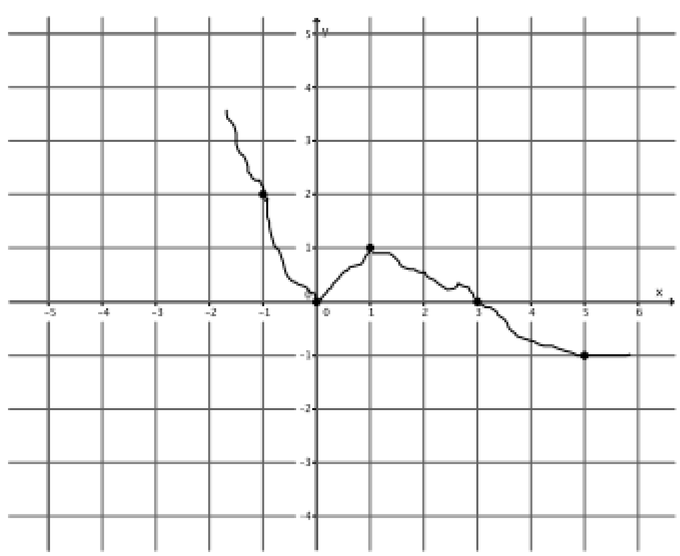
3. Compare the solutions you found in Exercise 1 with a partner. Add their solutions to your graph.

Is the prediction you made about the shape of the graph still true? Explain.

4. Compare the solutions you found in Exercise 2 with a partner. Add their solutions to your graph.

Is the prediction you made about the shape of the graph still true? Explain.

5. Joey predicts that the graph of $-x + 2y = 3$ will look like the graph shown below. Do you agree? Explain why or why not.



6. We have looked at some equations that appear to be lines. Can you write an equation that has solutions that do not form a line? Try to come up with one, and prove your assertion on the coordinate plane.

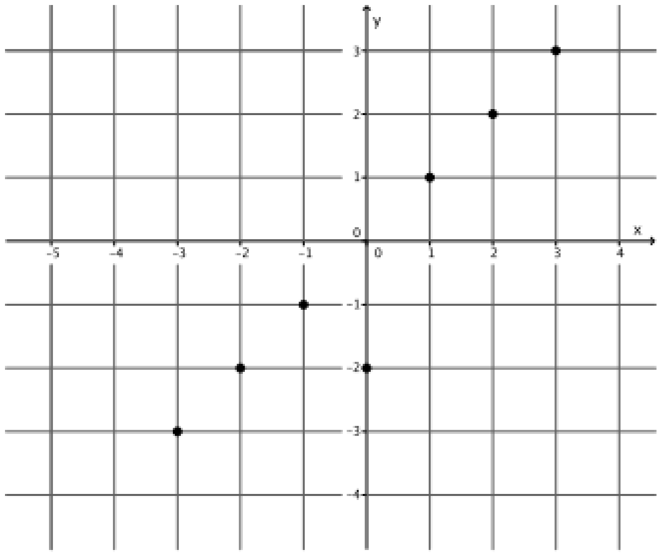
Summary Lesson 13:

Lesson 13 Independent Practice

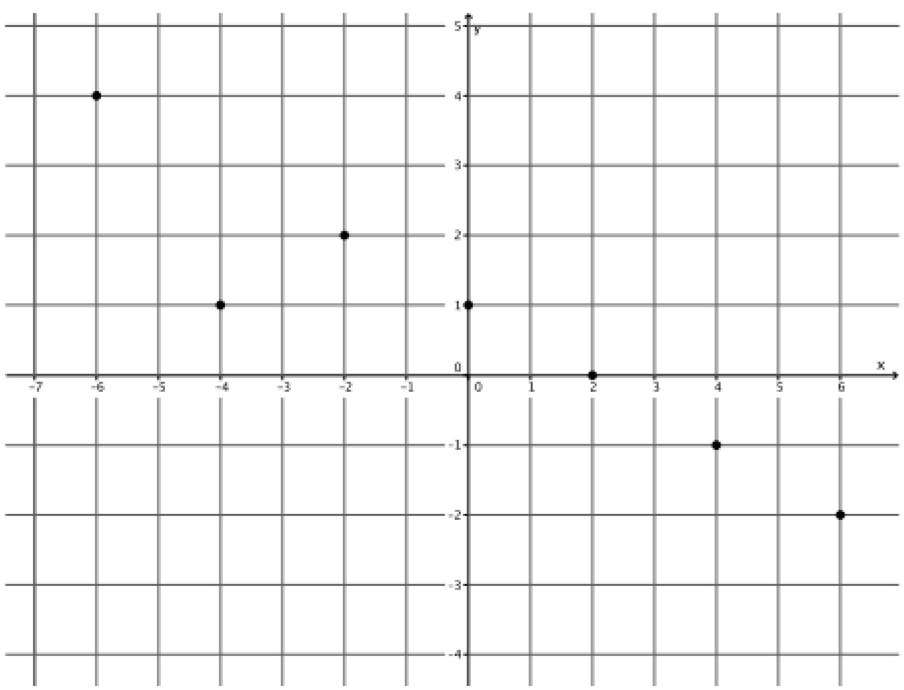
1. Find at least ten solutions to the linear equation $\frac{1}{2}x + y = 5$, and plot the points on a coordinate plane. What shape is the graph of the linear equation taking?

X	Linear Equation: $\frac{1}{2}x + y = 5$	y

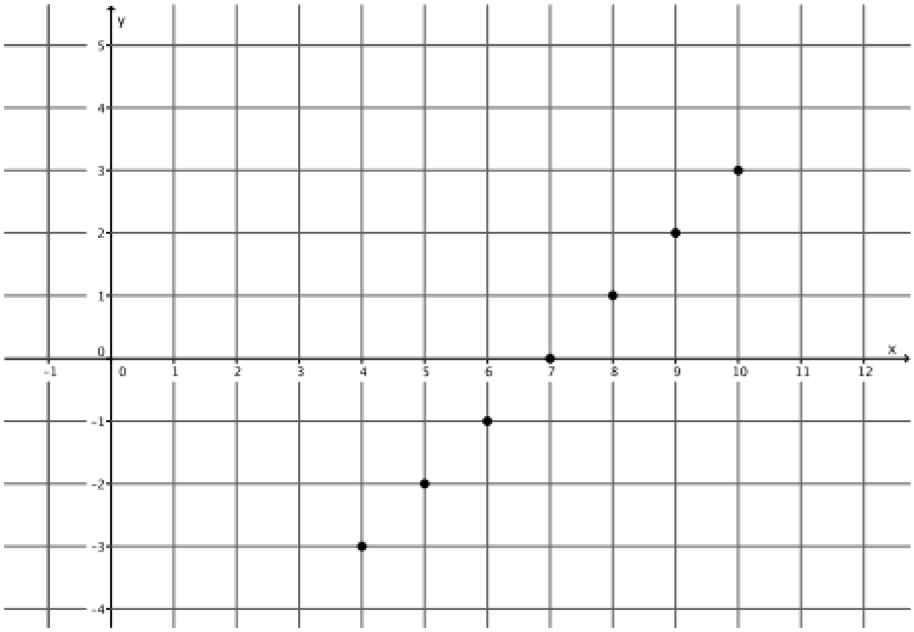
2. Can the following points be on the graph of the equation $x - y = 0$? Explain.



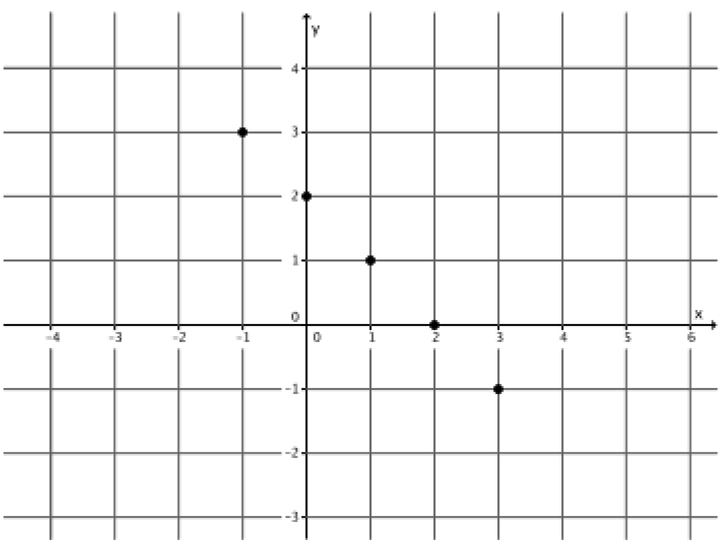
3. Can the following points be on the graph of the equation $x + 2y = 2$? Explain.



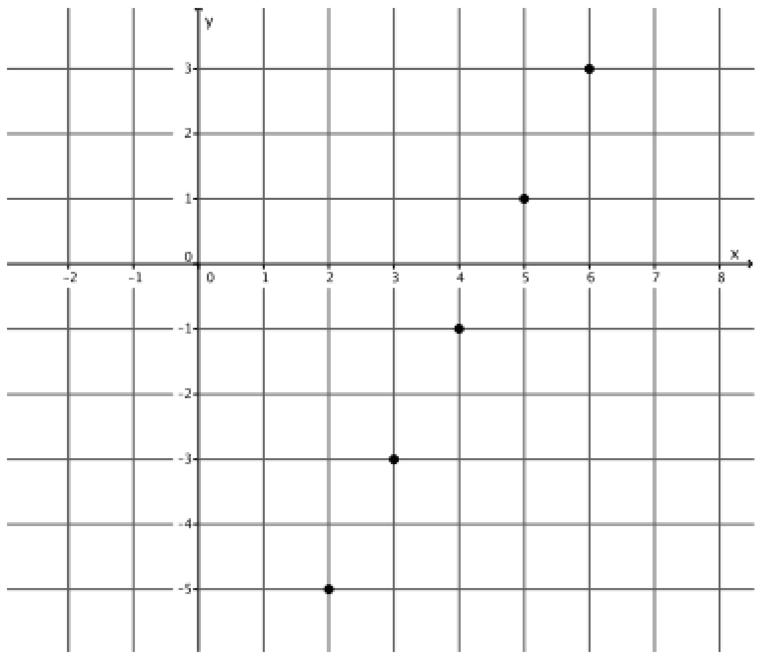
4. Can the following points be on the graph of the equation $x - y = 7$? Explain.



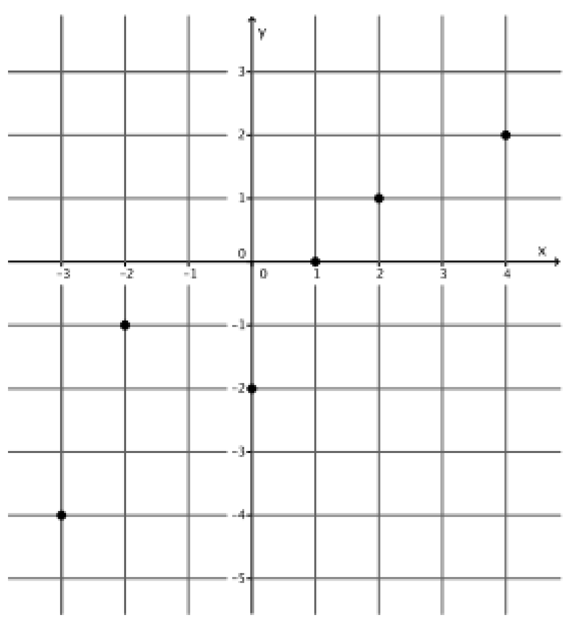
5. Can the following points be on the graph of the equation $x + y = 2$? Explain.



6. Can the following points be on the graph of the equation $2x - y = 9$? Explain.



7. Can the following points be on the graph of the equation $x - y = 1$? Explain.

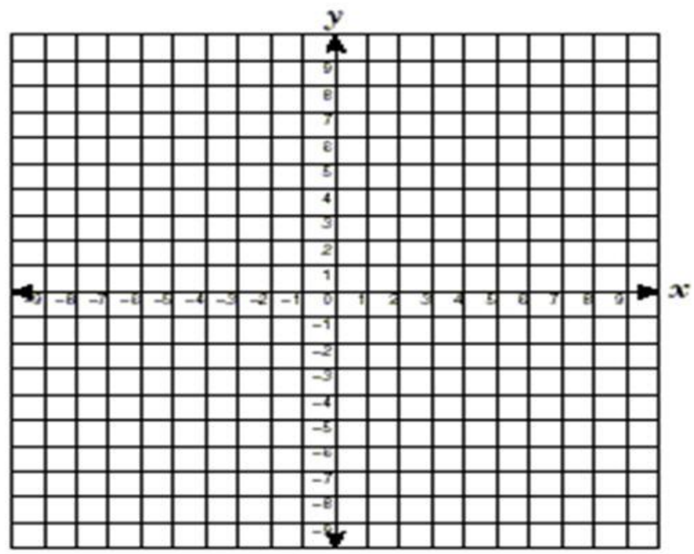


Lesson 14-Graph of a Linear Equation-Horizontal and Vertical Lines

Essential Questions:

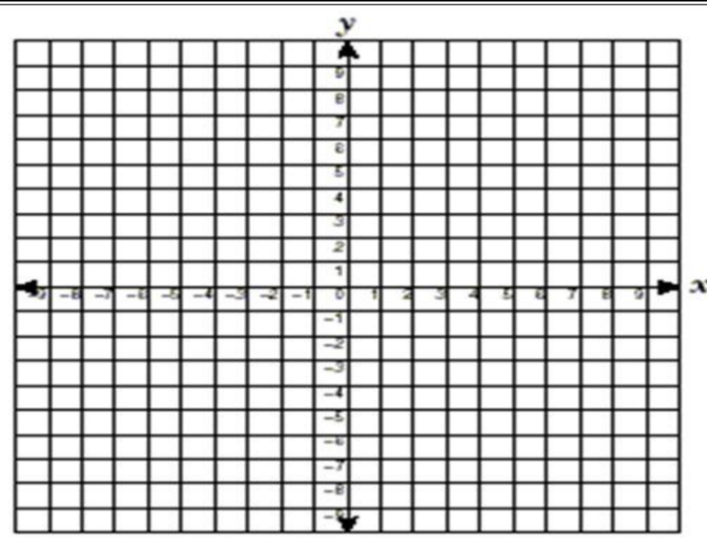
On Your Own
Exercises:

1. Find at least four solutions to graph the linear equation $1x + 2y = 5$.



2. Find at least four solutions to graph the linear equation

$$1x + 0y = 5.$$



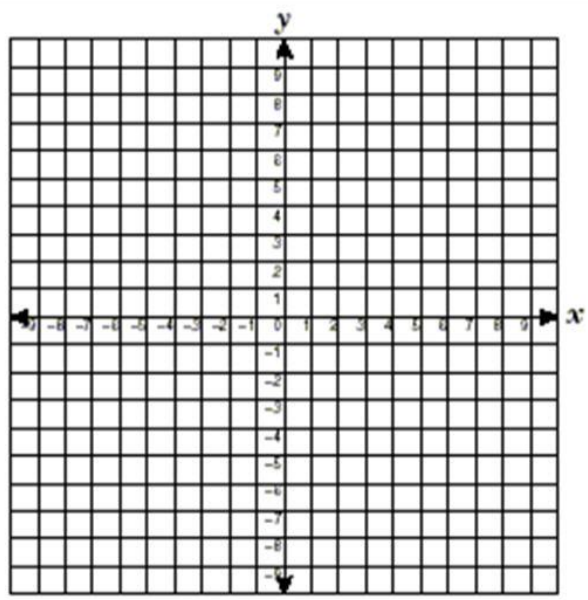
3. What was different about the equations in Exercises 1 and 2? What effect did this change have on the graph?

Discussion: Case 1

$$ax + by = c$$

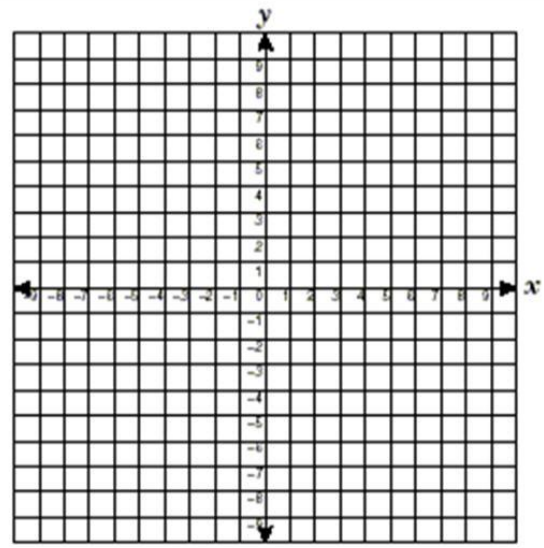
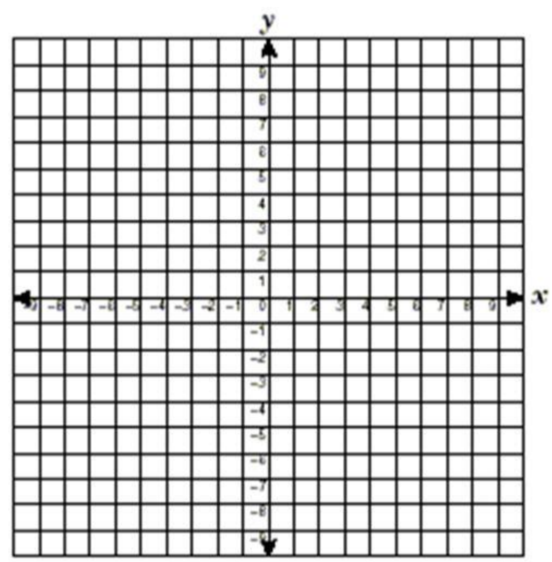
Where a, b, and c are constants and a = 1 and b=0

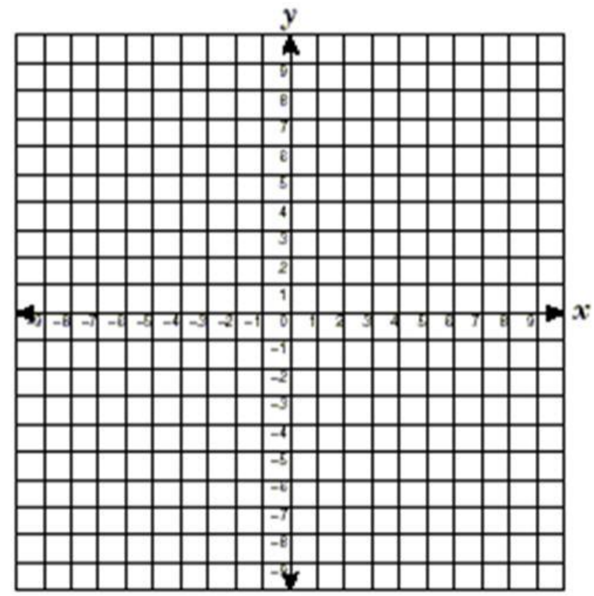
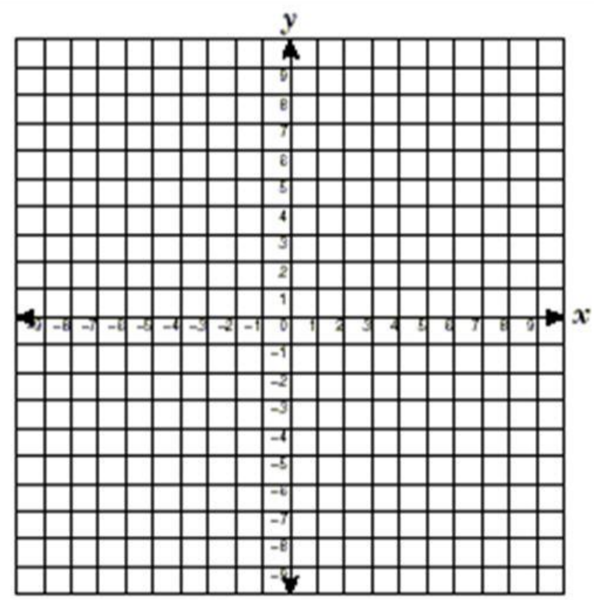
Graph the solutions on the coordinate grid.



Theorem:

On Your Own

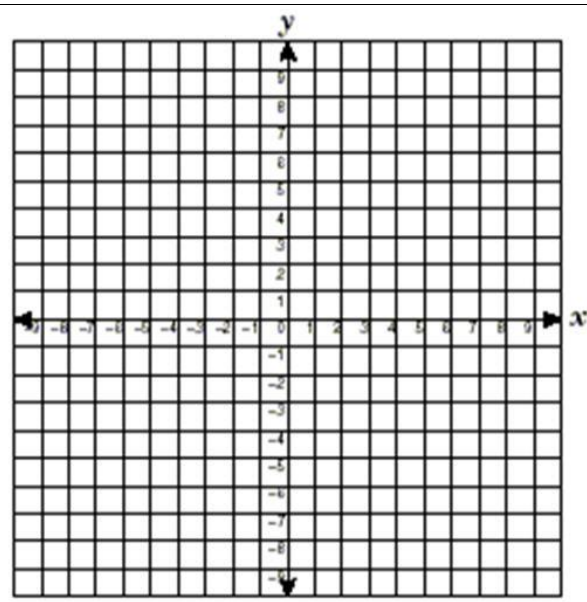
4. Graph the linear equation $x = -2$	
5. Graph the linear equation $x = 3$	
6. What will the graph of $x = 0$ look like?	

7. Find at least four solutions to graph the linear equation $2x + 1y = 2$.	
8. Find at least four solutions to graph the linear equation $0x + 1y = 2$	
9. What was different about the equations in Exercises 7 and 8? What effect did this change have on the graph?	

Discussion: Case 2

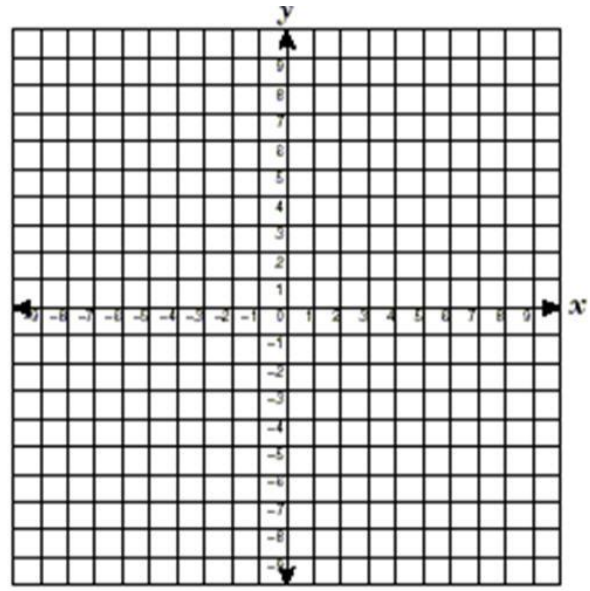
$$ax + by = c$$

Where a, b, and c are constants and a = 0 and b=1

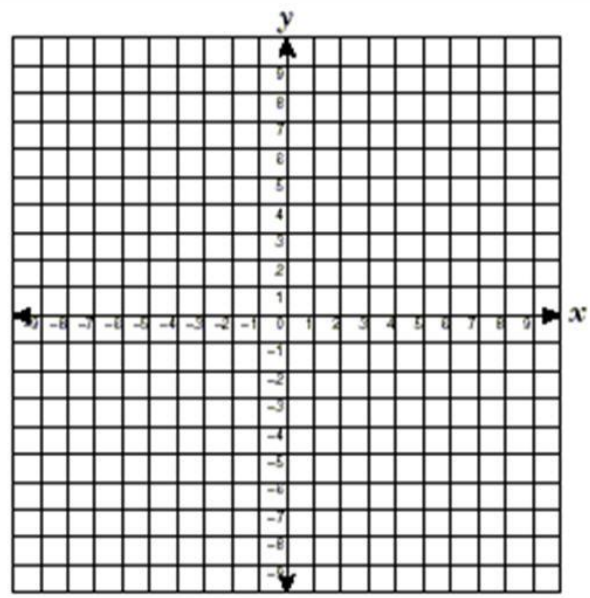
Graph the solutions on the coordinate plane	
Theorem:	

On Your Own

10. Graph the linear equation $y = -2$.



11. Graph the linear equation $y = 3$

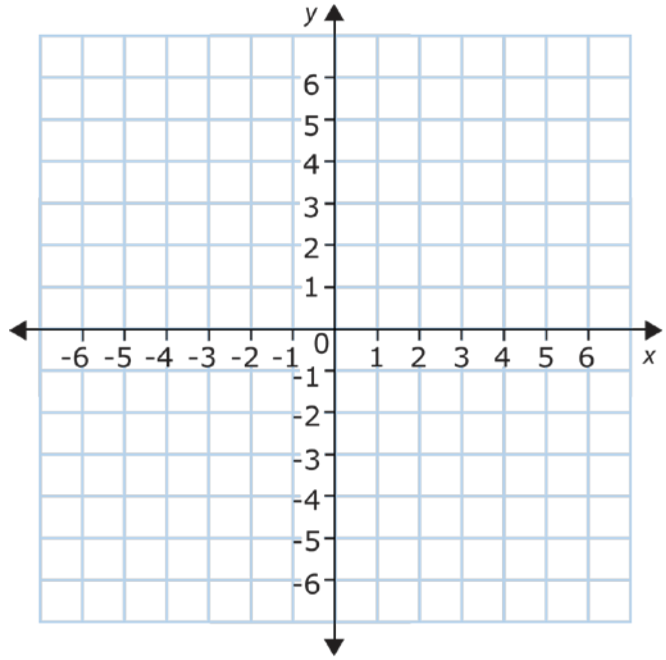


12. What will the graph of $y = 0$ look like?

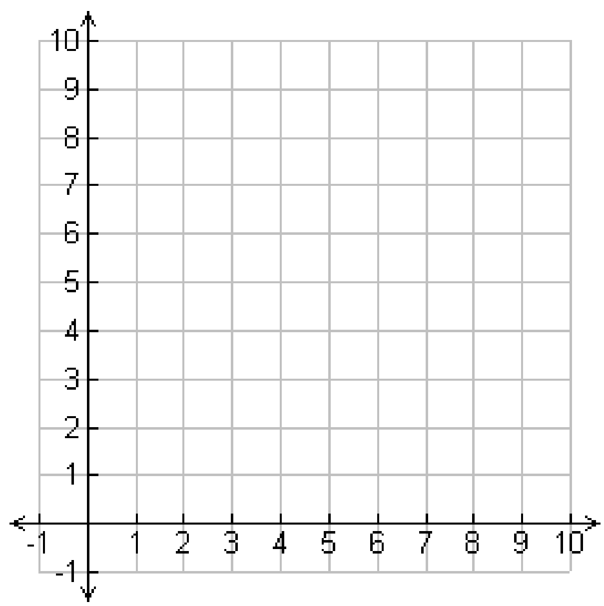
Summary Lesson 14:

Lesson 14 Independent Practice

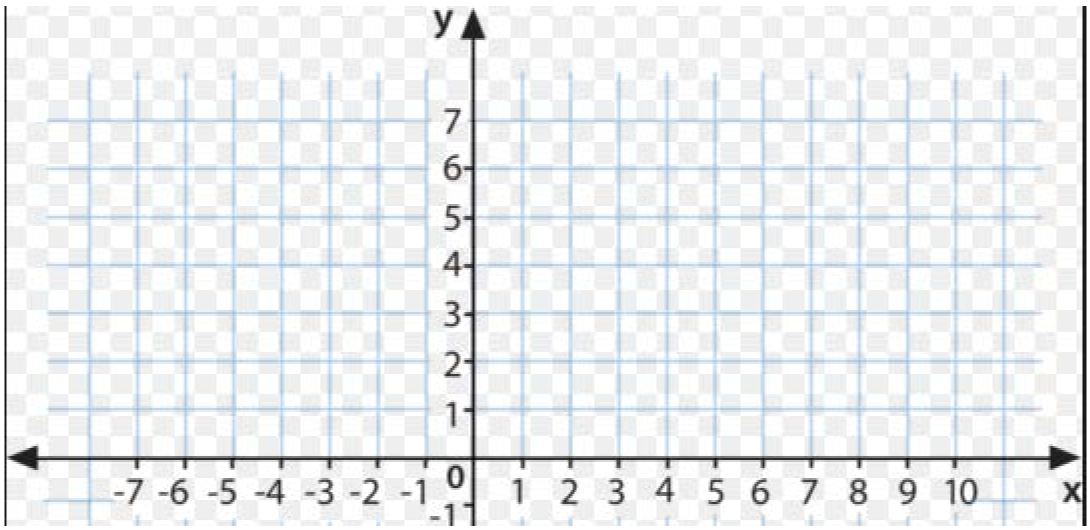
1. Graph the two-variable linear equation $ax + by = c$, where $a = 0$, $y = 1$, and $c = -4$.



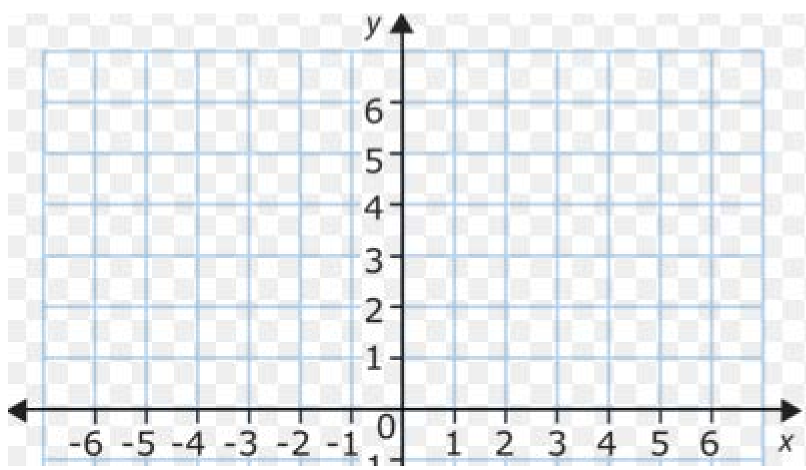
2. Graph the two-variable linear equation $ax + by = c$, where $a = 1$, $y = 0$, and $c = 9$.



3. Graph the linear equation $y = 7$.



4. Graph the linear equation $x = 1$.



5. Explain why the graph of a linear equation in the form of $y = c$ is the horizontal line, parallel to the x-axis passing through the point (0,c).

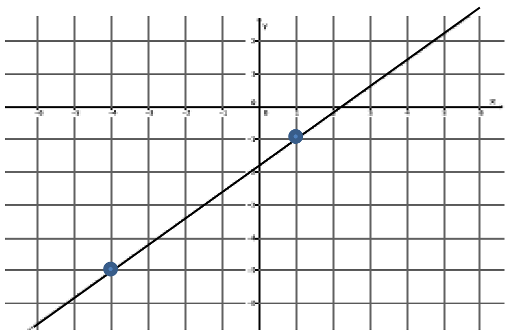
6. Explain why there is only one line with the equation $y = c$ that passes through the point (0, c).

Lesson 15 – The Slope of a Non-Vertical Line

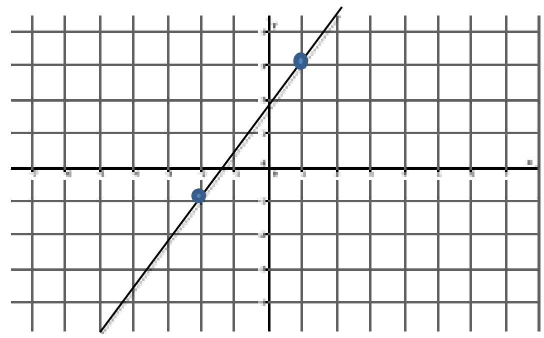
Essential Questions:

Opening Exercise:

Graph A



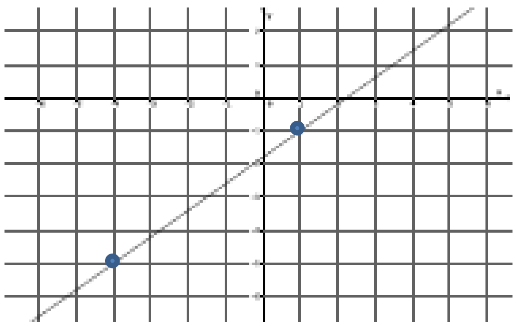
Graph B



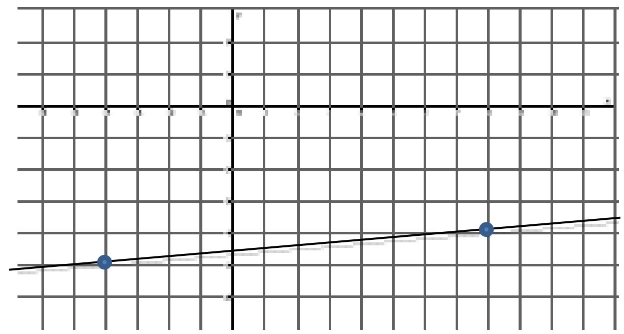
a. Which graph is steeper?	
b. Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.	
c. Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)	

Pair 1:

Graph A



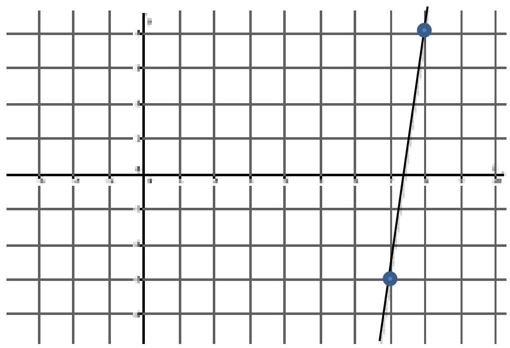
Graph B



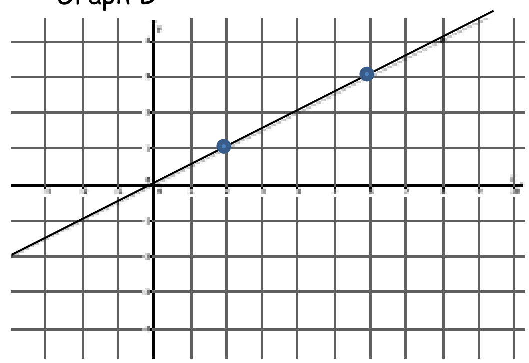
a. Which graph is steeper?	
b. Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.	
c. Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)	

Pair 2:

Graph A



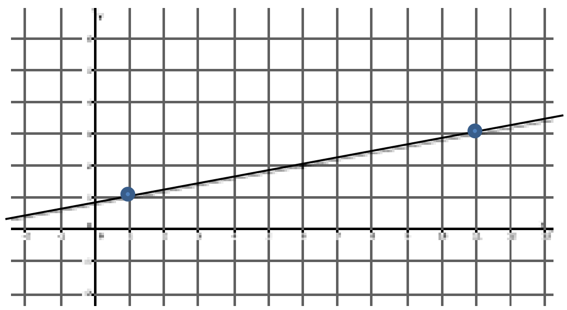
Graph B



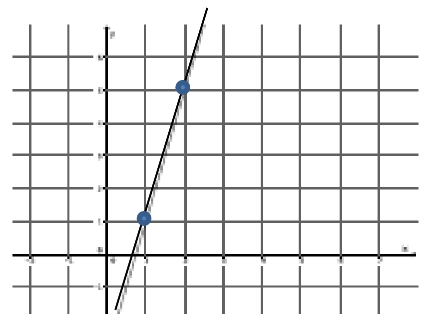
a. Which graph is steeper?	
b. Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.	
c. Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)	

Pair 3:

Graph A



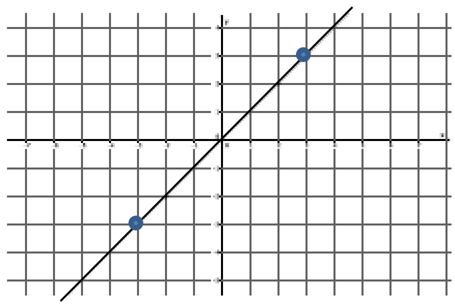
Graph B



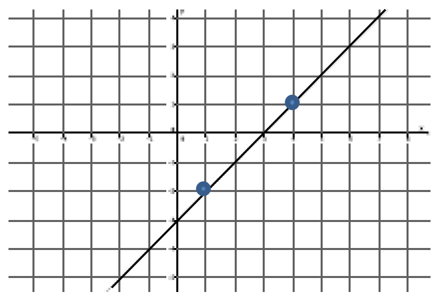
a. Which graph is steeper?	
b. Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.	
c. Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)	

Pair 4:

Graph A

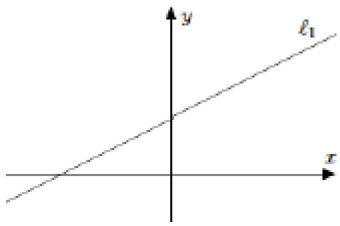


Graph B

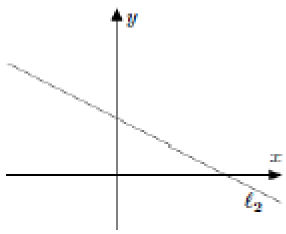


a. Which graph is steeper?	
b. Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.	
c. Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)	

Example 1 – two other types of non-vertical lines



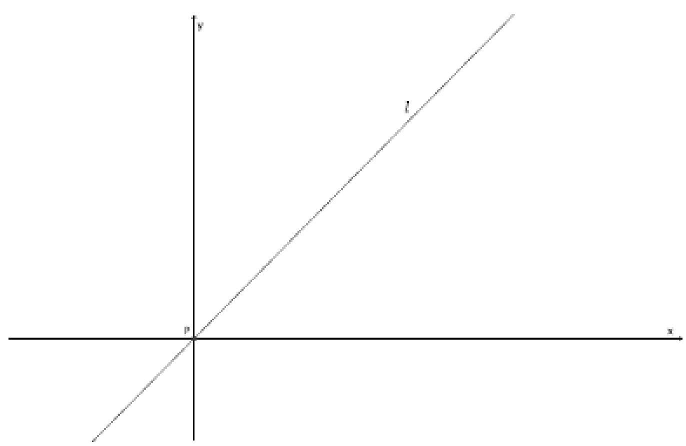
Left-to-right inclining



Left-to-right declining

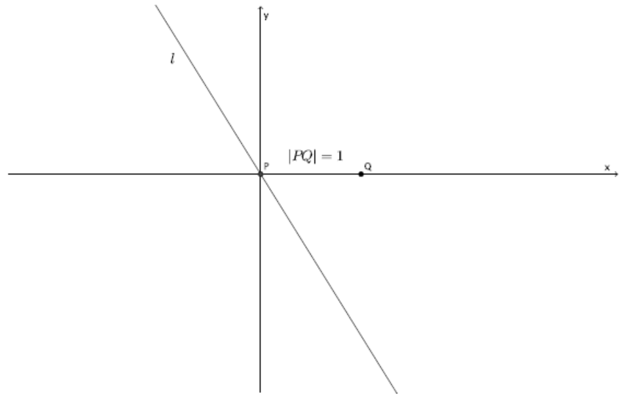
Example 2 – finding the number that will be the slope of the line

Describe slope for left to right inclining



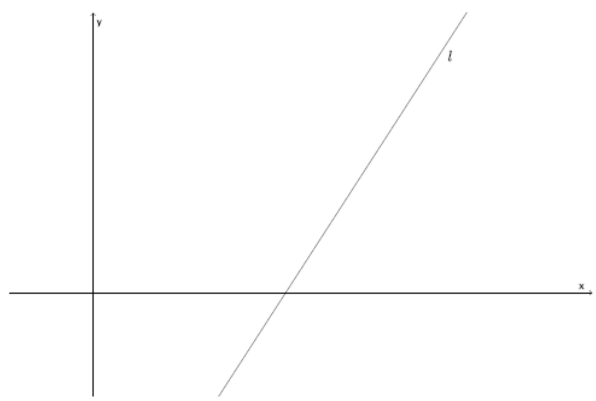
Example 3

Describe the slope for left to right declining:

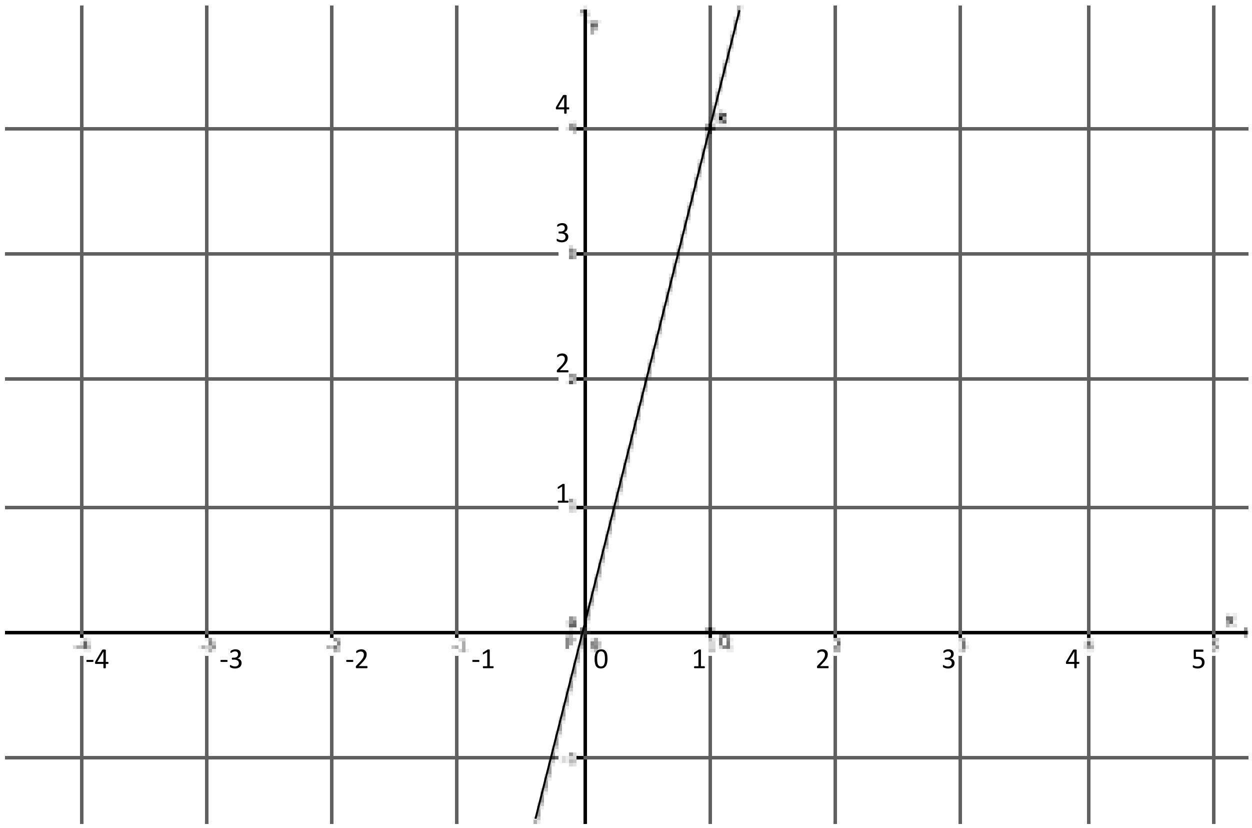


Example 4

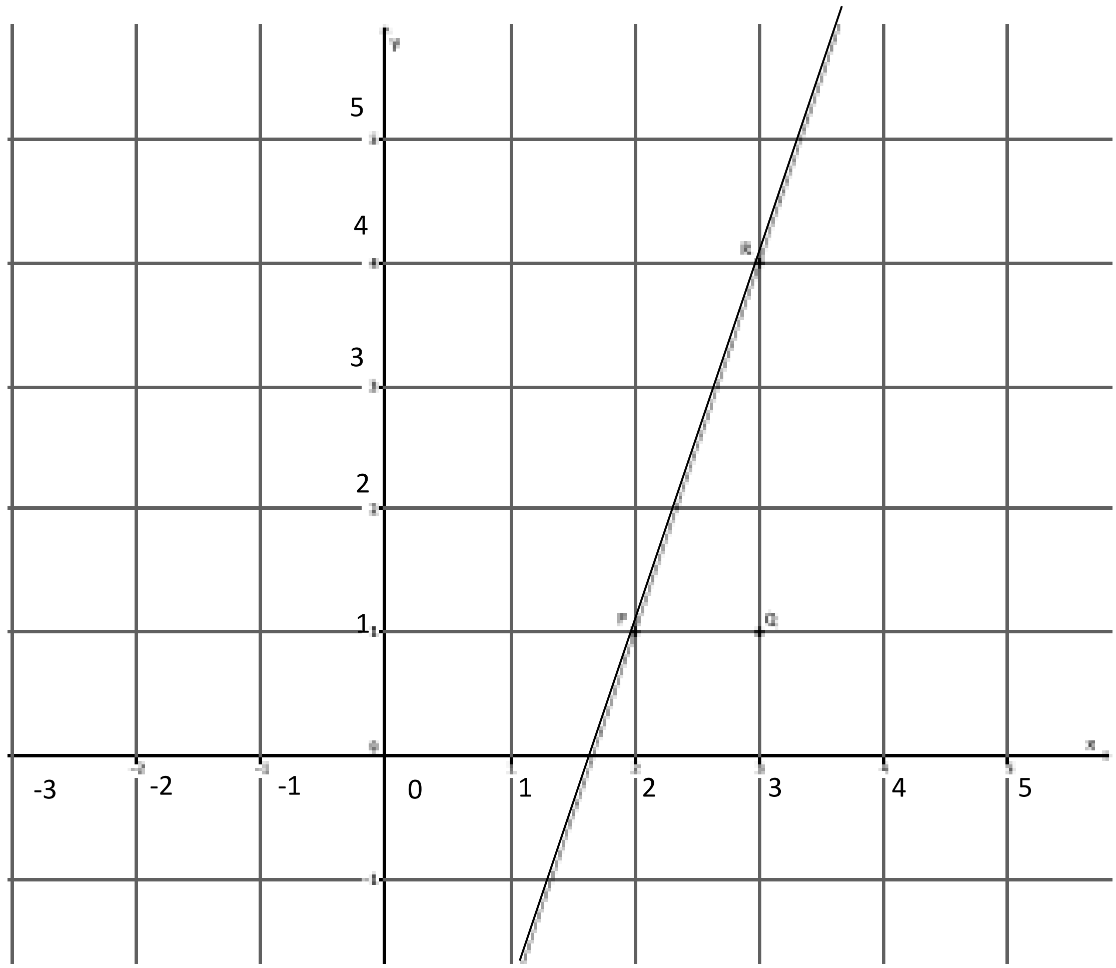
How do you describe the slope if the line does not go through the origin?



Exercises 1-6 – On your own



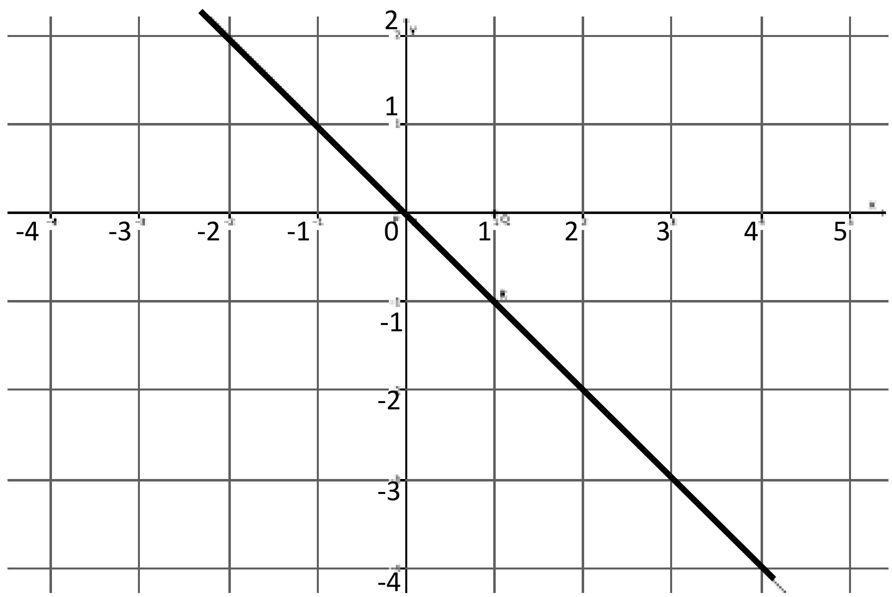
1. What is the slope of this non-vertical line?



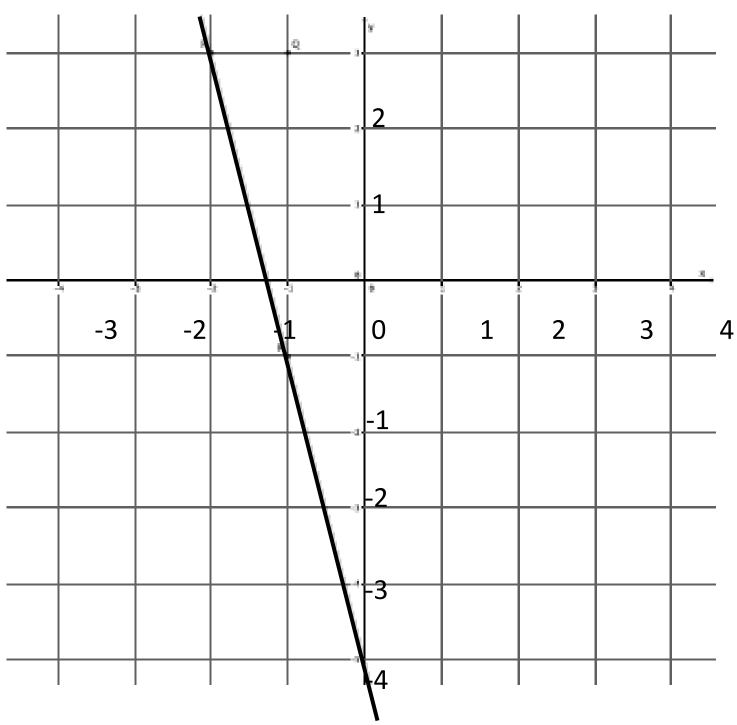
2. What is the slope of this non-vertical line?

3. Which of the lines in Exercises 1 and 2 is steeper? Compare the slopes of each of the lines. Is there a relationship between the steepness and slope?

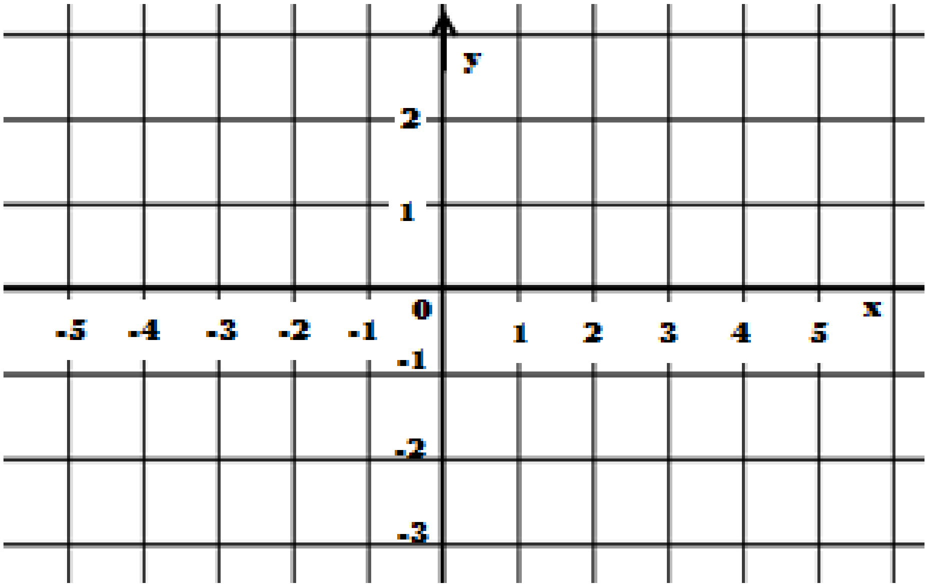
4. What is the slope of this non-vertical line?



5. What is the slope of this non-vertical line?



6. What is the slope of this non-vertical line?



Discussion:

What did you notice in exercise 6?	
In exercise 3, you were asked to compare the steepness of the graphs of two lines and then compare their slopes. What did you notice?	
Does the same relationship exist for lines with negative slopes? Look at Exercises 4 and 5.	
Generalize the idea of steepness.	

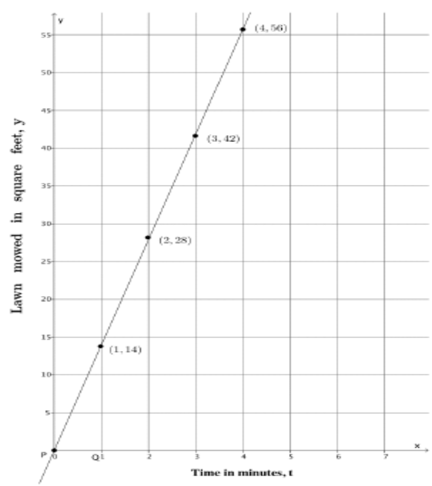
Example 5

Look again at the proportional relationships that we graphed in lesson 11. Here is the problem and the work that we did:

Pauline mows a lawn at a constant rate. Suppose she mowed a 35 sq.ft. lawn in 2.5 minutes.

t (time in minutes)	Linear equation: $y = \frac{35}{2.5}t$	y (area in square feet)
0	$y = \frac{35}{2.5}(0)$	0
1	$y = \frac{35}{2.5}(1)$	$\frac{35}{2.5} = 14$
2	$y = \frac{35}{2.5}(2)$	$\frac{70}{2.5} = 28$
3	$y = \frac{35}{2.5}(3)$	$\frac{105}{2.5} = 42$
4	$y = \frac{35}{2.5}(4)$	$\frac{140}{2.5} = 56$

Now, if we plot the points on the coordinate place and the origin of the graph is point P, Then we have the following graph:



What is the slope of this line? Explain.

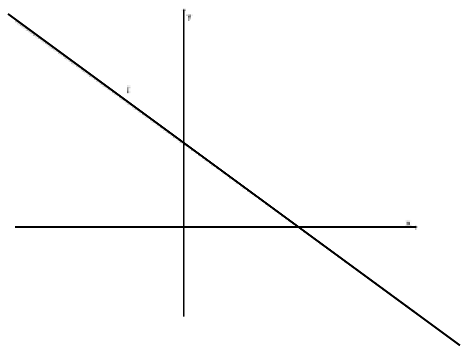
What is the rate of mowing the lawn?

This is a proportional relationship. What do you notice about the slope and the rate of change?

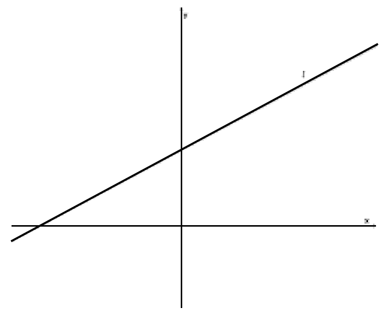
Summary:

Independent Practice:

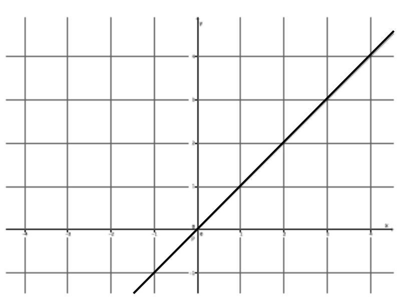
1. Does the graph of the line show a positive or a negative slope? Explain.



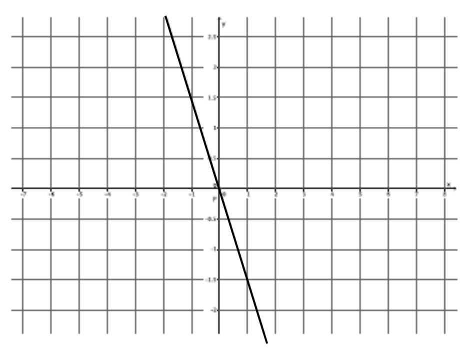
2. Does the graph of the line shown below have a positive or negative slope? Explain.



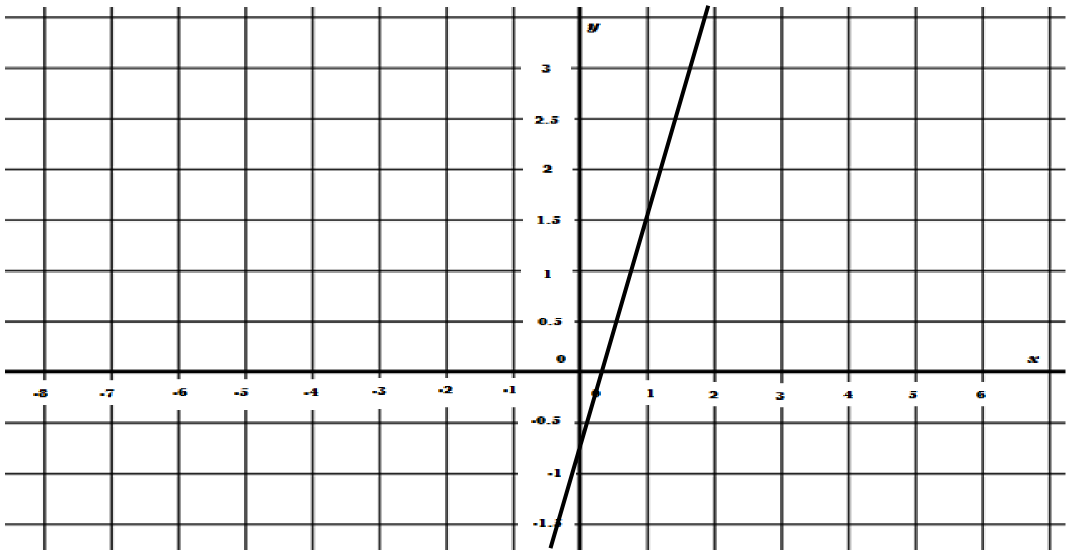
3. What is the slope of this non-vertical line?



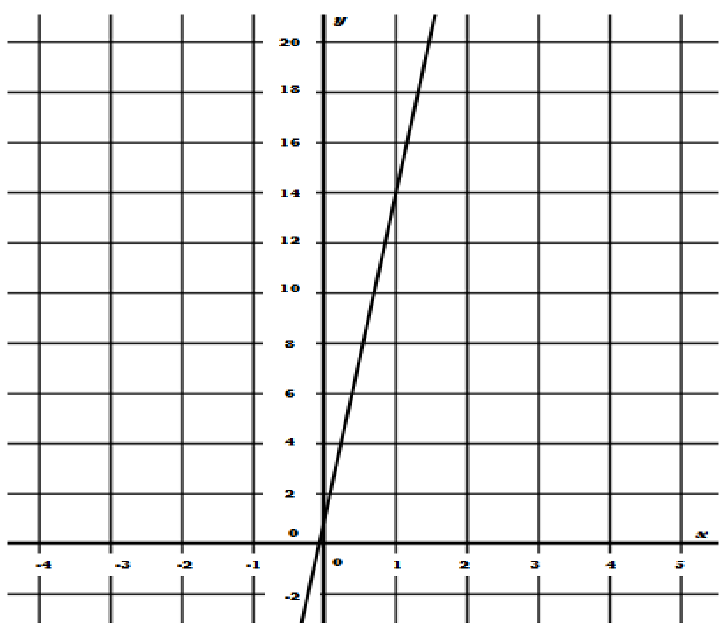
4. What is the slope of this non-vertical line?



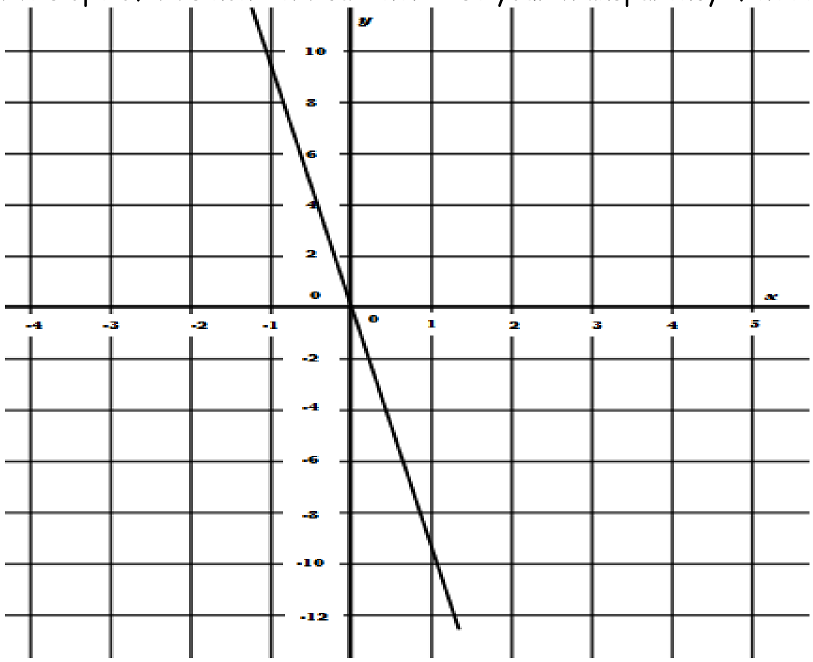
5. What is the slope of this non-vertical line? Use your transparency if needed.



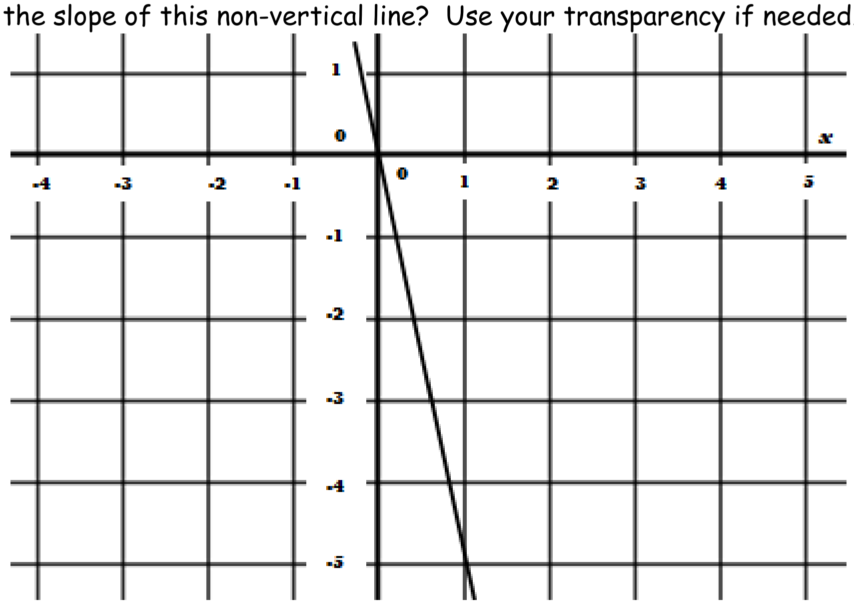
6. What is the slope of this non-vertical line? Use your transparency if needed.



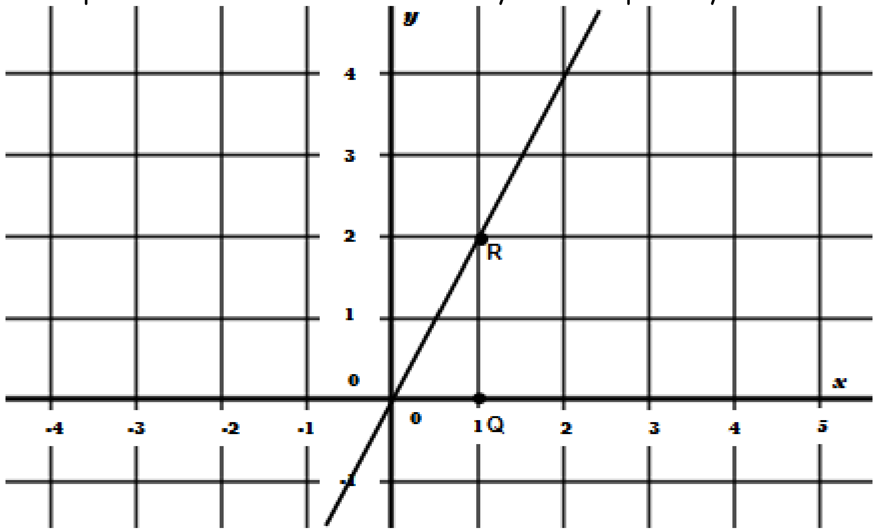
7. What is the slope of this non-vertical line? Use your transparency if needed.



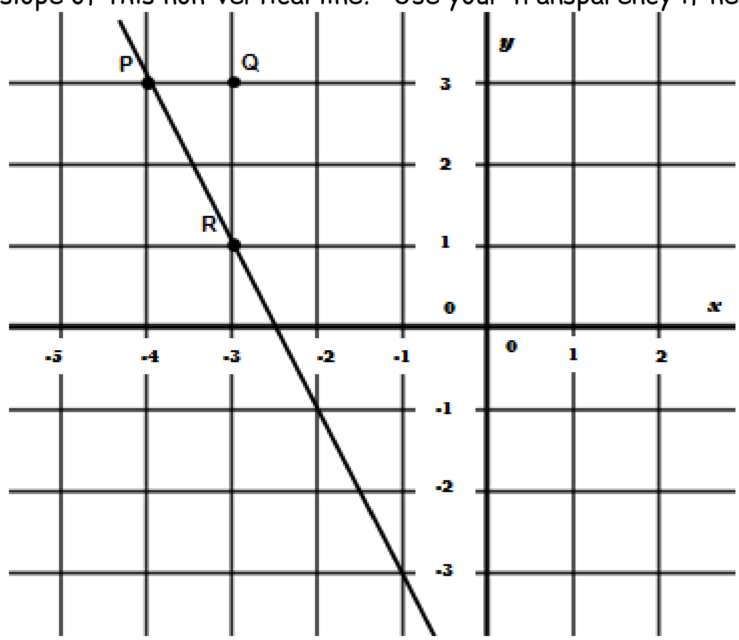
8. What is the slope of this non-vertical line? Use your transparency if needed.



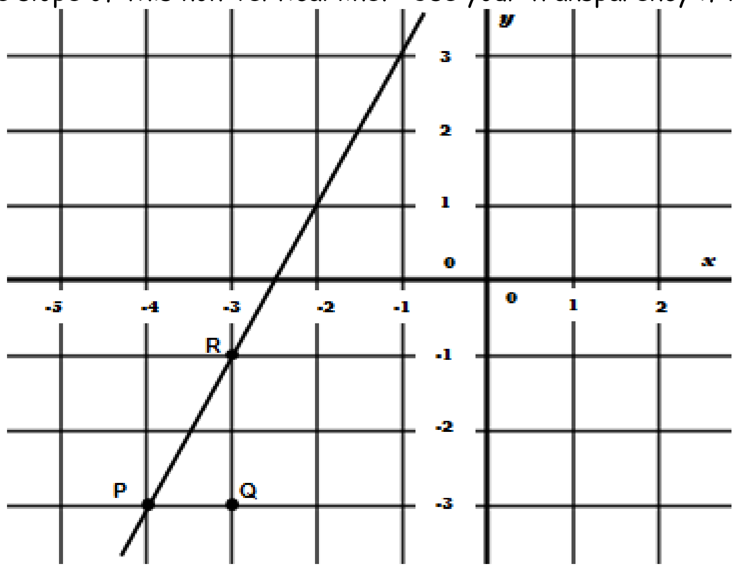
9. What is the slope of this non-vertical line? Use your transparency if needed.



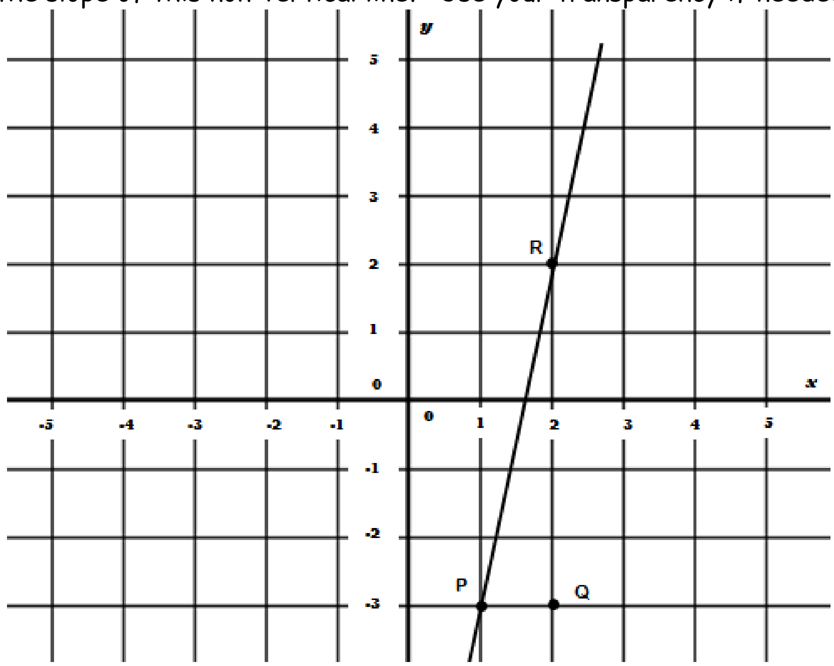
10. What is the slope of this non-vertical line? Use your transparency if needed.



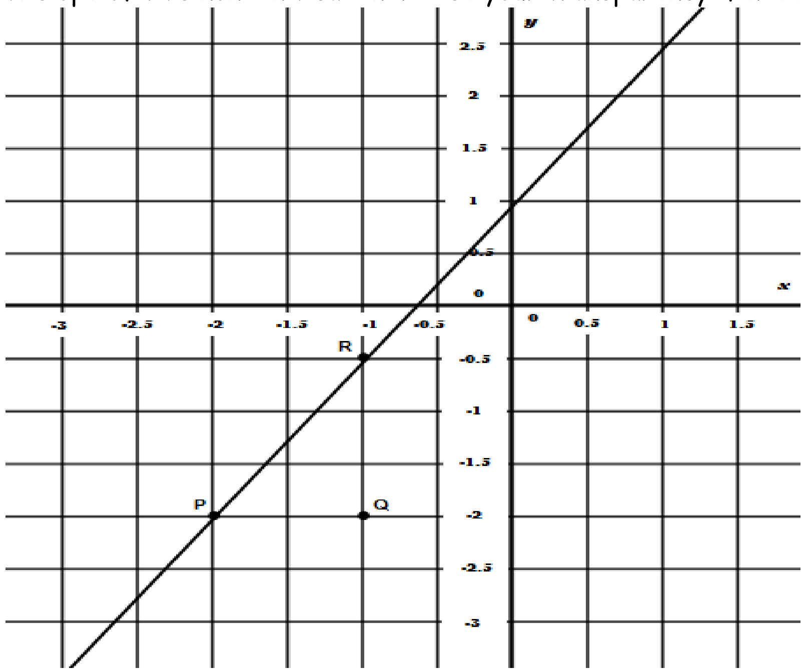
11. What is the slope of this non-vertical line? Use your transparency if needed.



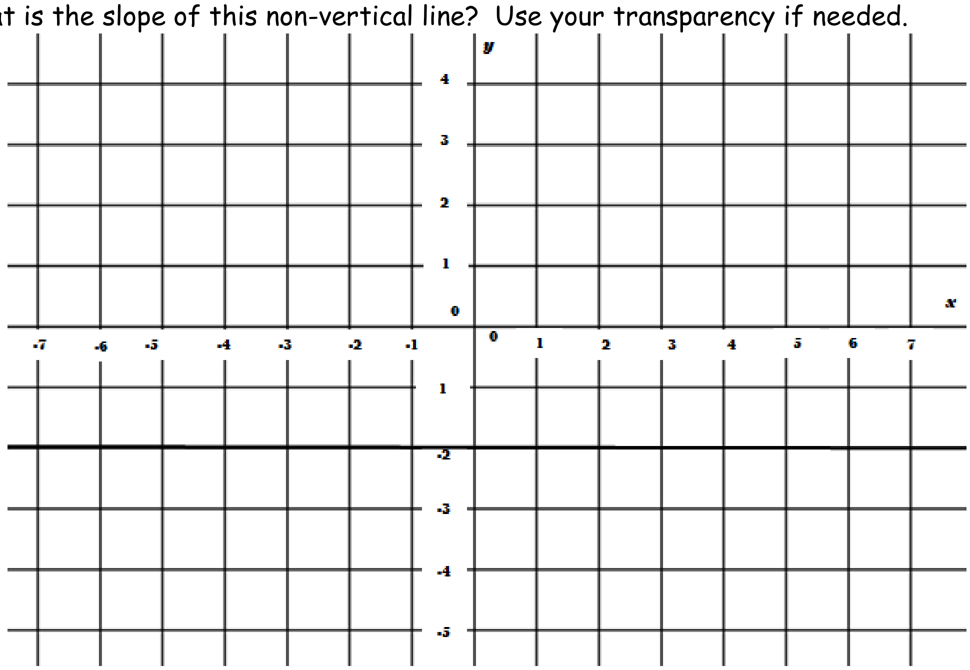
12. What is the slope of this non-vertical line? Use your transparency if needed.



13. What is the slope of this non-vertical line? Use your transparency if needed.



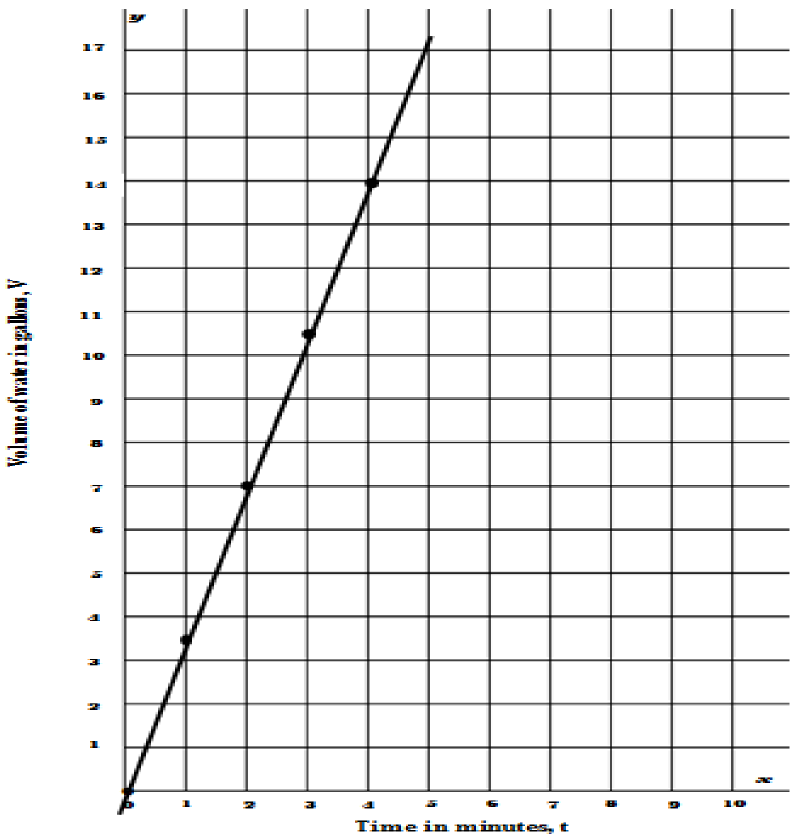
14. What is the slope of this non-vertical line? Use your transparency if needed.



15. In lesson 11, you did work below involving constant rate problems. Use the table and the graphs provided to answer the questions that follow.

Suppose the volume of water that comes out in 3 minutes is 10.5 gallons.

t (time in minutes)	Linear equation: $V = \frac{10.5}{3}t$	V (in gallons)
0	$V = \frac{10.5}{3}(0)$	0
1	$V = \frac{10.5}{3}(1)$	$\frac{10.5}{3} = 3.5$
2	$V = \frac{10.5}{3}(2)$	$\frac{21}{3} = 7$
3	$V = \frac{10.5}{3}(3)$	$\frac{31.5}{3} = 10.5$
4	$V = \frac{10.5}{3}(4)$	$\frac{42}{3} = 14$

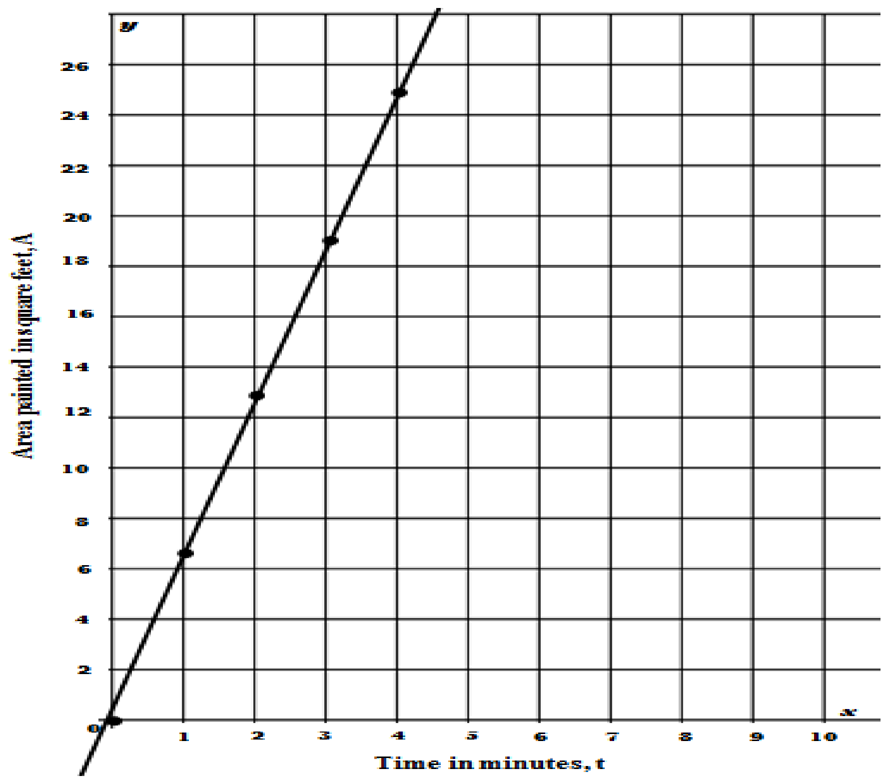


a. How many gallons of water flow out of the faucet per minute? In other words, what is the unit rate of water flow?

b. Assume that the graph of the situation is a line, as shown in the graph. What is the slope of the line?

16. Emily paints at a constant rate. She can paint 32 square feet in five minutes.

t (time in minutes)	Linear equation: $A = \frac{32}{5}t$	A (area painted in square feet)
0	$A = \frac{32}{5}(0)$	0
1	$A = \frac{32}{5}(1)$	$\frac{32}{5} = 6.4$
2	$A = \frac{32}{5}(2)$	$\frac{64}{5} = 12.8$
3	$A = \frac{32}{5}(3)$	$\frac{96}{5} = 19.2$
4	$A = \frac{32}{5}(4)$	$\frac{128}{5} = 25.6$

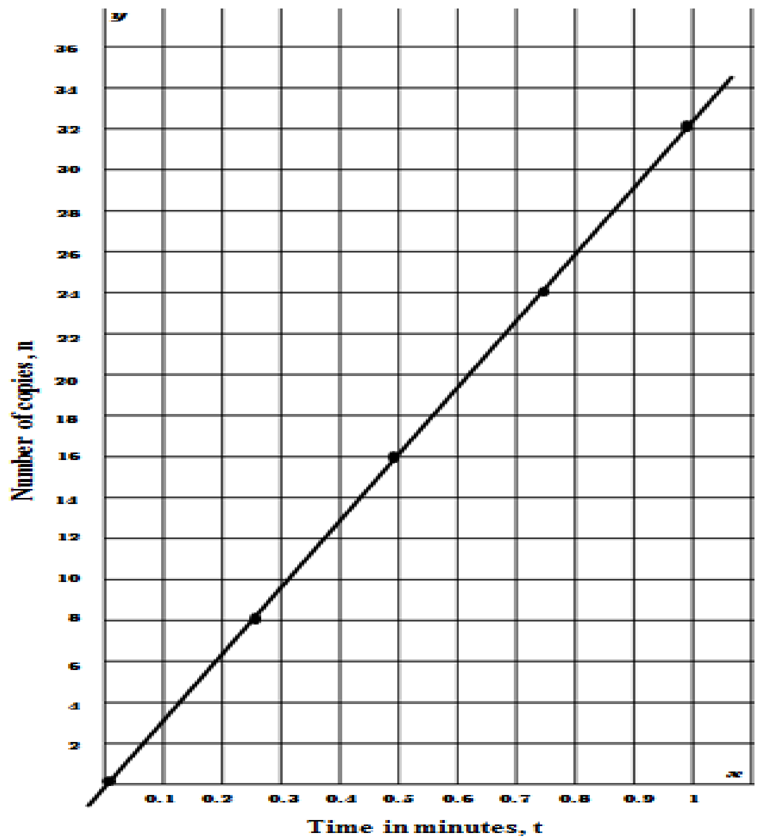


a. How many square feet can Emily paint in one minute? In other words, what is her unit rate of painting?

b. Assume that the graph of the situation is a line, as shown in the graph. What is the slope of the line?

17. A copy machine makes copies at a constant rate. The machine can make 80 copies in $2\frac{1}{2}$ minutes.

t (time in minutes)	Linear equation: $n = 32t$	n (number of copies)
0	$n = 32(0)$	0
0.25	$n = 32(0.25)$	8
0.5	$n = 32(0.5)$	16
0.75	$n = 32(0.75)$	24
1	$n = 32(1)$	32



a. How many copies can the machine make each minute? In other words, what is the unit rate of the copy machine?

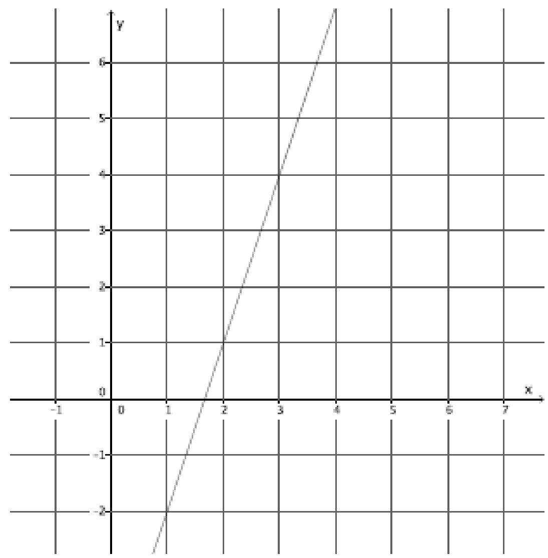
b. Assume that the graph of the situation is a line, as shown in the graph. What is the slope of the line?

Lesson 16: The Computation of the Slope of a Non-Vertical Line

Essential Questions:

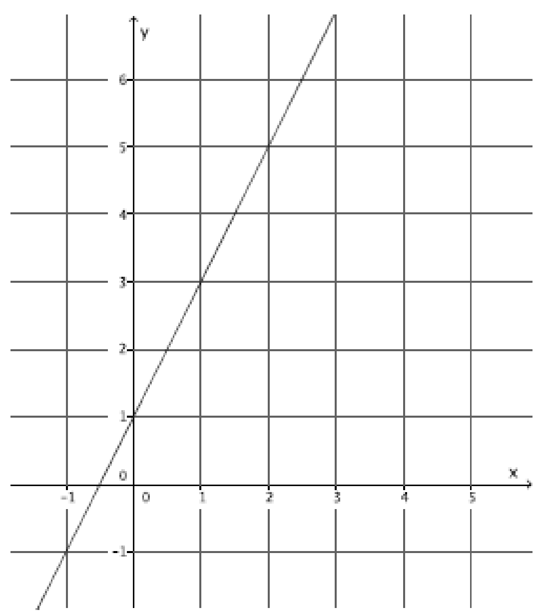
Example 1:

Using what you learned in the last session, determine the slope of the line with the following graph:



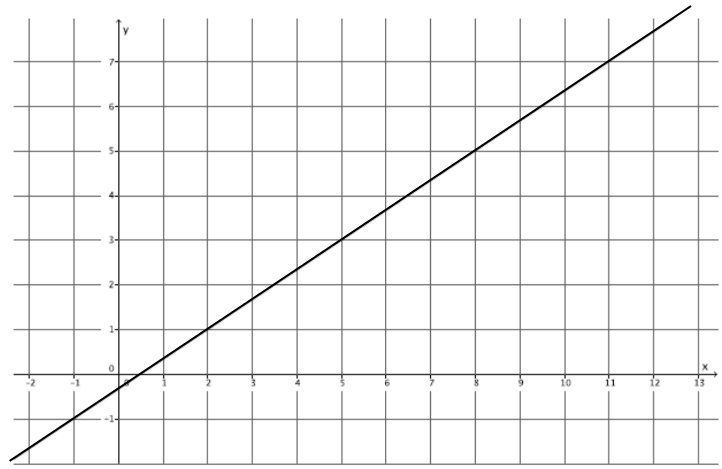
Example 2:

Using what you learned in the last session, determine the slope of the line with the following graph:



Example 3:

What is different about this line compared to the last two examples?

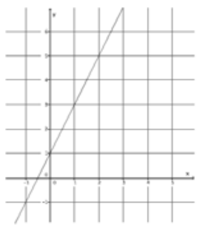


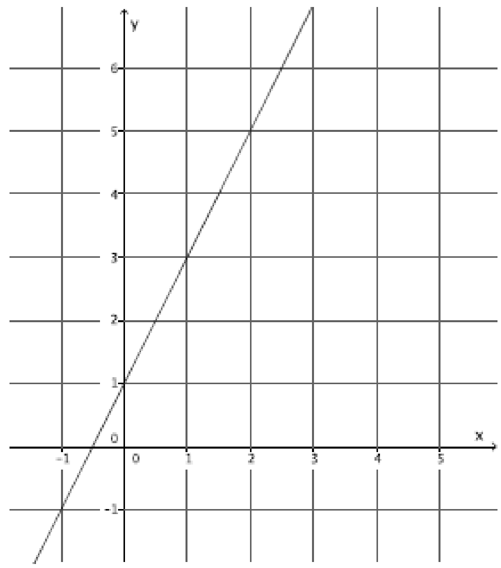
Discussion:

Based on what we learned in the last lesson, what do we know about unit rate of a problem and the slope?

Discussion:

Let's take a closer look at example 2:





Are there points on the line with integer coordinates that we could use to help us determine the slope of the line?

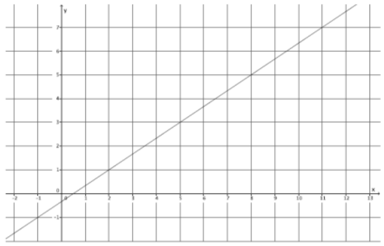
What is the ratio of one of the slope triangles?

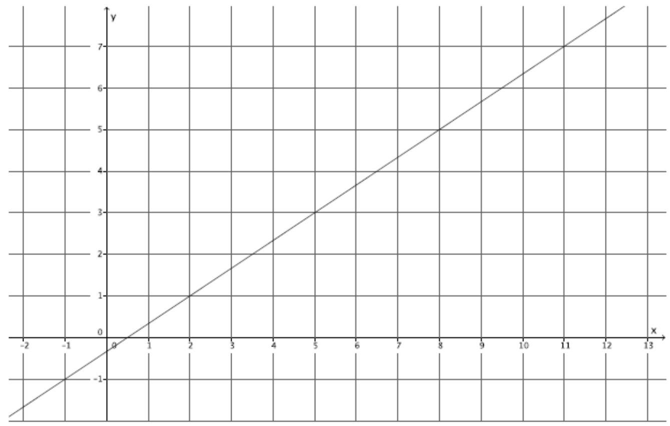
Try a different slope triangle. What is the ratio of the larger slope triangle?

What do you notice about the ratio of the smaller slope triangle and the ratio of the larger slope triangle?

Are the slope triangles in this diagram similar? How do you know?

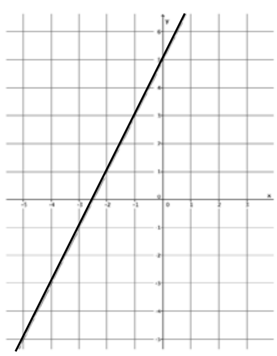
Let's look again at Example 3:

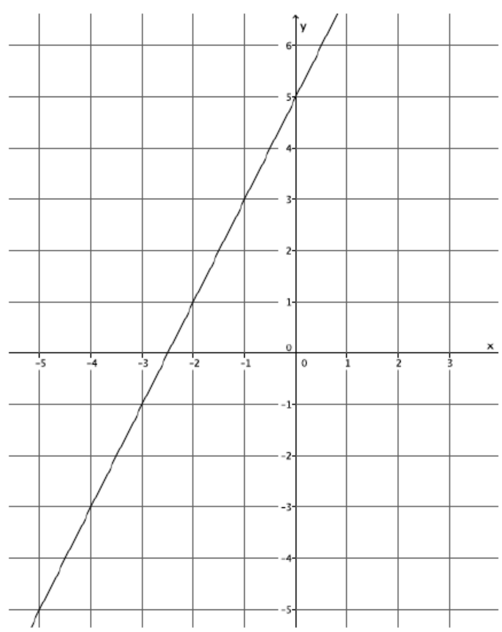




What do you think the slope of the line is? Explain.

Exercise 1





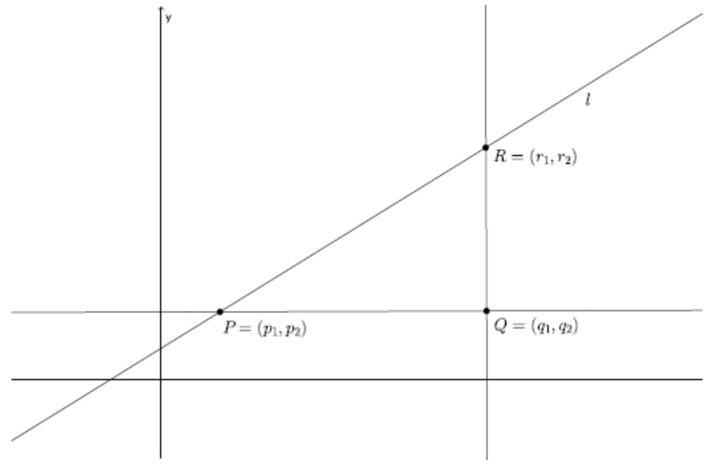
a. Select any two points on the line to label as P and R.

b. Identify the coordinates of point P and R.

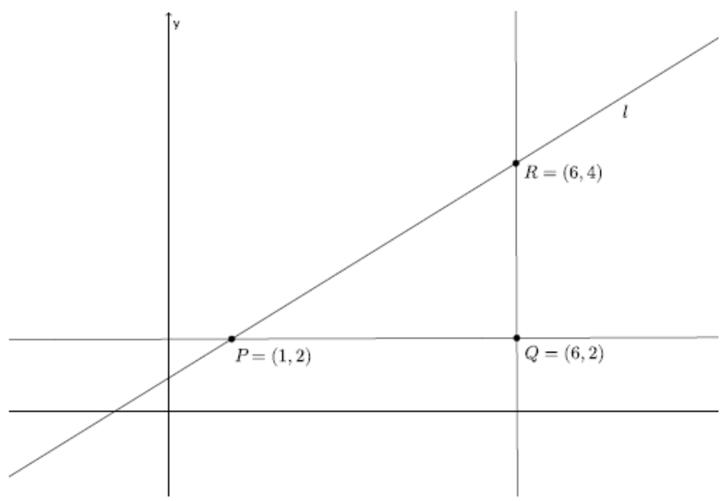
c. Find the slope of the line using as many different points as you can. Identify the points and show your work to the right.

Discussion

Can the slope of the line be found using any two points?
How do you know?



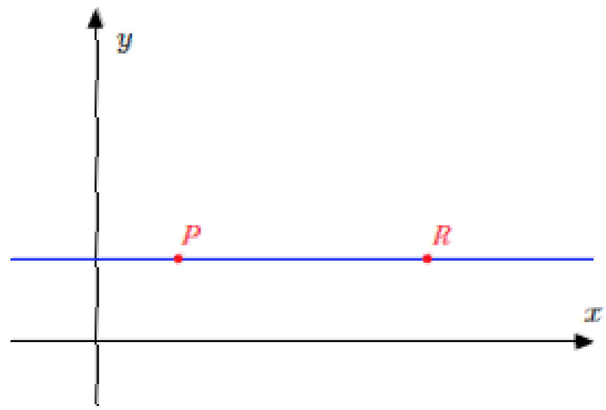
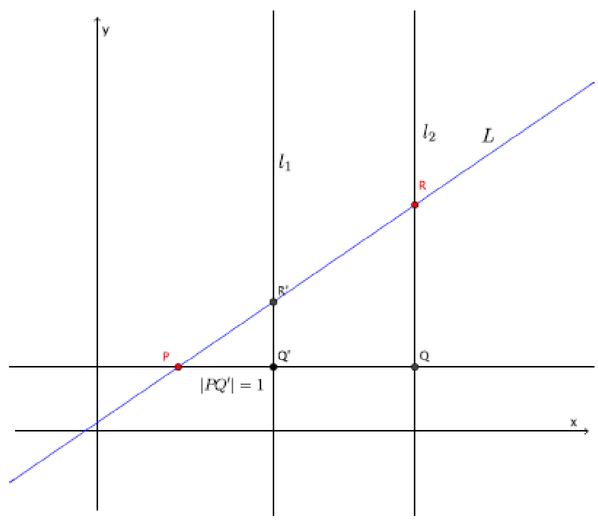
Let's look at an example with coordinates rather than variables:



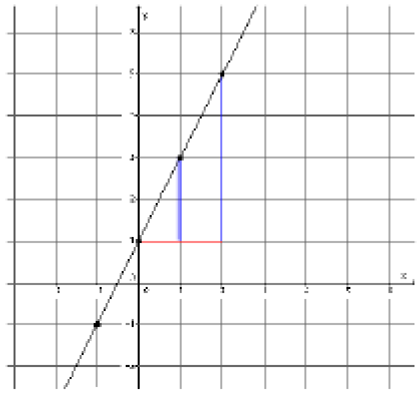
What do you notice about the y-coordinates of points P and Q?

What do you notice about the x-coordinates of points R and Q?

By substitution we can show the slope formula as:

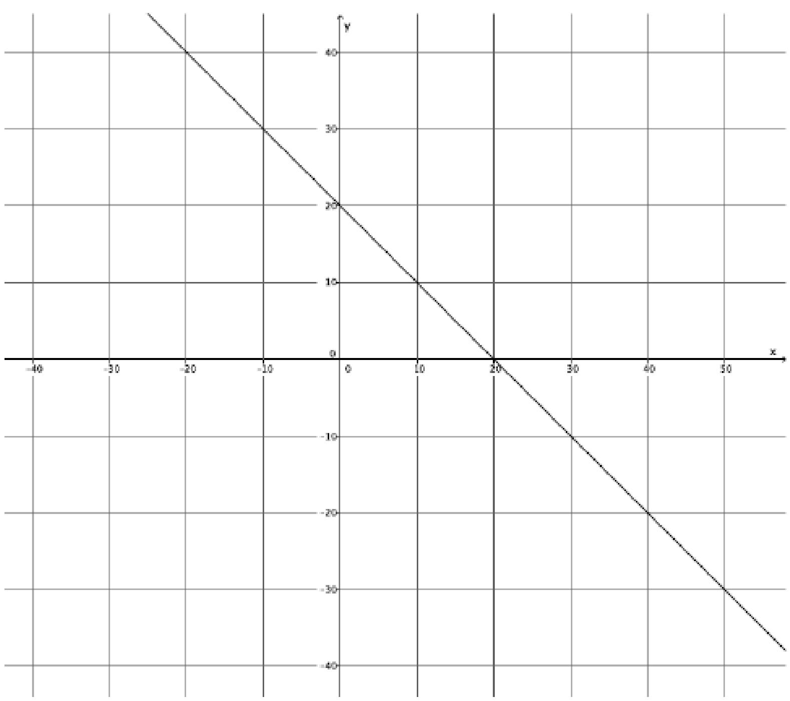
Translate the slope formula into words.	
Suppose we are given a horizontal line. Based on our work in the last lesson, what do we expect the slope of this line to be? Determine the slope of the line using the slope formula	
Now for the general case. We want to show that we can choose any two points P and R to find the slope, not just a point like R' where we have fixed the horizontal distance at 1.	

Lesson 16 Summary: Us the diagram below to help you summarize today's lesson.

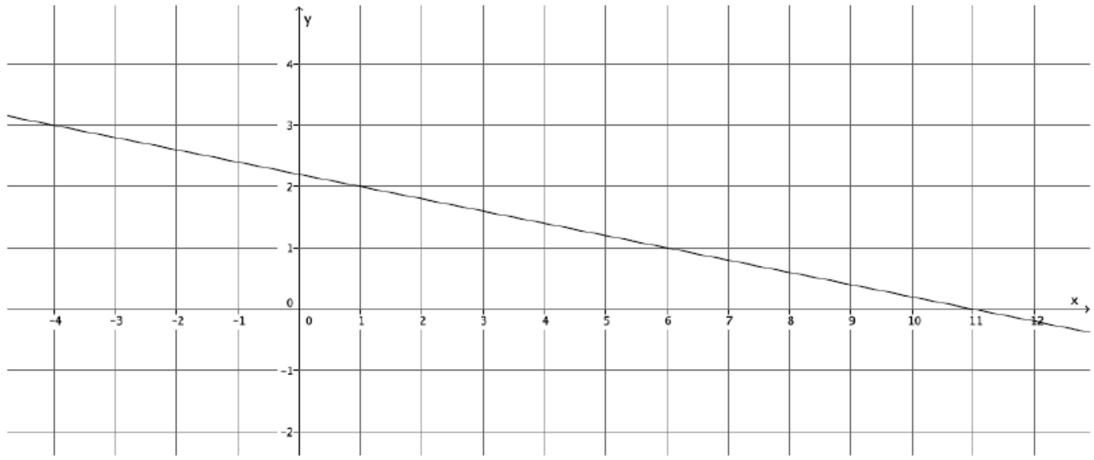


Independent Practice:

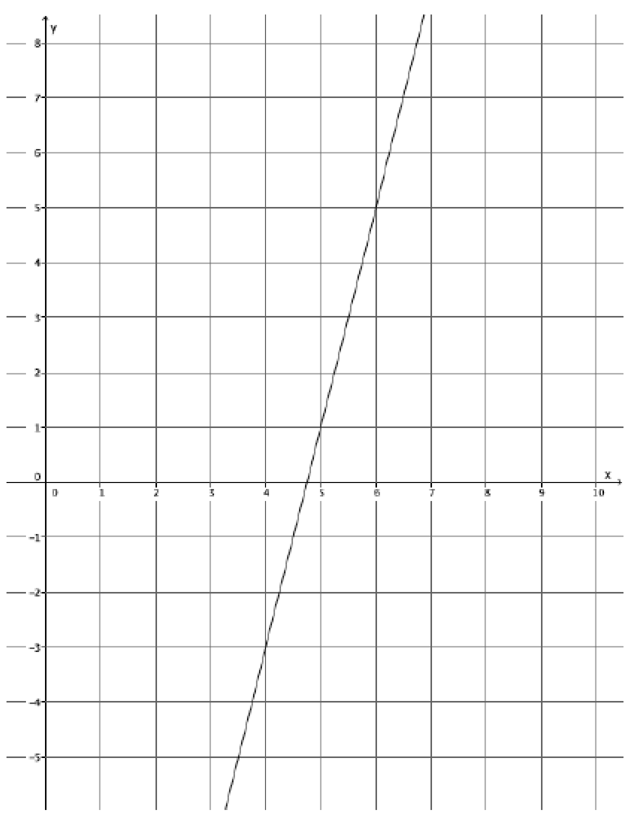
1. Calculate the slope of the line using two different pairs of points. Show your work.



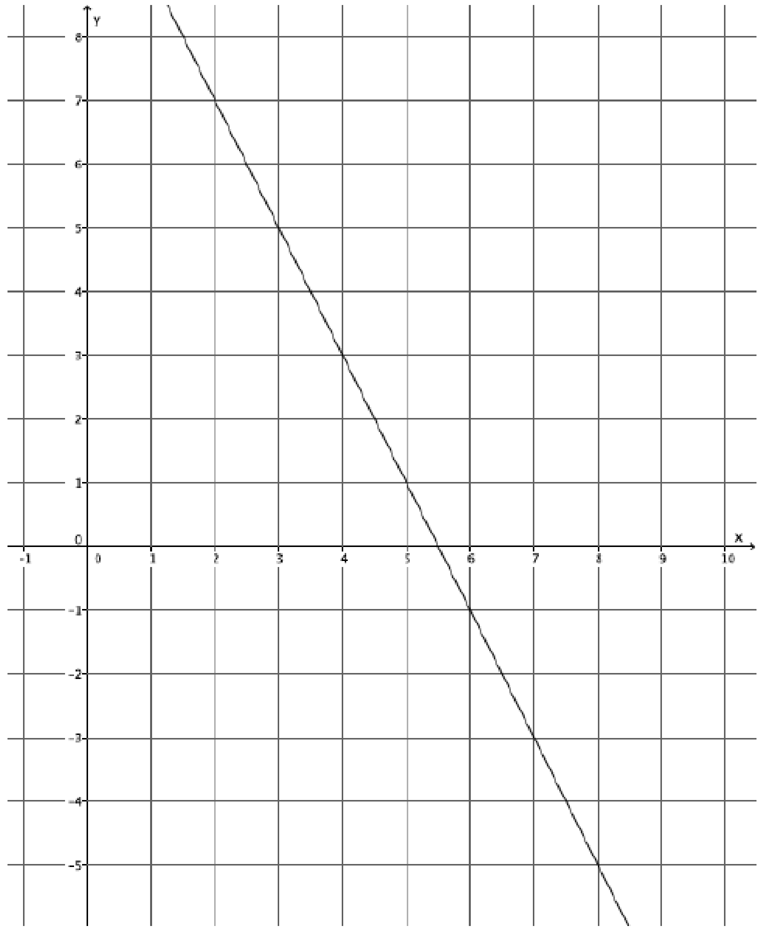
2. Calculate the slope using two different pairs of points. Show your work



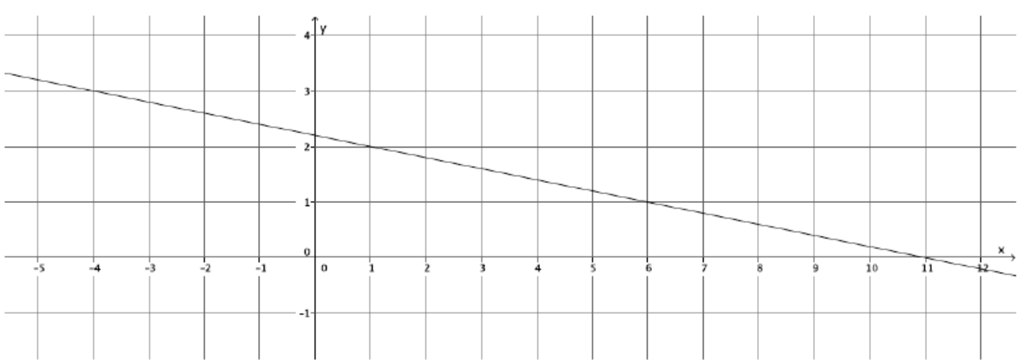
3. Calculate the slope of the line using two different pairs of points. Show your work.



4. Calculate the slope of the line using two different pairs of points. Show your work.

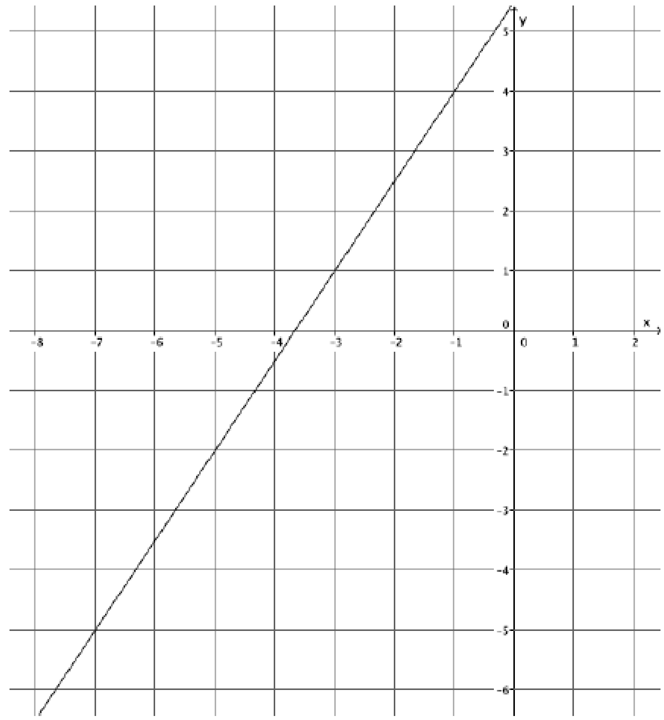


5. Calculate the slope of the line using two different pairs of points. Show your work



6. Calculate the slope of the line using two different pairs of points.

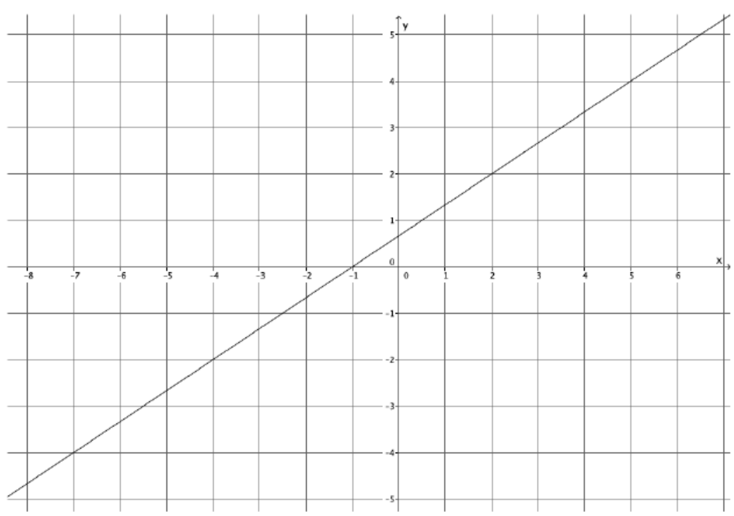
a. Select any two points on the line to compute the slope.



b. Select two different points on the line to calculate the slope.

c. What do you notice about your answers in parts (a) and (b)? Explain:

7. Calculate the slope of the line in the graph below. Show your work.



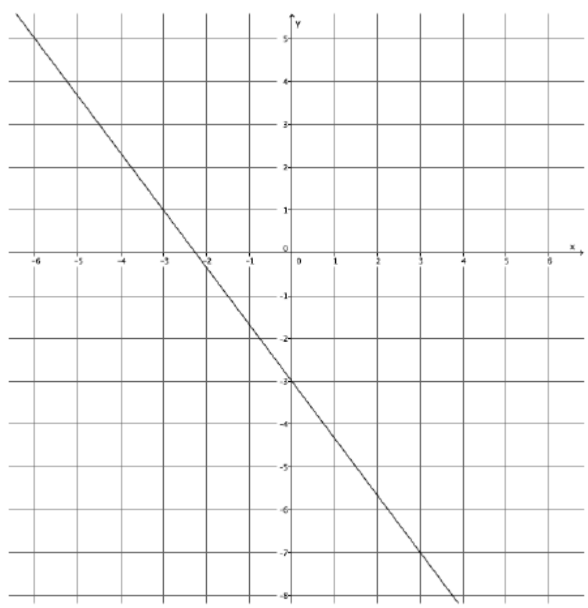
8. Your teacher tells you that a line goes through the points (-6, $\frac{1}{2}$) and (-4, 3).

a. Calculate the slope of this line.

b. Do you think the slope will be the same if the order of the points is reversed? Verify by calculating the slope and explain your result.

9. Use the graph to complete parts A – C.

a. Select any two points on the line to calculate the slope.

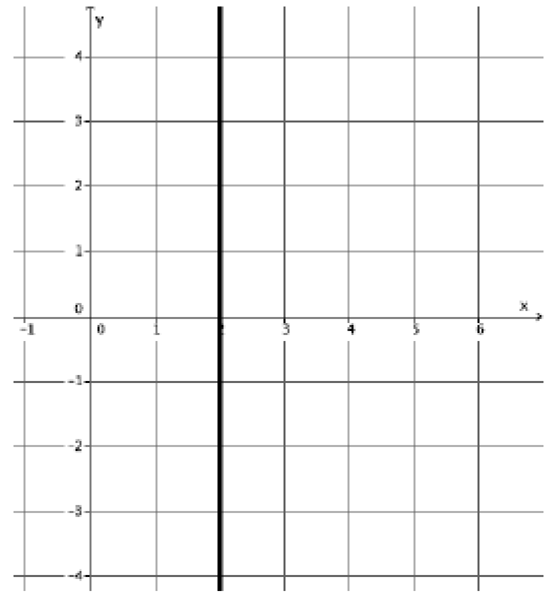


b. Compute the slope again, this time reversing the order of the coordinates.

c. What do you notice about the slopes you computed in parts A and B?

d. Why do you think you got the result you did in part C?

10. Each of the lines in the lesson was non-vertical. Consider the slope of a vertical line, $x = 2$. Select any two points on the line to calculate the slope. Based on your answer, why do you think the slope focuses only on non-vertical lines?



Challenge:

11. A certain line has a slope of $\frac{1}{2}$. Name two points that may be on the line.

Lesson 17: The Line Joining Two distinct Points of the Graph $y=mx+b$ has Slope m

Essential Questions:

Exercises 1-3: Work independently

Exercise 1

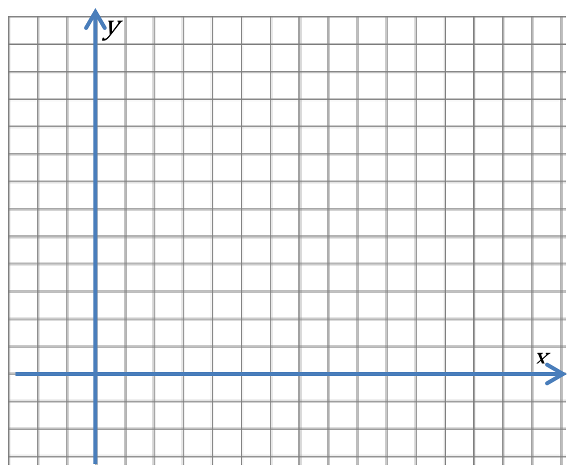
Find at least three solutions to the equation

$y=2x$,

and graph the solutions as points on the coordinate plane.

Connect the points to make a line.

Find the slope of the line.



Exercise 2

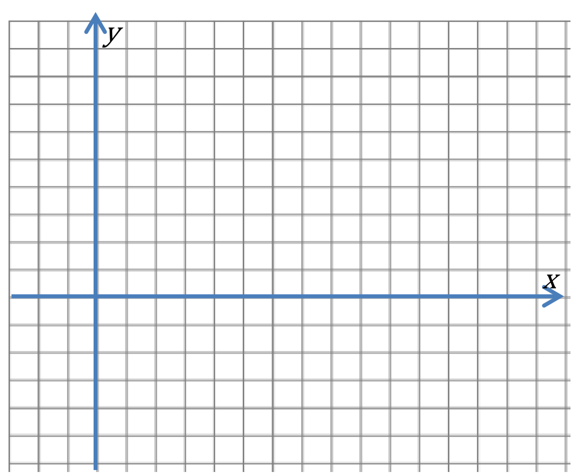
Find at least three solutions to the equation

$y = 3x - 1$,

and graph the solutions as points on the coordinate plane.

Connect the points to make a line.

Find the slope of the line.



Exercise 3

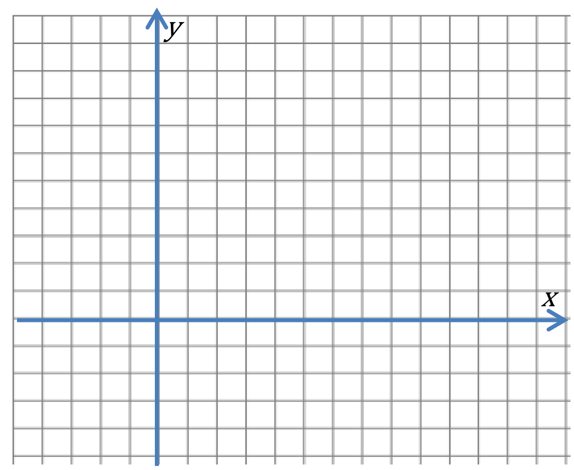
Find at least 3 solutions to the equation

Y = 3x + 1

And graph the solutions as points on the coordinate plane.

Connect the points to make a line.

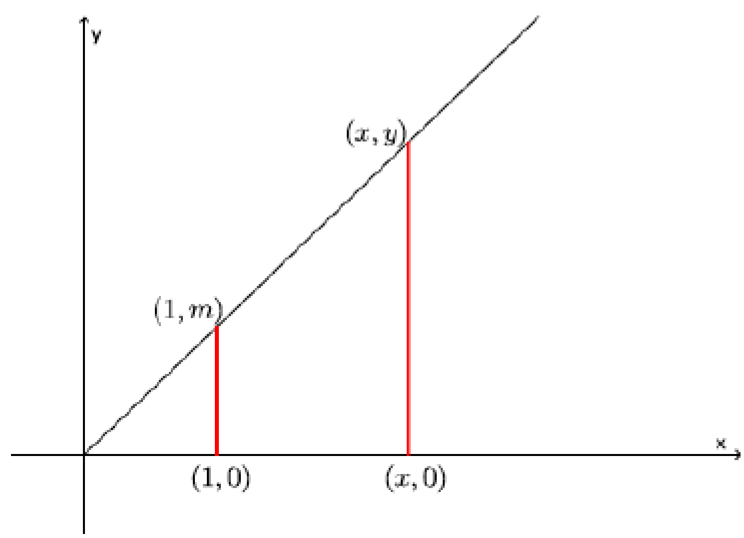
Find the slope of the line.



Discussion – proof of slope

Recall facts about slope from lesson 15.

Examine the diagram and think of how we could prove that $\frac{y}{m} = \frac{x}{1}$



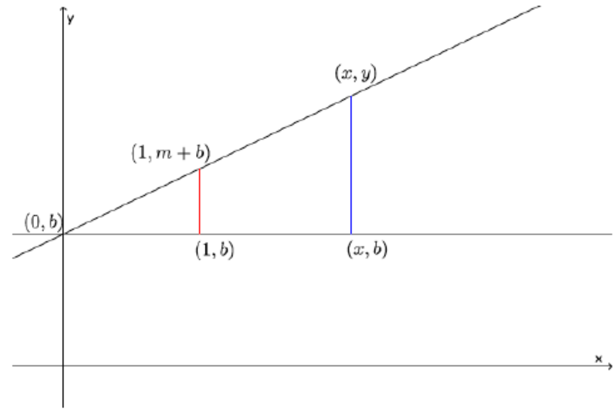
Do we have similar triangles? Explain

What is the slope of the line? Explain.

Write the ratio of the corresponding sides. Then, solve for y.

Therefore, the slope of the graph …

How does the diagram to the right compare to the graph of y=mx that we just worked on?



Generalize the slope any two point of the graph of
y = 3x-1

Our goal, ultimately, is to prove that the graph of a linear equation is a line

Discussion

When an equation is in the form
$y = mx + b$
The slope is easily identifiable

If the equation is in the form
$ax + by = c$
what should we do?

Solve the following equation for y
$8x + 2y = 6$

Exercises 4-11 – work independently

4. The graph of the equation

$y = 7x - 3$

has what slope?

5. The graph of the equation

$y = -\frac{3}{4}x - 3$

has what slope?

6. You have \$20 savings at the bank. Each week you add \$2 to your savings. Let y represent the total amount of money you have saved at the end of x weeks. Write an equation to represent this situation. Identify the slope of the equation. What does this number represent?

7. A friend is training for the marathon. She can run 4 miles in 28 minutes. Assume she runs at a constant rate. Write an equation to represent the total distance, y, your friend can run in x minutes. Identify the slope of the equation. What does that number represent?	
8. Four boxes of pencils cost \$5. Write an equation that represents the total cost, y, for x boxes of pencils. What is the slope of the equation? What does that number represent?	
9. Solve the following equation for y, and then identify the slope of the line: $9x - 3y = 15$	
10. Solve the following equation for y, and then identify the slope of the line: $5x + 9y = 8$	
11. Solve the following equation for y, and then identify the slope of the line: $ax + by = c$	

Lesson 17 Summary

Independent work – Lesson 17

1. Solve the following equation for y: $-4x + 8y = 24$ (put the equation into $y = mx + b$ format). Then answer the questions that follow.

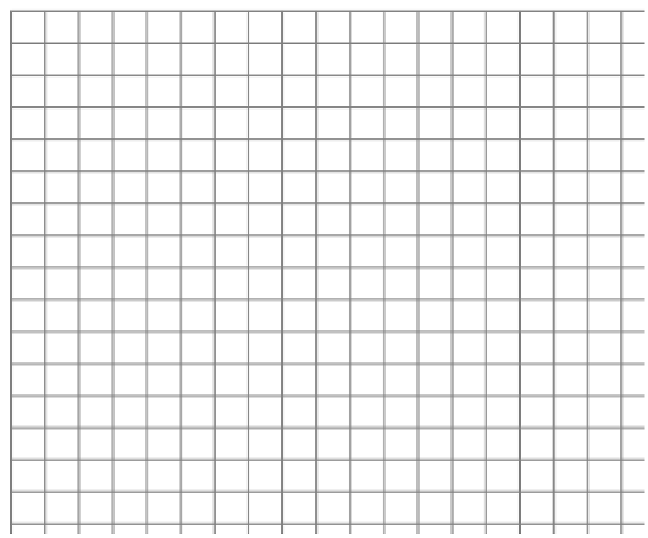
____

a. Based on the transformed equation, what is the slope of the linear equation?

b. Complete the table to find solutions to the linear equation:

x	Transformed linear equation	y
-2		
0		
2		
4		

c. Graph the points on the coordinate plane.



d. Find the slope between any two points.

e. The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of the equation.

f. Note the location (ordered pair) that describes where the line intersects the y-axis.

2. Solve the following equation for y: $9x + 3y = 21$ (put the equation into $y = mx + b$ format). Then answer the questions that follow.

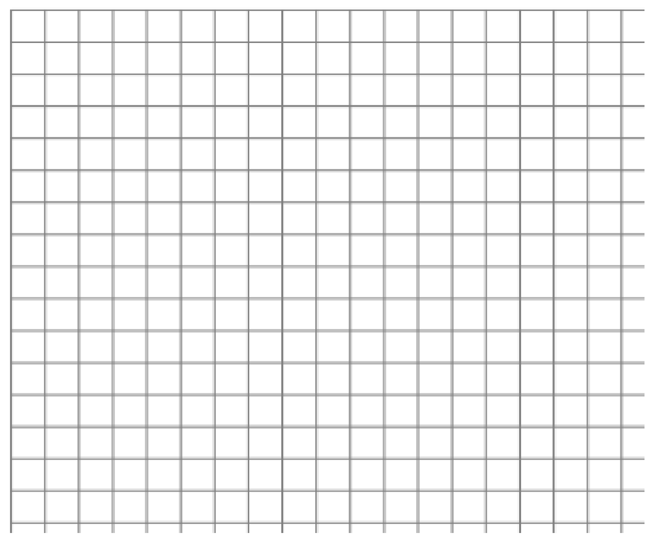
---

a. Based on the transformed equation, what is the slope of the linear equation?

b. Complete the table to find solutions to the linear equation:

x	Transformed linear equation	y
-1		
0		
1		
2		

c. Graph the points on the coordinate plane.



d. Find the slope between any two points.

e. The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of the equation.

f. Note the location (ordered pair) that describes where the line intersects the y-axis.

3. Solve the following equation for y: $2x + 3y = 6$. Then answer the questions that follow.

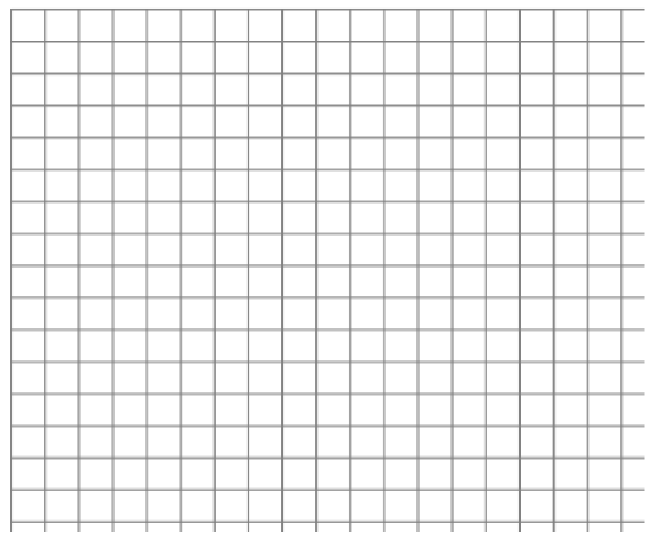
____

a. Based on your transformed equation, what is the slope of the equation>

b. Complete the table to find solutions to the linear equation.

x	Transformed linear equation	y
-6		
-3		

c. Graph the points on the coordinate plane.



d. Find the slope between any two points.

e. The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of the equation.

f. Note the location (ordered pair) that describes where the line intersects the y-axis.

4. Solve the following equation for y: $5x - y = 4$. Then, answer the questions that follow.

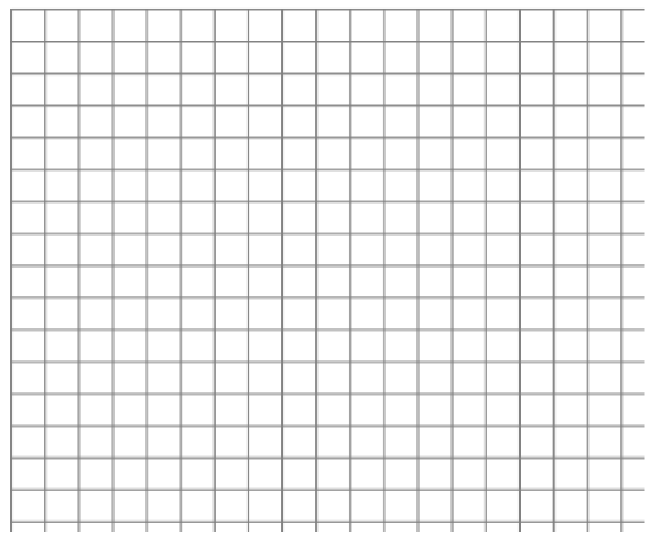
---

a. Based on your transformed equation, what is the slope?

b. Complete the table to find solutions to the linear equation.

x	**Transformed linear equation**	**y**

c. Graph the points on the coordinate plane.



d. Find the slope between any two points.

e. The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of the equation.

f. Note the location (ordered pair) that describes where the line intersects the y-axis.

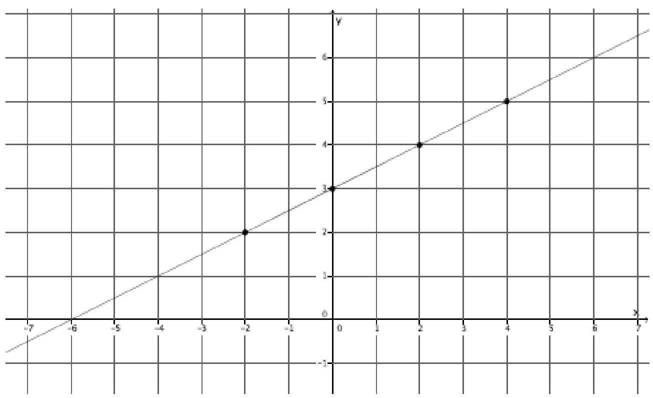
Lesson 18: There Is Only One Line Passing Through a Given Point with a Given Slope

Esential Questions:

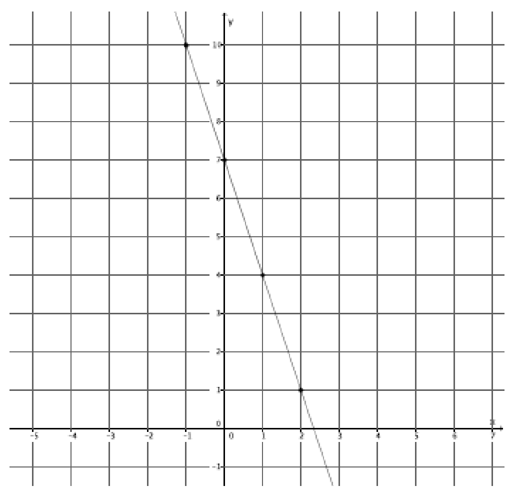
Opening Exercise

Examine each of the graphs and their equations below. Identify the coordinates of the point where the line intersects the y-axis.
Describe the relationship between the point and the equation y = m x + b.

a. $y = \frac{1}{2}x + 3$

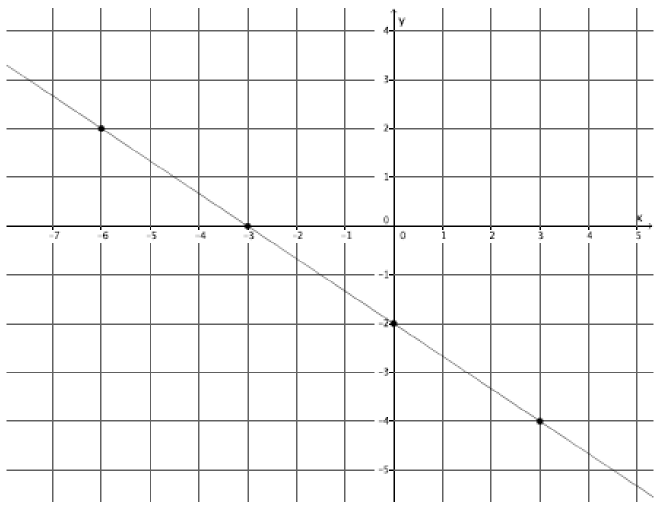


b. $y = -3x + 7$

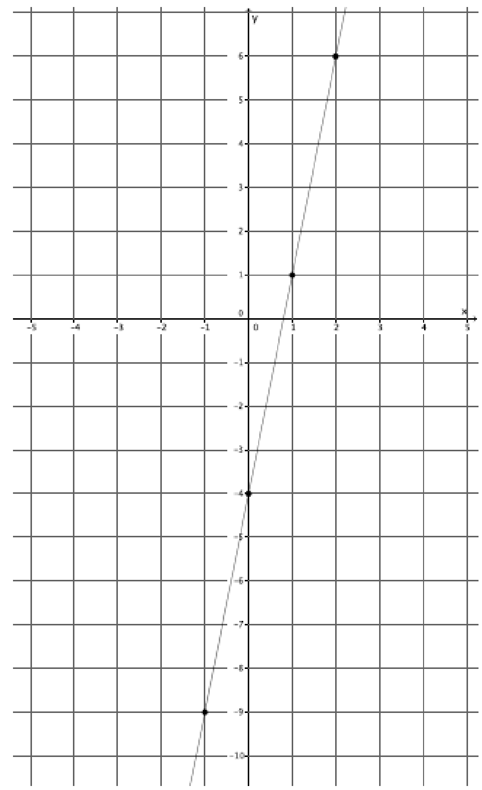


Examine each of the graphs and their equations below. Identify the coordinates of the point where the line intersects the y-axis.
Describe the relationship between the point and the equation y = m x + b.

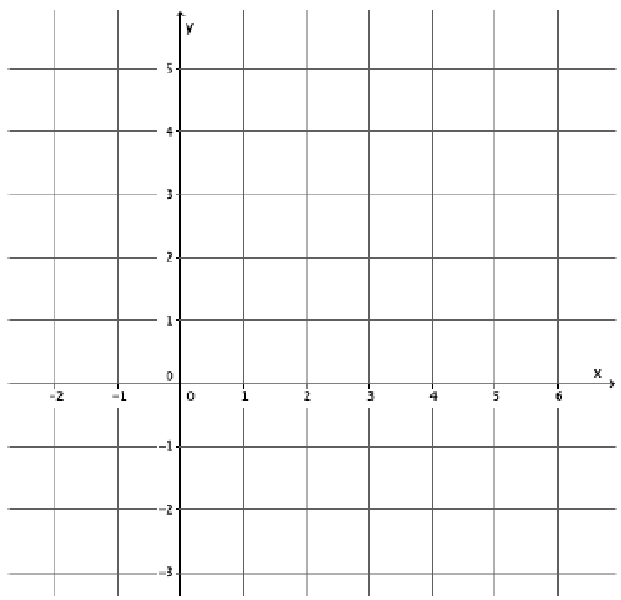
c. $y = -\frac{2}{3}x - 2$



d. $y = 5x - 4$



What do you notice about the value of b in relation to the point where the graph of the equation intersected the y-axis?	

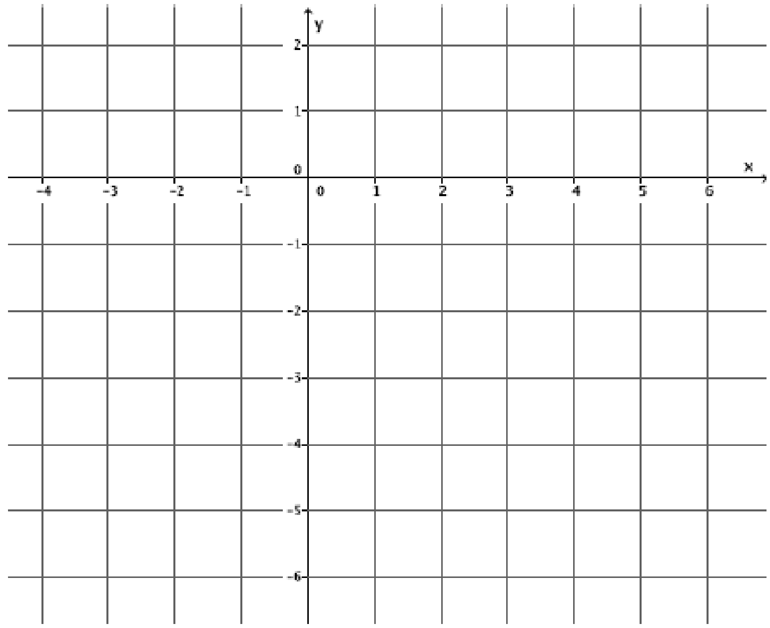
When a linear equation is in the form $y = mx + b$ it is said to be in slope intercept form. Why do you think it is called slope intercept form?	
Example 1	
Graph the equation $y = \frac{2}{3}x + 1$ Name the slope and y-intercept.	

Example 2

Graph the equation

$y = -\frac{3}{4}x - 2$

Name the slope and y-intercept.

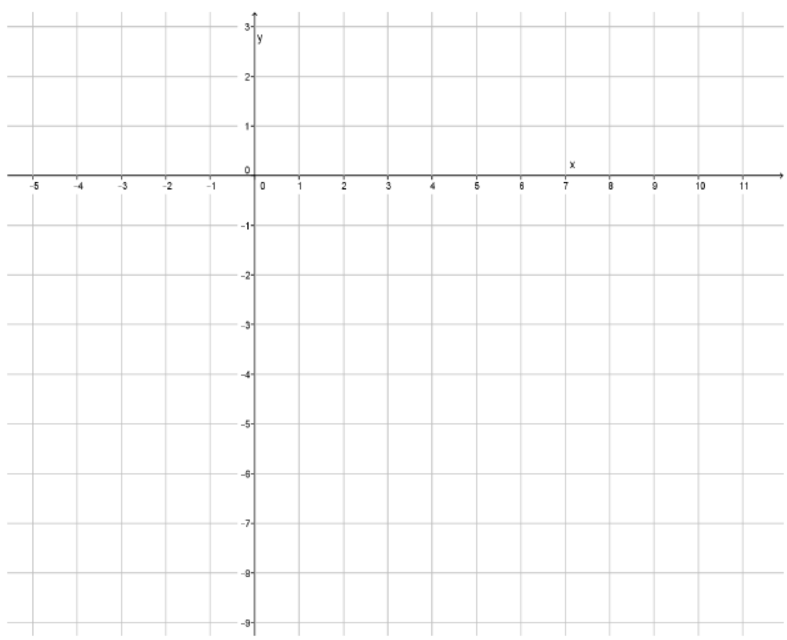


Example 3

Graph the equation

$y = 4x - 7$

Name the slope and y-intercept.



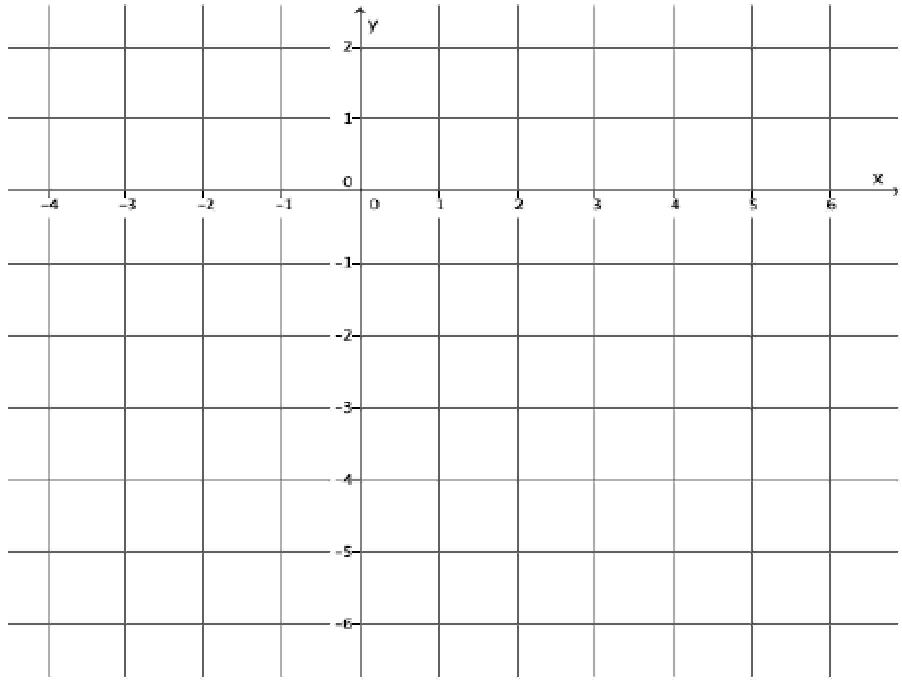
Exercises 1-4

1. Graph the equation

$y = \frac{5}{2}x - 4$

a. Name the slope and y-intercept

b. Graph the known point and then use the slope to find a second point before drawing the line.

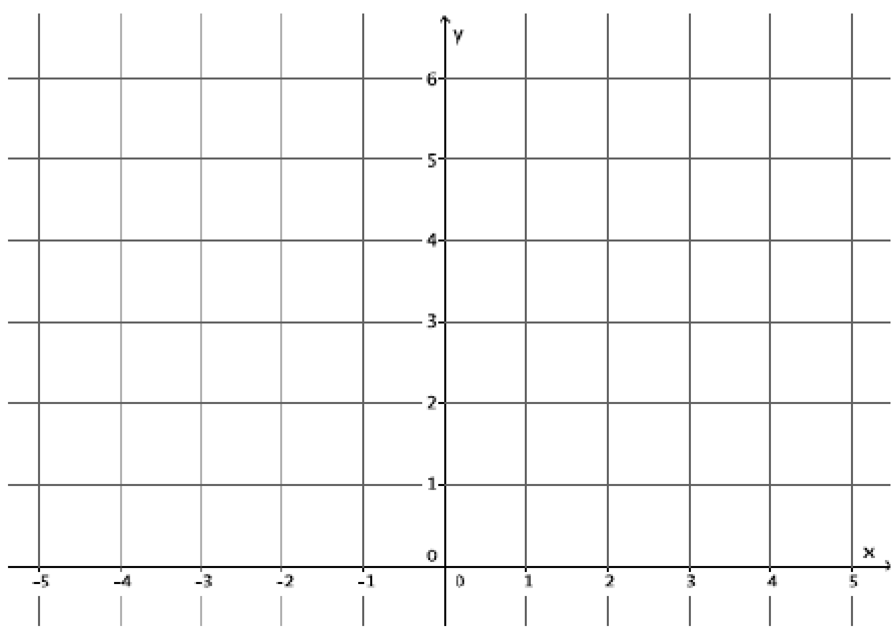


2. Graph the equation

$y = -3x + 6$

a. Name the slope and y intercept

b. Graph the known point and then use the slope to find a second point before drawing the line.



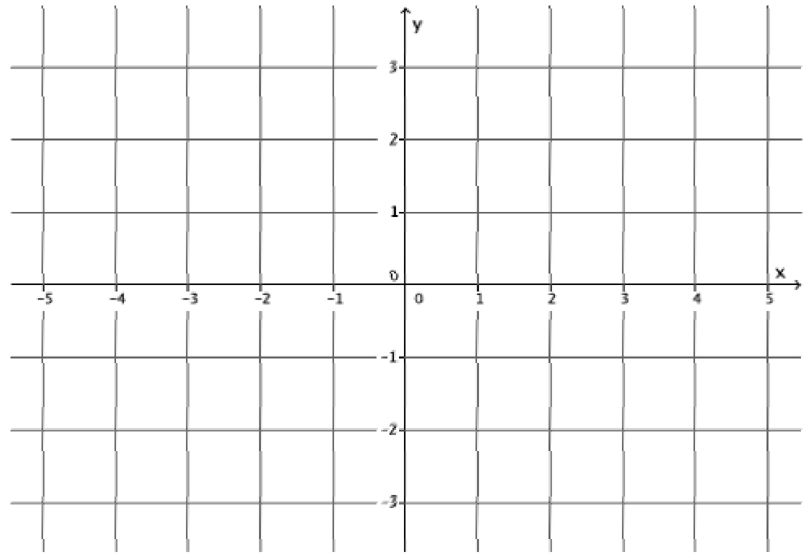
3. The equation
$y = 1x + 0$
can be simplified to $y = x$.

Graph the equation

$y = x$

a. Name the slope and y-intercept

b. Graph the known point and then use the slope to find a second point before drawing the line.

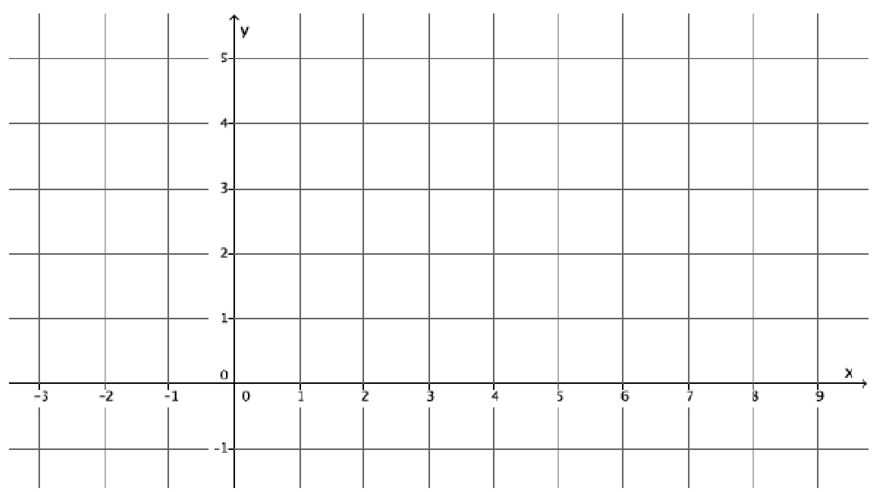


4. Graph the point (0, 2)

a. Find another point on the graph using the slope, $m = \frac{2}{7}$.

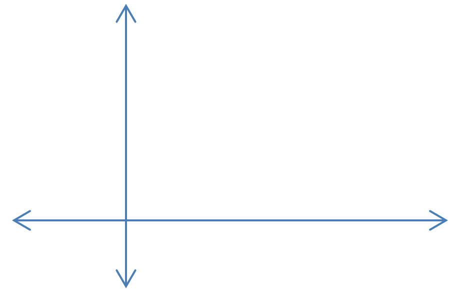
b. Connect the points to make the line.

c. Draw a different line that goes through the point (0,2) with the slope $m = \frac{2}{7}$. What do you notice?



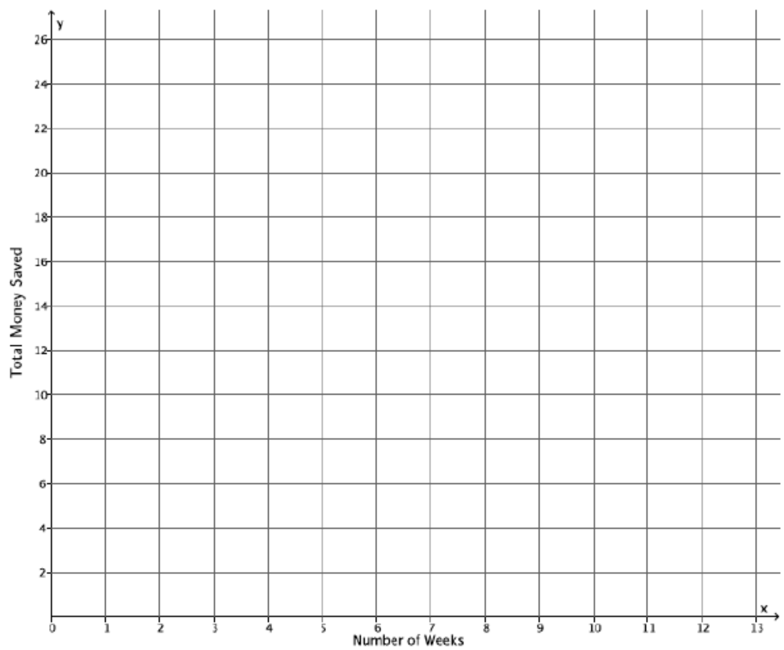
Discussion

How can we show that two straight lines that have the same slope and pass through the same point are the same line?



Exercise 5-6 Complete Independently

5. A bank put $10 into a saving account when you opened the account. Eight weeks later, you have a total of $24. Assume you saved the same amount every week.



a. If y is the total amount of money in the savings account and x represents the number of weeks, write an equation in the form y=mx+b that describes the situation.

b. Identify the slope and y-intercept. What do these numbers represent>

c. Graph the equation on the coordinate plane.

d. Could any other line represent this situation? For example, could a line through point (0, 10) with slope $\frac{7}{5}$ represent the amount of money you save each week? Explain.

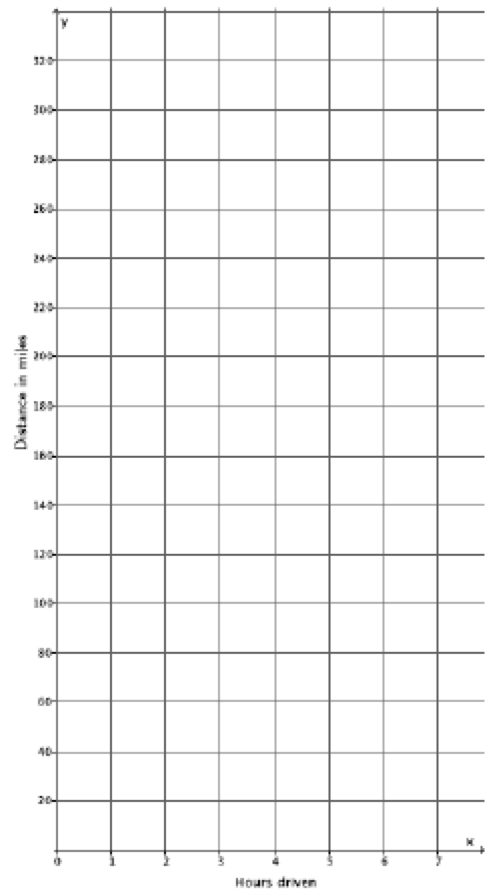
6. A group of friends are on a road trip. So far, they have driven 120 miles. They continue their trip and drive at a constant rate of 50 miles per hour.

a. Let y represent the total distance traveled in x hours. Write an equation to represent the total number of miles driven in x hours.

b. Identify the slope and y intercept. What do these numbers represent?

c. Graph the equation on a coordinate plane.

d. Could any other line represent this situation? For example could a line through point (0, 120) with slope 75 represent the total distance the friends drive? Explain.



Summary Lesson 18

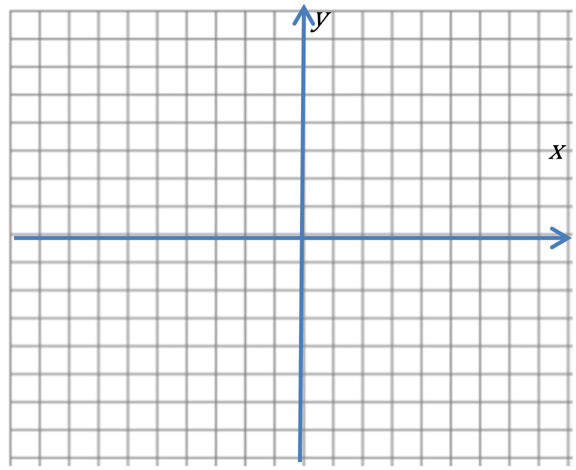
Lesson 18 – Independent Practice

Graph each equation on a separate pair of x and y axes.

1. Graph the equation $y = \frac{4}{5}x - 5$.

a. Name the slope and y-intercept.

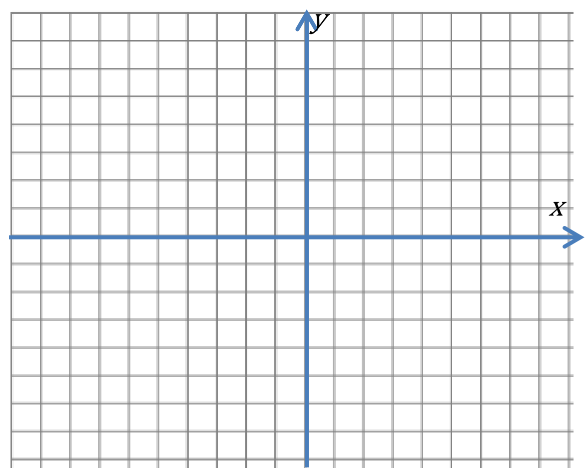
b. Graph the known point, and then use the slope to find a second point before drawing the line.



2. Graph the equation $y = x + 3$.

a. Name the slope and y-intercept.

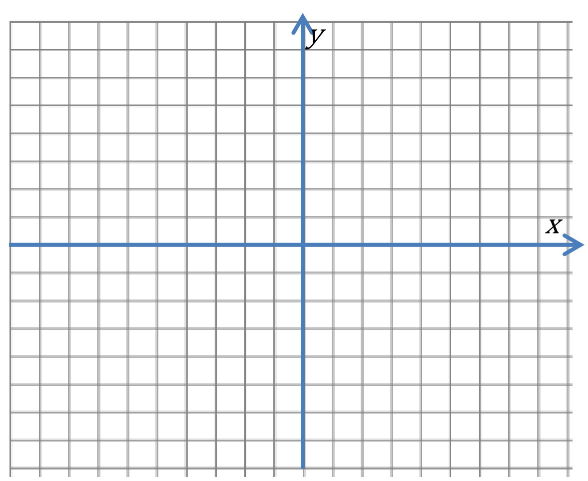
b. Graph the known point, and then use the slope to find a second point before drawing the line.



3. Graph the equation $y = -\frac{4}{3}x + 4$.

a. Name the slope and y-intercept.

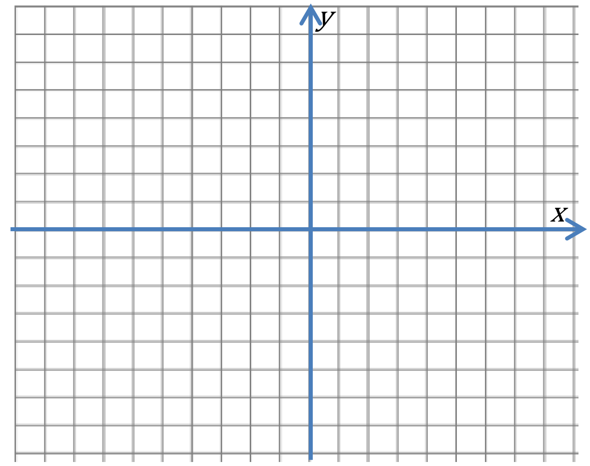
b. Graph the known point, and then use the slope to find a second point before drawing the line.



4. Graph the equation $y = \frac{5}{2}x$.

a. Name the slope and y-intercept.

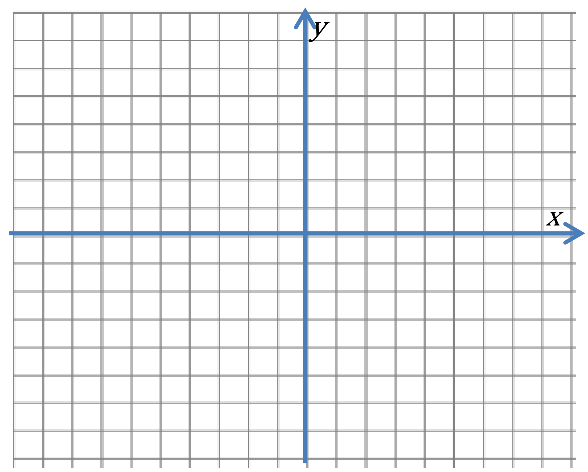
b. Graph the known point, and then use the slope to find a second point before drawing the line.



5. Graph the equation $y = 2x - 6$.

a. Name the slope and y-intercept.

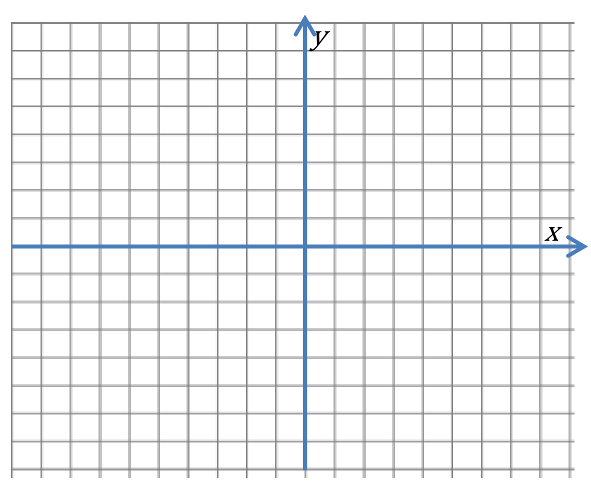
b. Graph the known point, and then use the slope to find a second point before drawing the line.



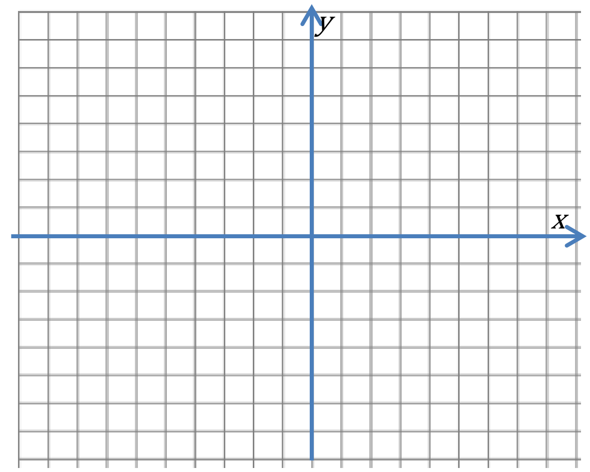
6. Graph the equation $y = -5x + 9$.

a. Name the slope and y-intercept.

b. Graph the known point, and then use the slope to find a second point before drawing the line.



7. Graph the equation $y = \frac{1}{3}x + 1$.

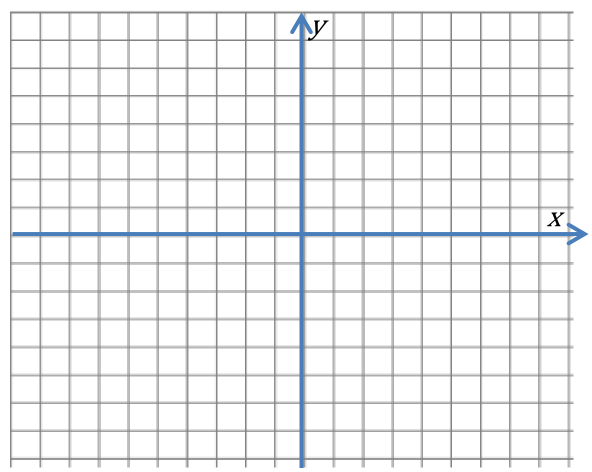


a. Name the slope and y-intercept.

b. Graph the known point, and then use the slope to find a second point before drawing the line.

8. Graph the equation $5x + 4y = 8$.

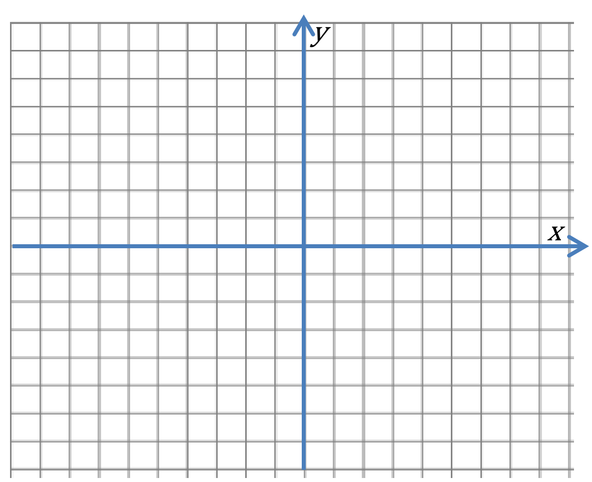
(hint: Transform the equation so that it is of the form $y = mx+b$)



a. Name the slope and y-intercept.

b. Graph the known point, and then use the slope to find a second point before drawing the line.

9. Graph the equation $-2x + 5y = 30$.



a. Name the slope and y-intercept.

b. Graph the known point, and then use the slope to find a second point before drawing the line.

10. Let l and l' be two lines with the same slope m passing through the same point P. Show that there is only one line with a slope m, where $m<0$, passing through the given point P. Draw a diagram if needed.

Lesson 19: The Graph of a Linear Equation in Two Variables is a Line

Essential Questions:

Exercise 1

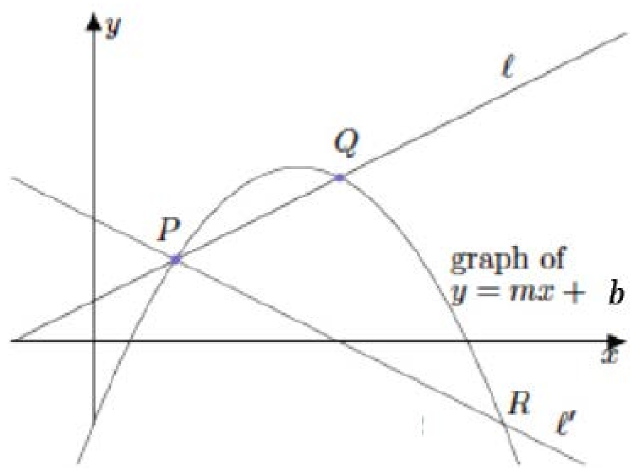
Theorem: The graph of a linear equation y = mx +b is a non-vertical line with slope m and passing through (0,b), where b is a constant

1. Prove the theorem by completing parts (a)-(c). Given two distinct points, P and Q, on the graph of y=mx+b and let l be the line passing through P and Q. You must show the following:

(1) Any point of the graph of y=mx+b is on line l and

(2) Any point on the line l is on the graph y=mx+b

a. Proof of (1): Let R be any point on the graph of y=mx+b. Show that R is on l. Begin by assuming it is not. Assume the graph looks like the diagram below where R is on l'.

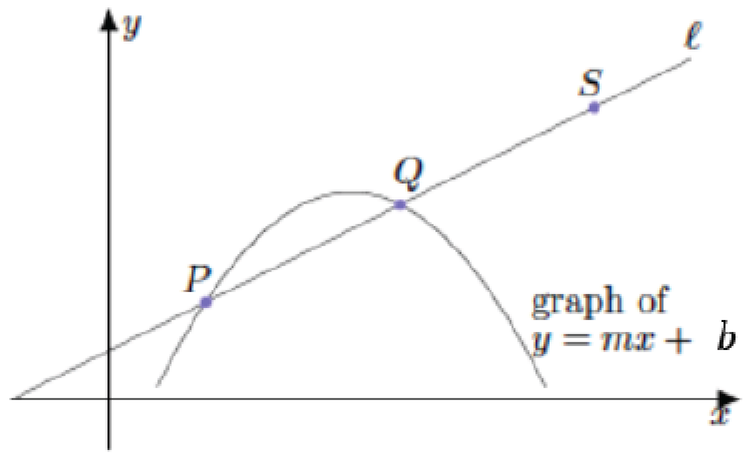


What is the slope of line l?

What is the slope of line l'?

What can you conclude about line l and l'? Explain.

b. Proof of (2): Let S be any point on line l, as shown.

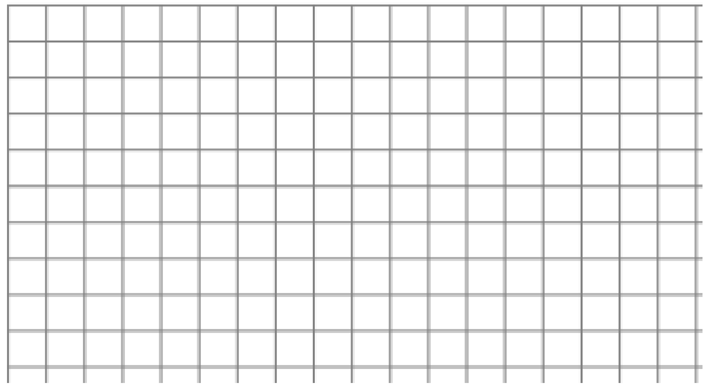


Show that S is a solution to y=mx+b. Hint: Use point (0,b)

c. Now that you have shown that any point on the graph of y=mx+b is on line in part (a), and any point on line l is on the graph of y=mx+b in part (b), what can you conclude about the graphs of linear equations?

Exercise 2-8

2. Use x=4 and x= -4 to find two solutions to the equation x + 2y = 6. Plot the solutions as points on the coordinate plane.



a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation x + 2y = 6

b. When x = 1, what is the value of y? Does this solution appear to be a point on the line?

c. When x = -3, what is the value of y? Does this solution appear to be on the line?

d. is the point (3, 2) on the line?

e. Is the point (3, 2) a solution to the linear equation x + 2y = 6.

3. Use $x = 4$ and $x = 1$ to find two solutions to the equation $3x - y = 9$. Plot the solutions as points on the coordinate plane, and connect the points to make a line.

a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $3x - y = 9$.

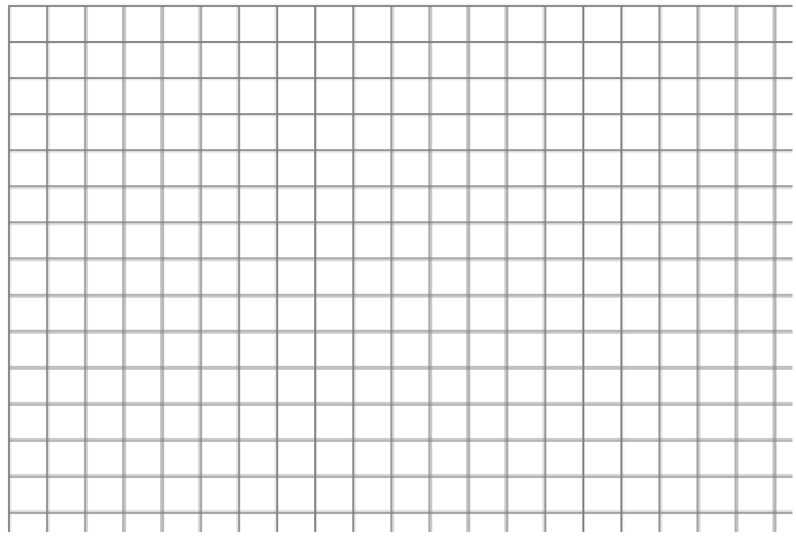
b. When $x = 4.5$, what is the value of y? Does this solution appear to be a point on the line?

c. When $x = \frac{1}{2}$, what is the value of y? Does this solution appear to be a point on the line?

d. Is the point $(2, 4)$ on the line?

e. Is the point $(2, 4)$ a solution to the linear equation $3x - y = 9$.

4. Use $x = 3$ and $x = -3$ to find two solutions to the equation $2x + 3y = 12$. Plot the solutions as points on the coordinate plane, and connect the points to make a line.



a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $2x + 3y = 12$

b. When $x = 2$, what is the value of y? Does this solution appear to be a point on the line?

c. When $x = -2$, what is the value of y? Does this solution appear to be a point on the line?

d. Is the point $(8,-3)$ on the line?

e. Is the point $(8, -3)$ a solution to the linear equation $2x + 3y = 12$?

5. Use $x = 4$ and $x = -4$ to find two solutions to the equation $x - 2y = 8$. Plot the solutions as points on the coordinate plane and connect the points to make a line.

a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $x - 2y = 8$.

b. When $x = 7$, what is the value of y? Does this solution appear to be a point on the line?

c. When $x = -3$, what is the value of y? Does this solution appear to be appoint on the line?

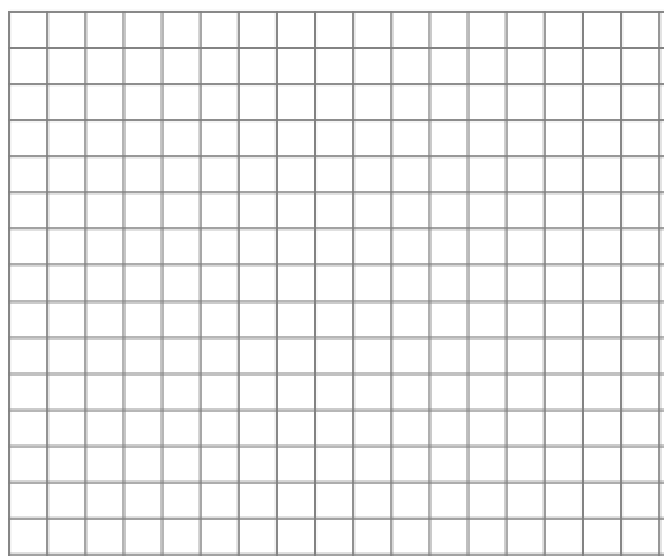
d. Is the point $(-2, -3)$ on the line?

e. Is the point $(-2, -3)$ a solution to the linear equation $x - 2y = 8$.

6. Based on your work in Exercises 2-5, what conclusions can you draw about the points on a line and solutions to a linear equation?	
7. Based on your work in Exercises 2-5, will a point that is not a solution to a linear equation be a point on the graph of a linear equation? Explain.	
8. Based on your work in Exercises 2-5, what conclusions can you draw about the graph of a linear equation?	

Example 1

Graph the equation
$2x + 3y = 9$

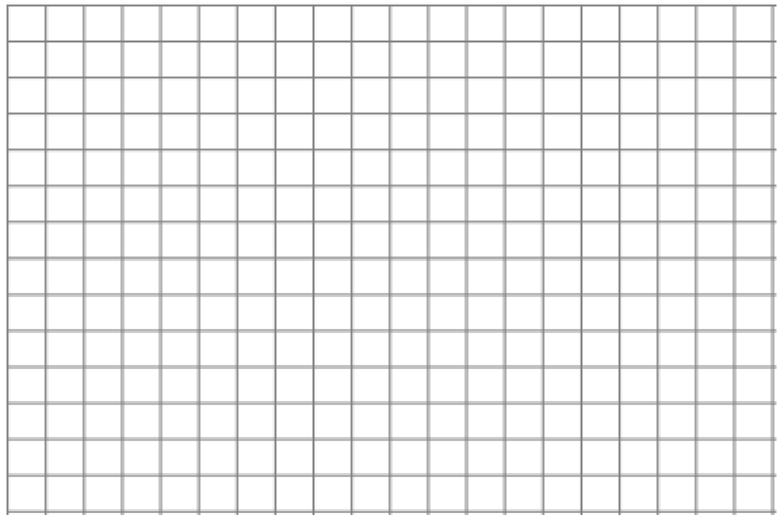


Exercises 9-11

9. Graph the equation $-3x + 8y = 24$ using intercepts.

10. Graph the equation $x - 6y = 15$ using intercepts.

11. Graph the equation $4x + 3y = 21$ using intercepts.



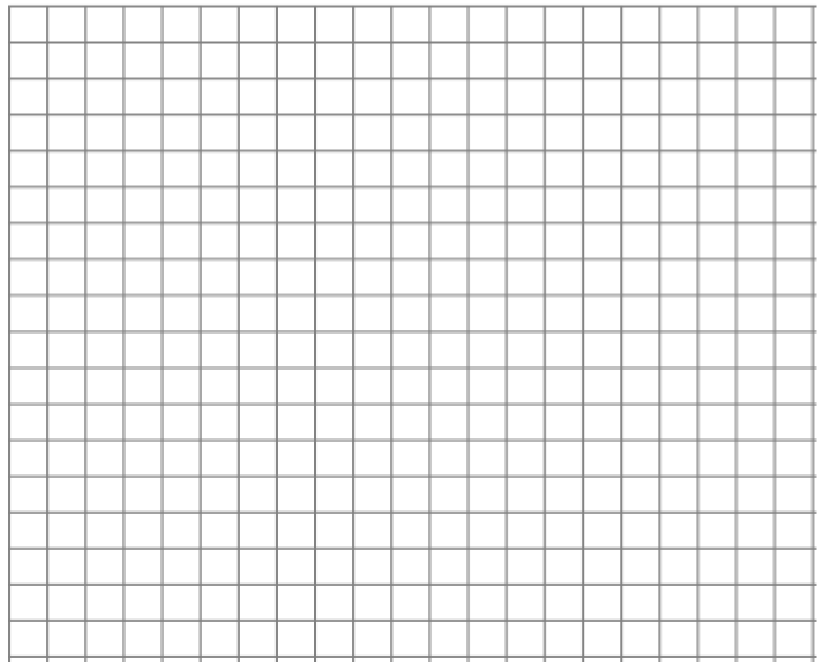
Lesson 19 summary:

Independent Work

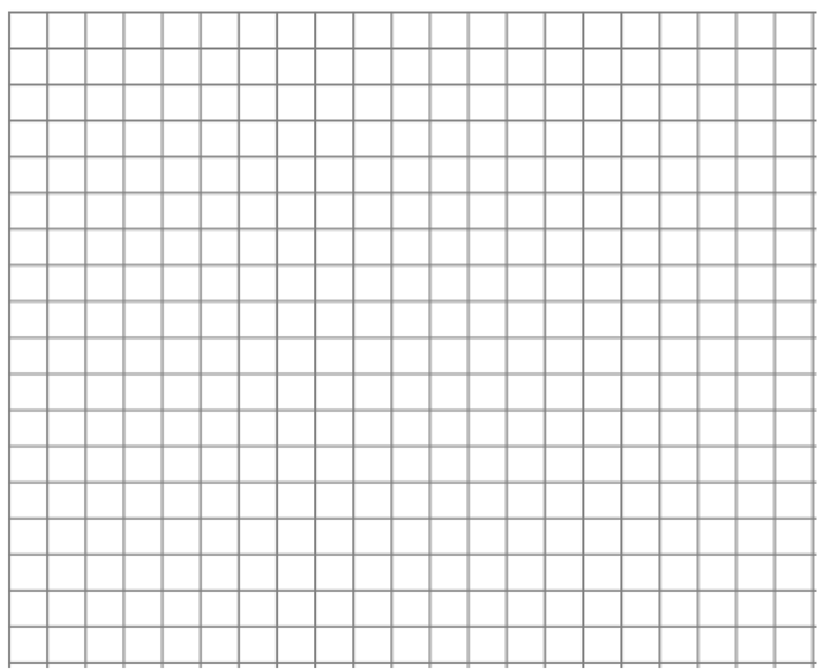
1. Graph the equation: $y = -6x + 12$

2. Graph the equation: $9x + 3y = 18$

3. Graph the equation: $y = 4x + 2$



4. Graph the equation: $y = -\frac{5}{7}x + 4$



5. Graph the equation: $\frac{3}{4}x + y = 8$

6. Graph the equation: $2x - 4y = 12$

7. Graph the equation: $y = 3$. What is the slope of the graph of this line?

8. Graph the equation: $x = -4$. What is the slope of the graph of this line?

9. Is the graph of $4x + 5y = \frac{3}{7}$ a linear equation? Explain.

10. Is the graph of $6x^2 - 2y = 7$ a linear equation? Explain.

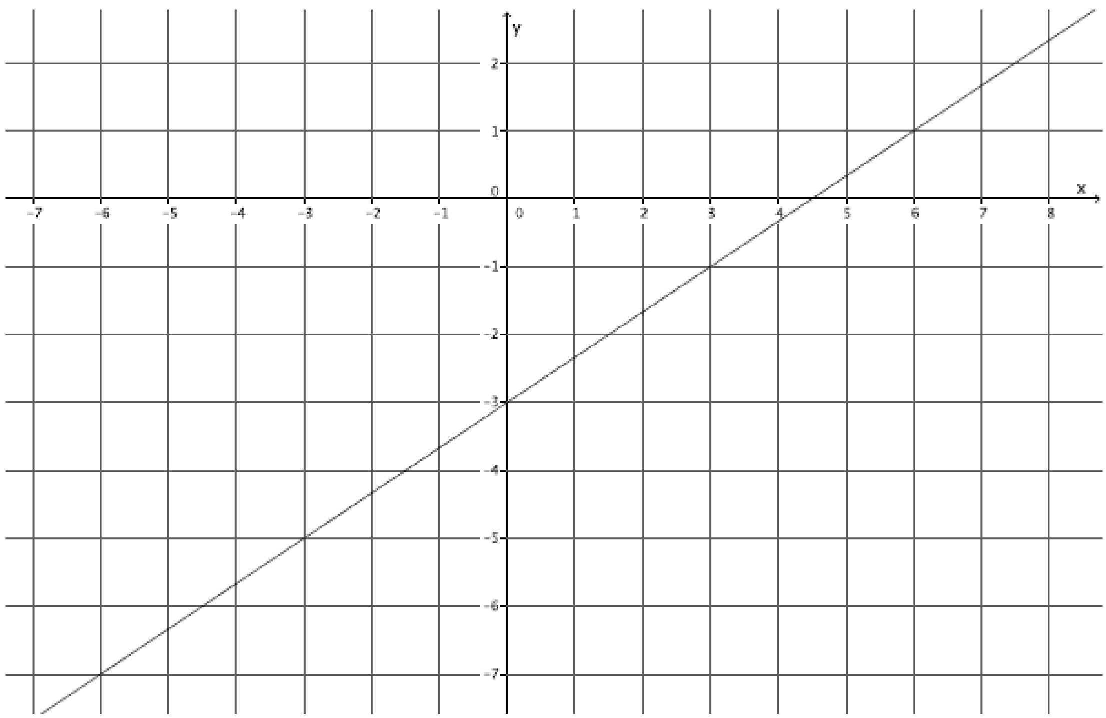
Lesson 20: Every Line is a Graph of a Linear Equation

Essential Questions:

Opening Exercise

We know that the graph of every linear equation is a line, can we say that every line is the graph of a linear equation?

Figure 1:



What equation does the line represent?

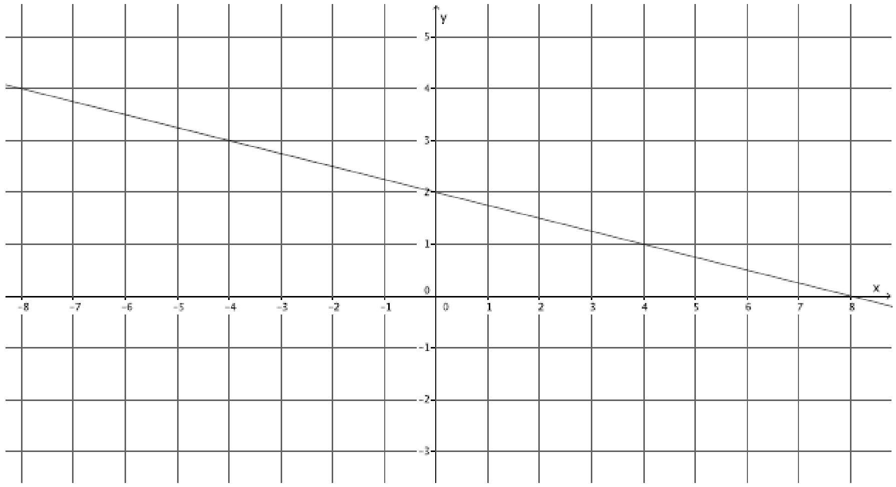


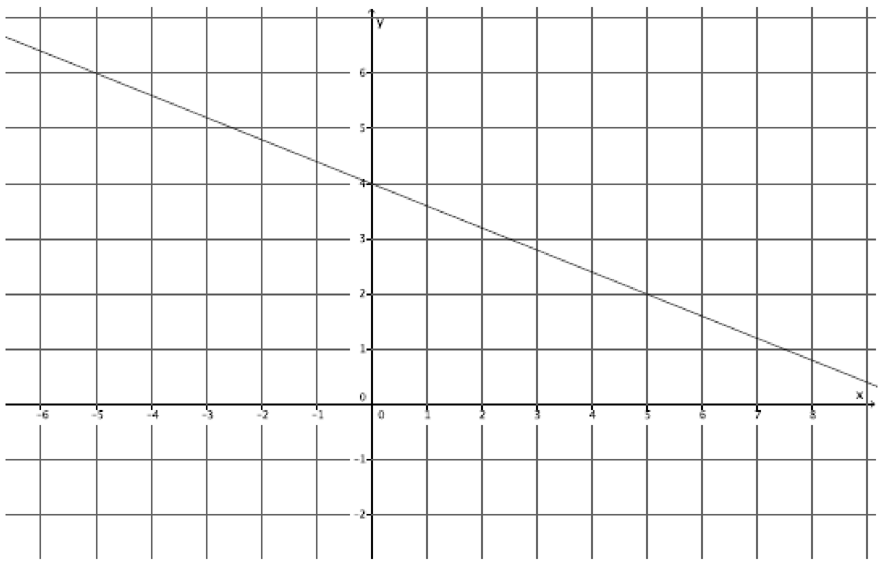
Figure 2:

What is the equation for the line in figure 2?

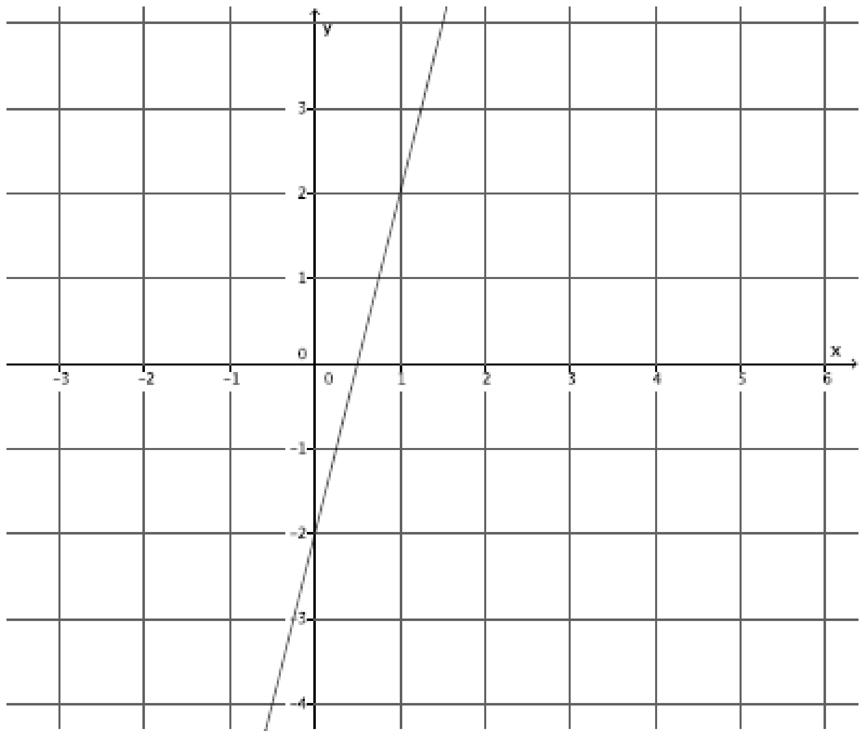
Example 1

Given a line we want to be able to write the equation that represents it.

Which form of a linear equation do you think will be most valuable for this task, the standard form **ax + by = c** or slope-intercept form **y = mx + b?**

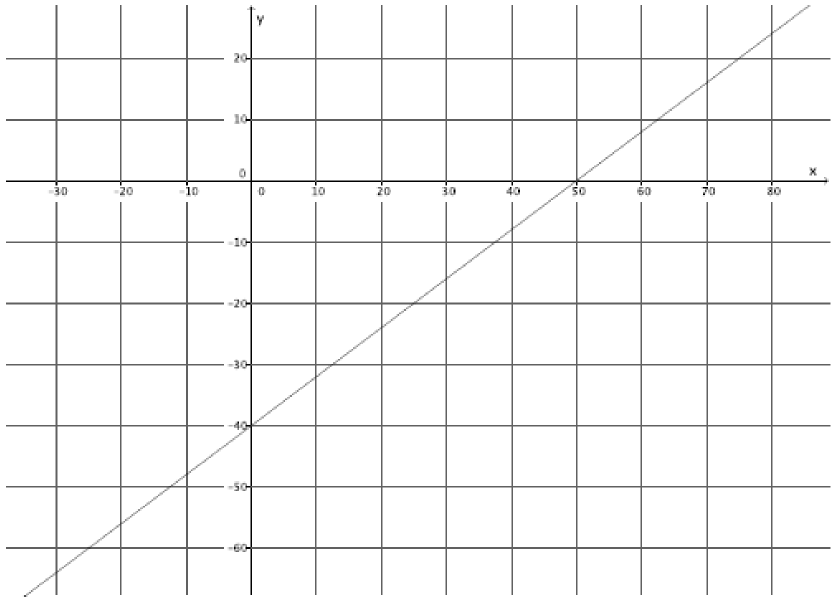


Example 2



Example 2 - work	
What is the y-intercept of the line?	
What is the slope of the line?	
Write the equation for the line (example 2).	

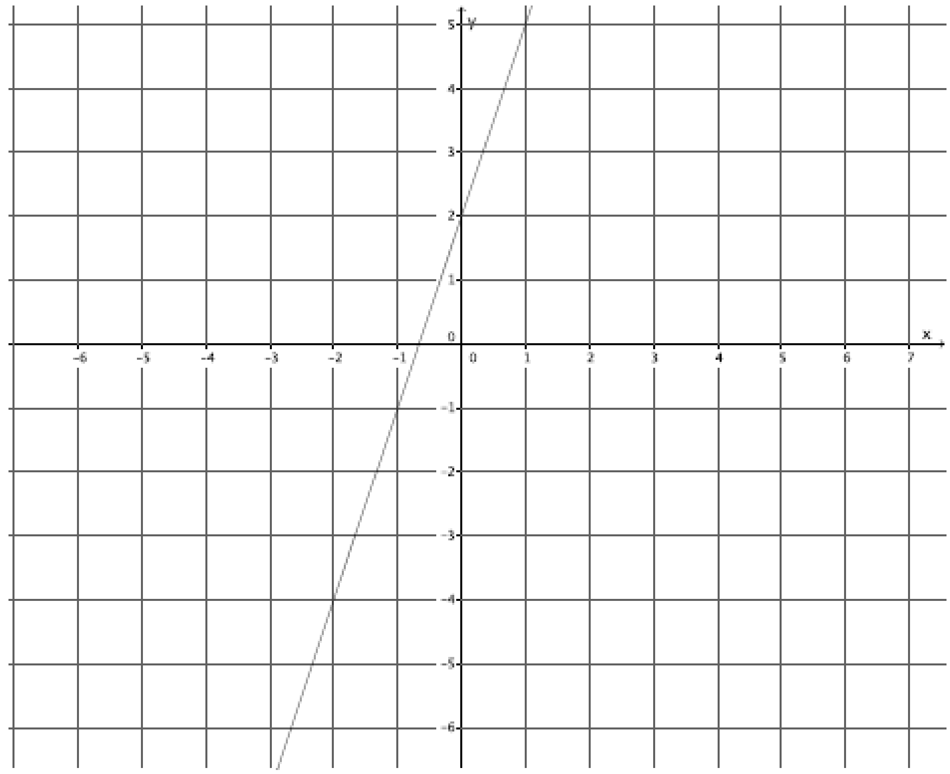
Example 3



Example 3-work	
What is the y-intercept of the line?	
What is the slope of the line?	
What are the steps to finding slope?	
What is the line of the graph?	

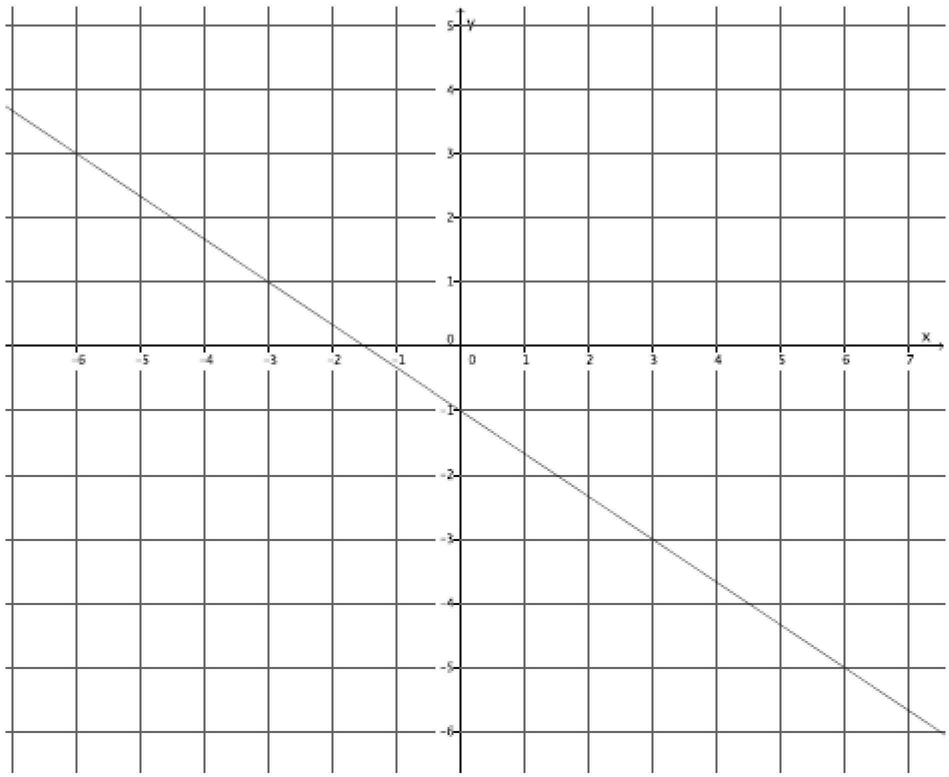
Exercises 1-6 Complete independently

1. Write the equation that represents the line shown.



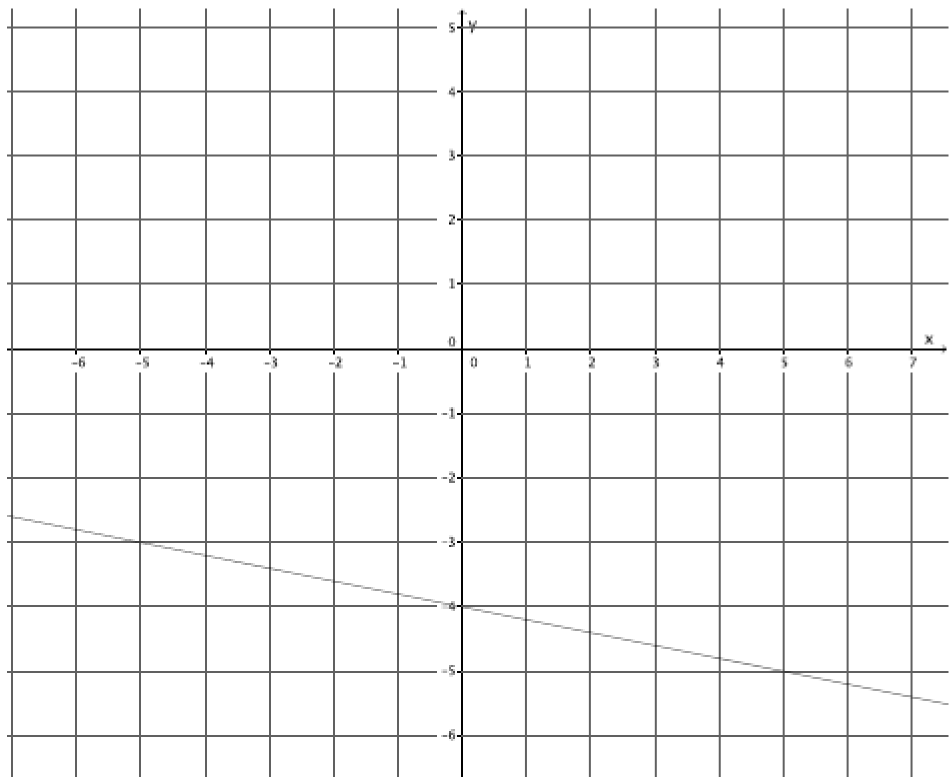
Use the properties of equality to change the equation from slope-intercept form, $y=\mathrm{mx}+b$, to standard form, $ax+by=c$, where a, b, and c are integers, and a is not negative.

2. Write the equation that represents the line shown.



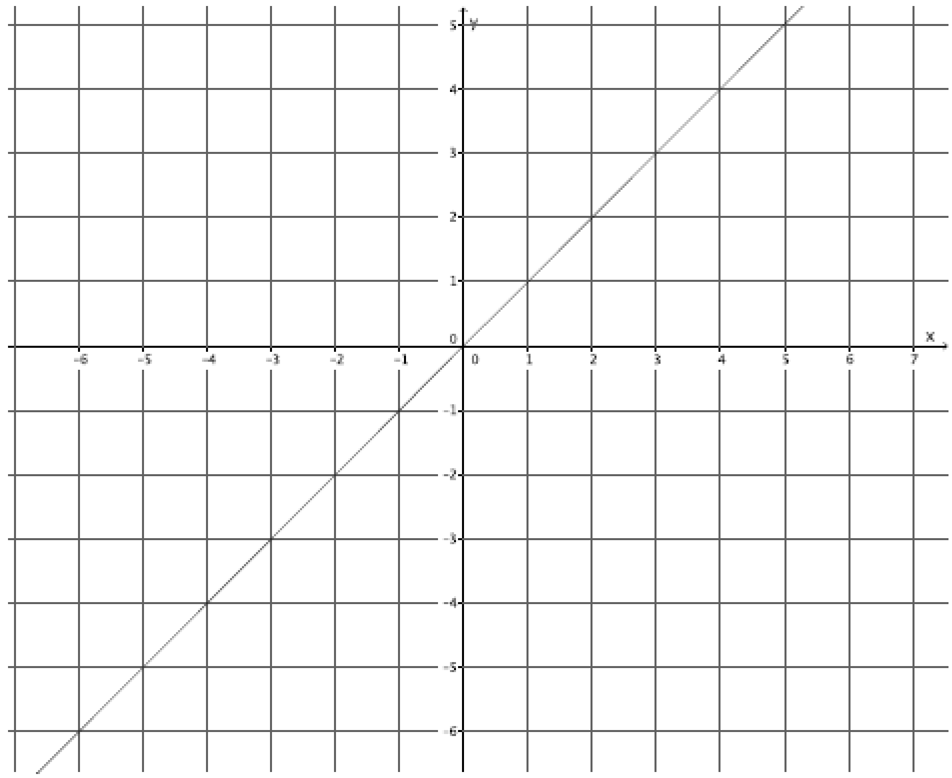
Use the properties of equality to change the equation from slope-intercept form, $y=mx+b$, to standard form, $ax+by=c$, where a, b, and c are integers, and a is not negative.

3. Write the equation that represents the line shown.



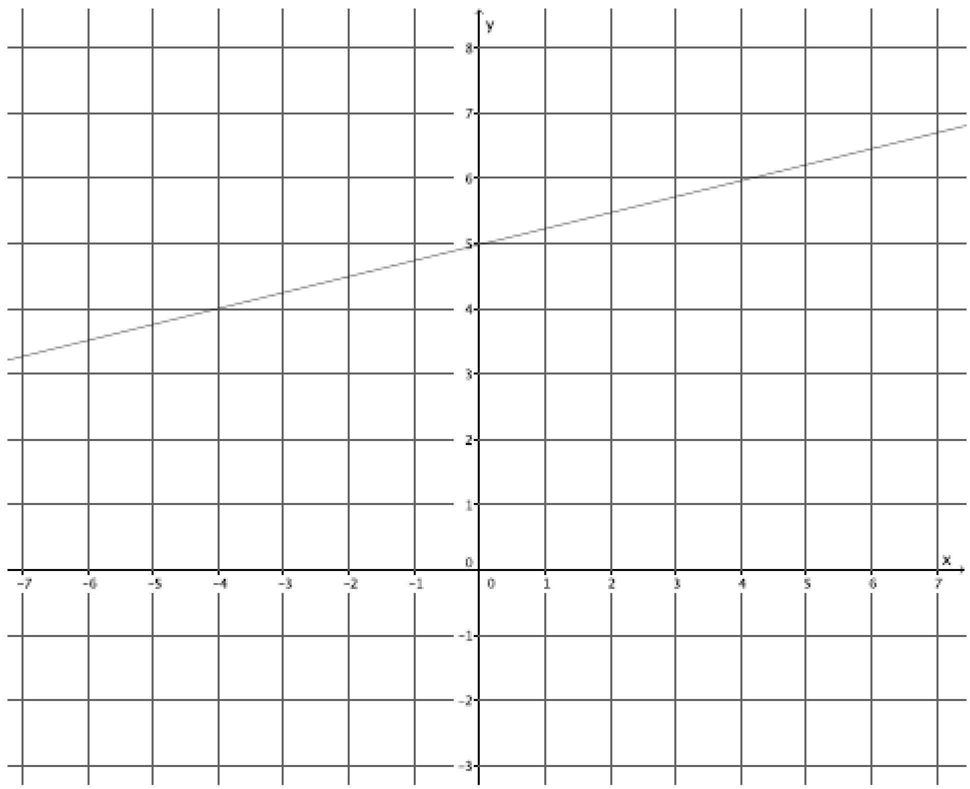
Use the properties of equality to change the equation from slope-intercept form, $y = \mathrm{mx} + b$, to standard form, $a\mathrm{x} + by = c$, where a, b, and c are integers, and a is not negative.

4. Write the equation that represents the line shown.



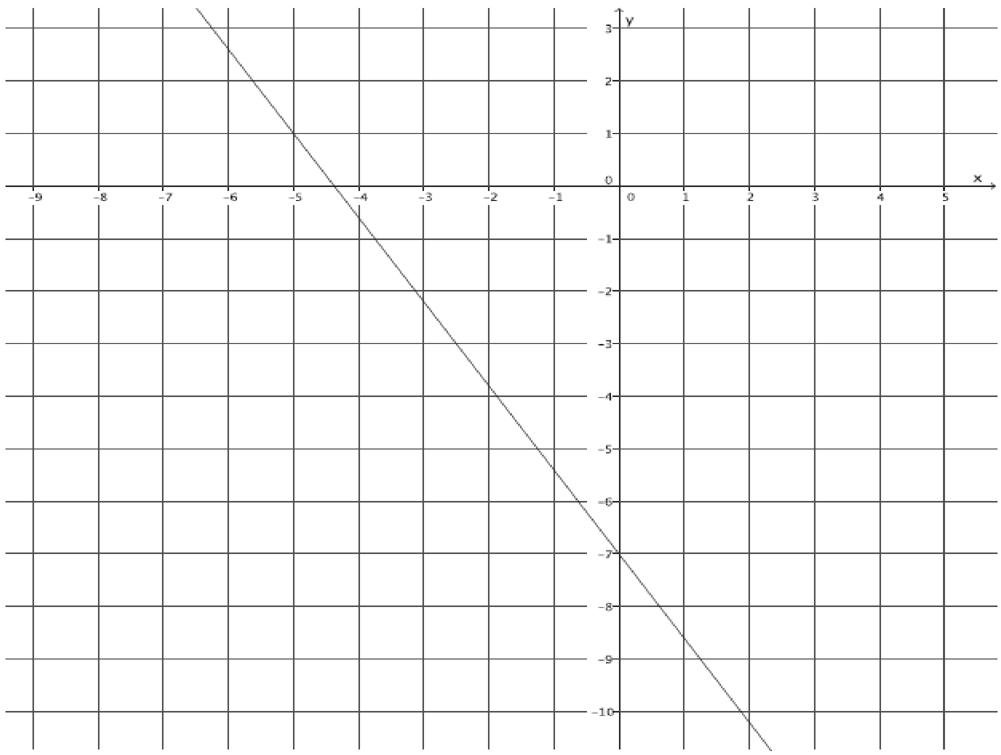
Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a, b, and c are integers, and a is not negative.

5. Write the equation that represents the line shown.



Use the properties of equality to change the equation from slope-intercept form, $y=mx+b$, to standard form, $ax+by=c$, where a, b, and c are integers, and a is not negative.

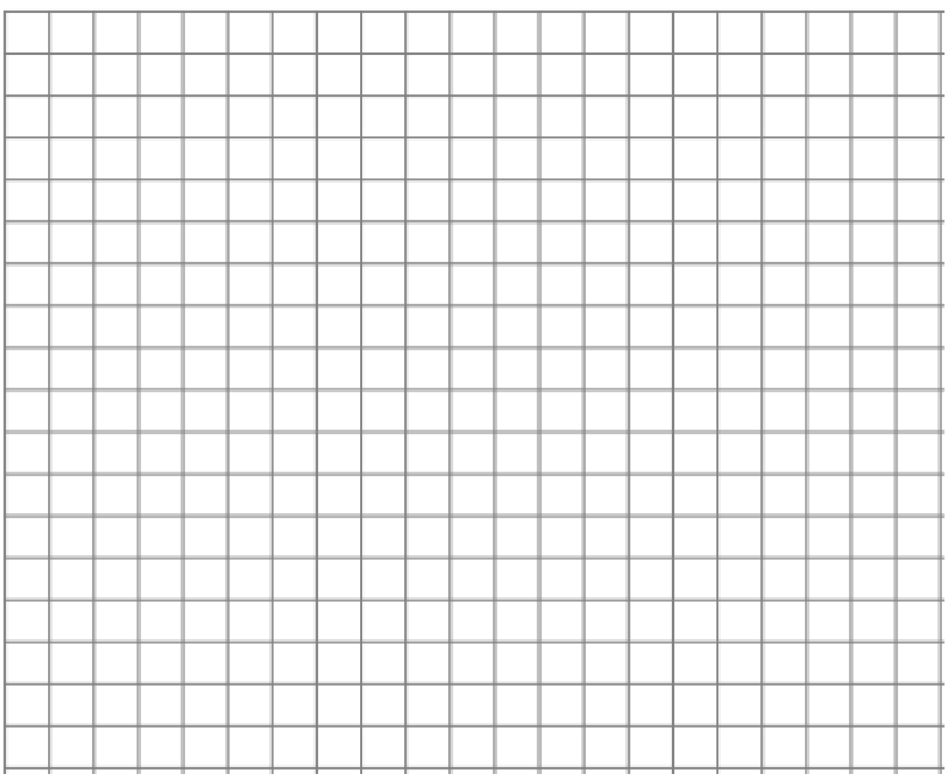
6. Write the equation that represents the line shown.



Use the properties of equality to change the equation from slope-intercept form, y=mx +b, to standard form, ax+by=c, where a, b, and c are integers, and a is not negative.

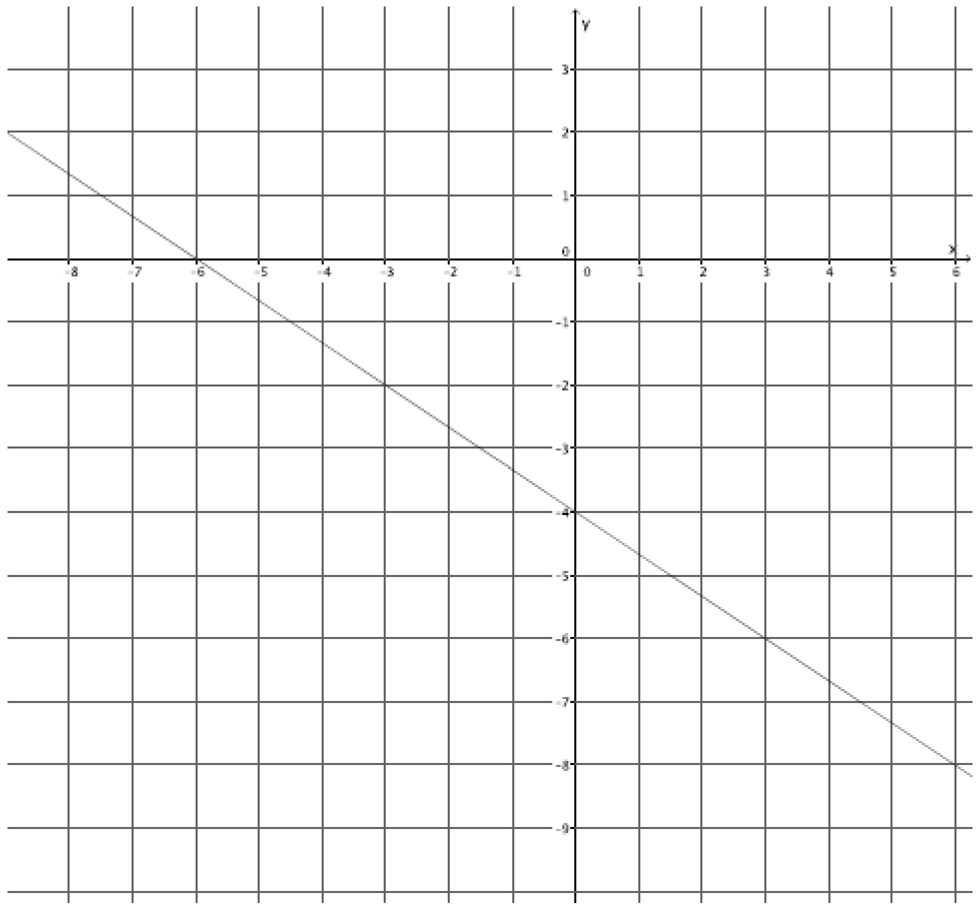
Summary:

Example:



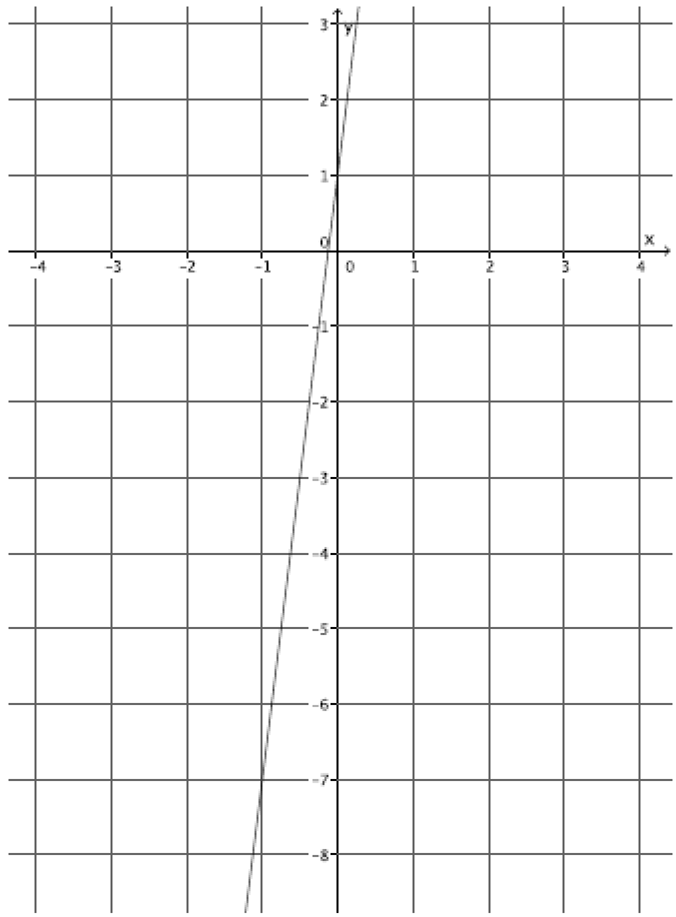
Independent Practice

1. Write the equation that represents the line shown.



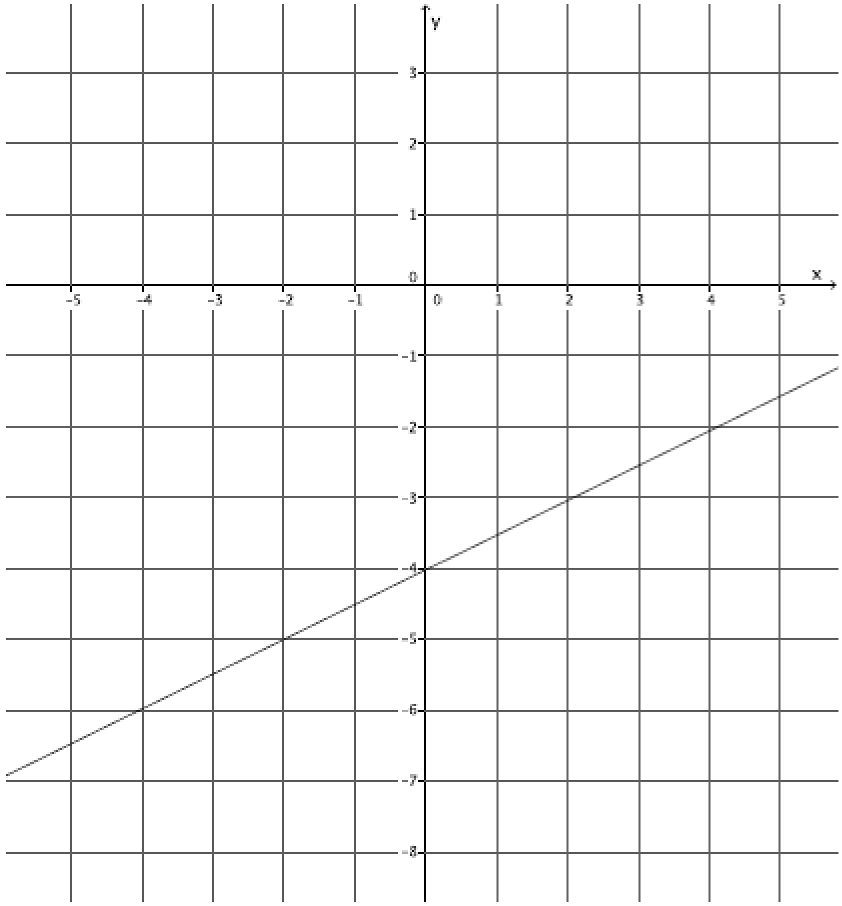
Use the properties of equality to change the equation from slope-intercept form, $y = \mathrm{mx} + b$, to standard form, $ax + by = c$, where a, b, and c are integers, and a is not negative.

2. Write the equation that represents the line shown.



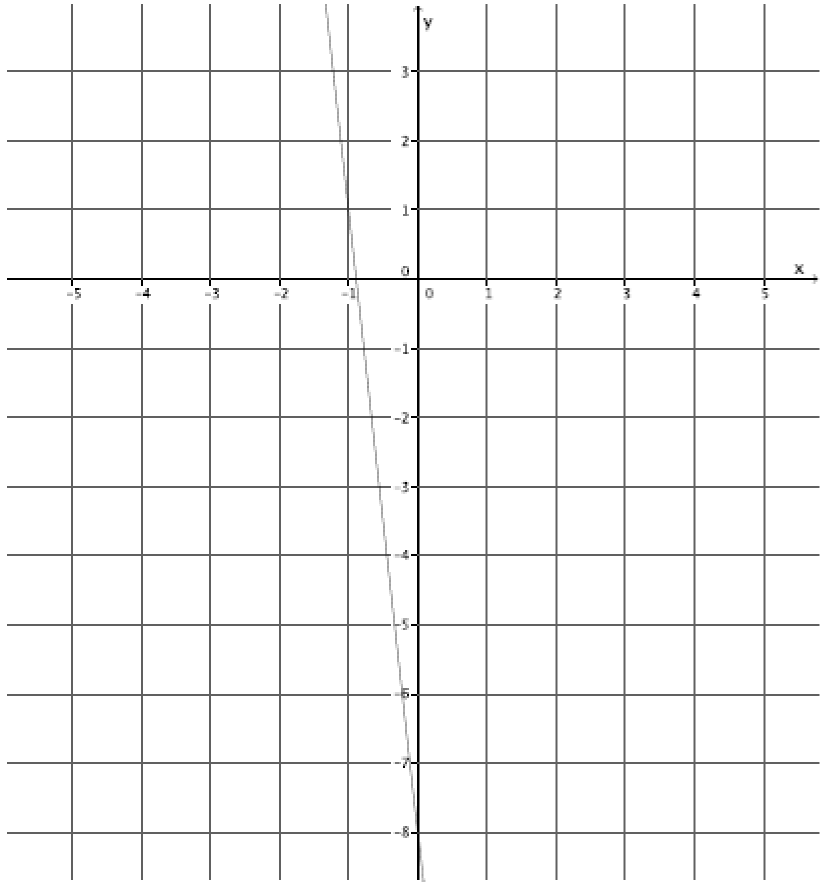
Use the properties of equality to change the equation from slope-intercept form, $y=\mathrm{mx}+b$, to standard form, $ax+by=c$, where a, b, and c are integers, and a is not negative.

3. Write the equation that represents the line shown.



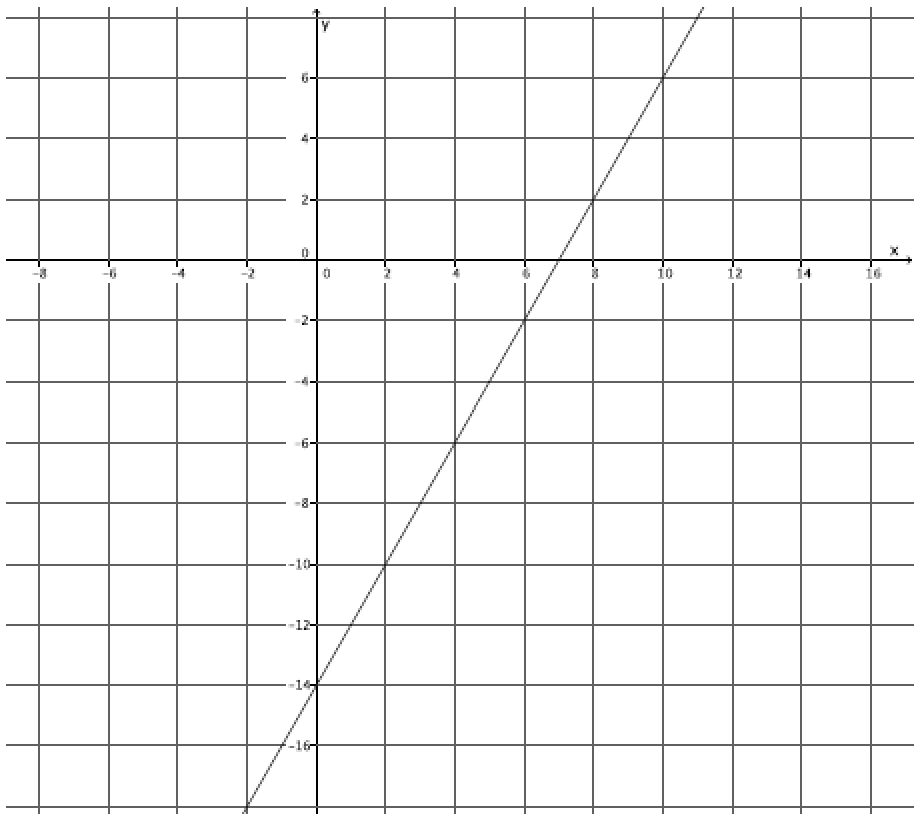
Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a, b, and c are integers, and a is not negative.

4. Write the equation that represents the line shown.



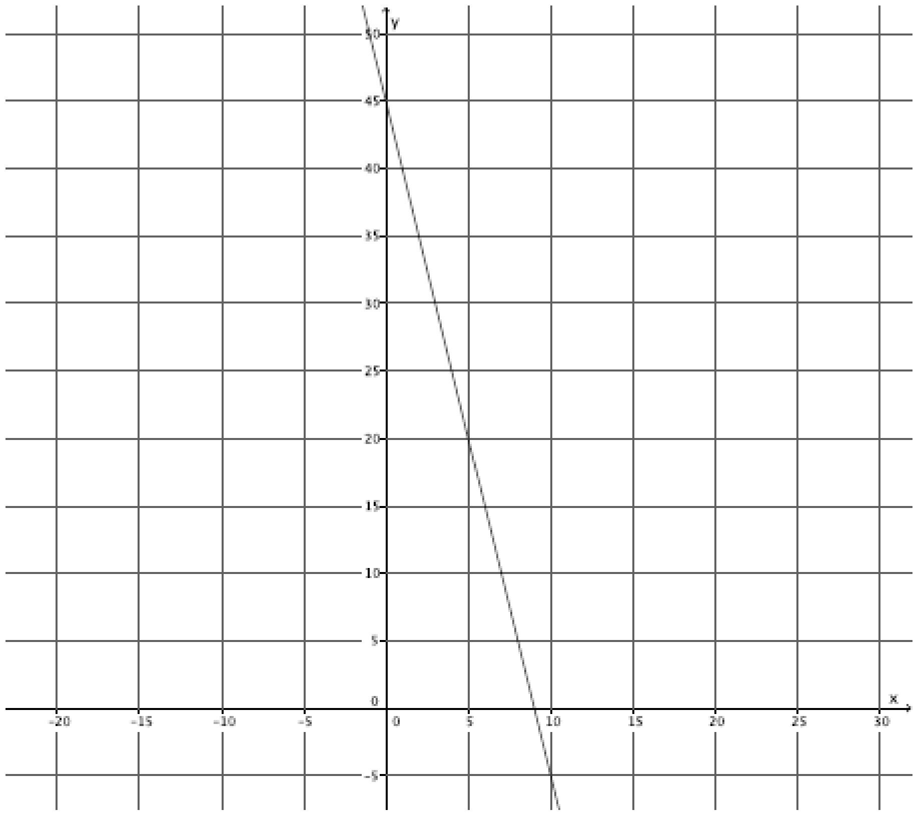
Use the properties of equality to change the equation from slope-intercept form, $y=\mathrm{mx}+b$, to standard form, $ax+by=c$, where a, b, and c are integers, and a is not negative.

5. Write the equation that represents the line shown.



Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a, b, and c are integers, and a is not negative.

6. Write the equation that represents the line shown.



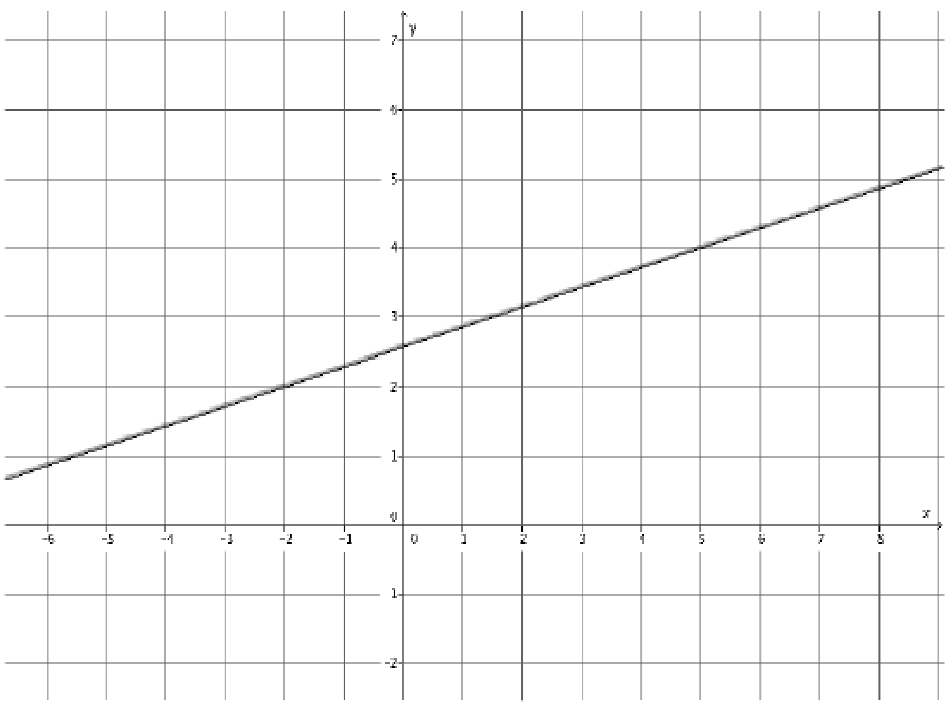
Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a, b, and c are integers, and a is not negative.

Lesson 21: Some Facts about Graphs of Linear Equations in Two Variables

Essential Questions:

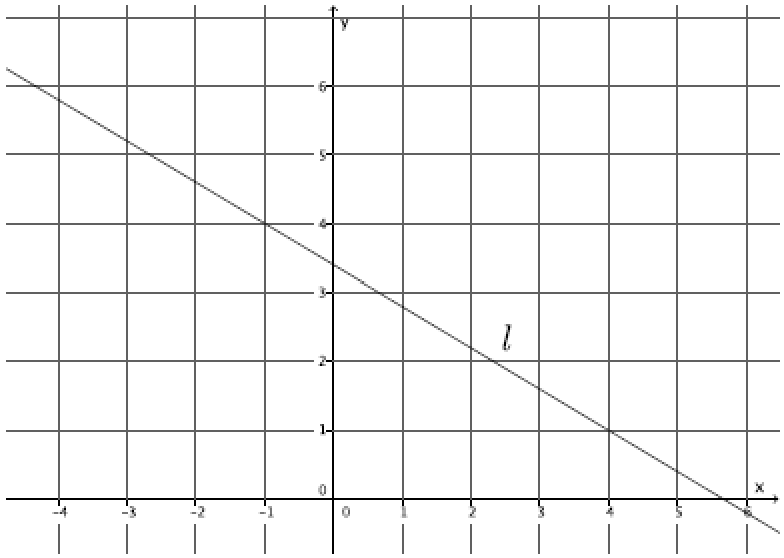
Example 1

Let line l be given in the coordinate plane. What linear equation is the graph of line l?



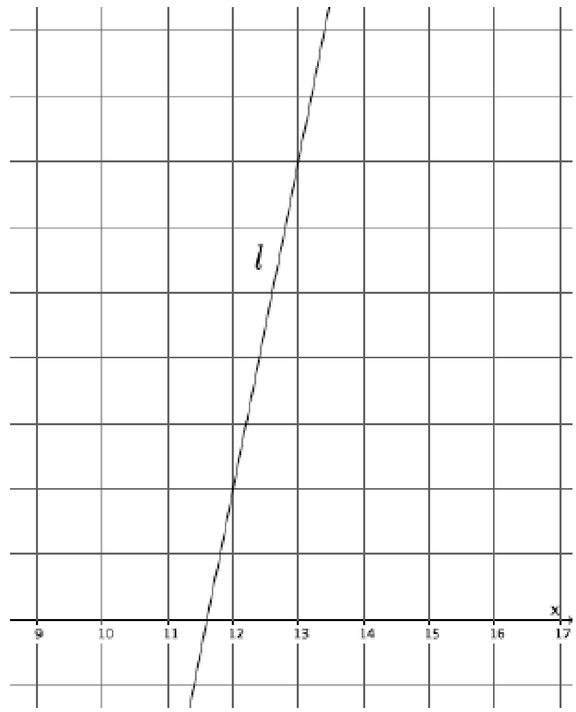
Example 2

Let l be given in the coordinate plane.



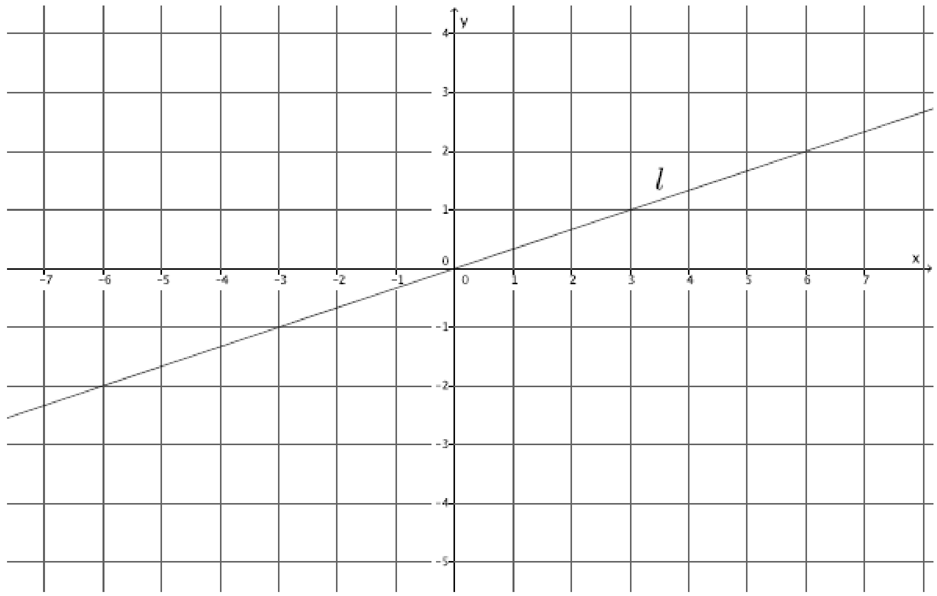
Write the equation of the line.	
Transform the equation so that it is written in standard form.	

Example 3
Let a line l be given in the coordinate plane.



What is the equation of the line?	
Write the equation of line l in standard form.	

Example 4

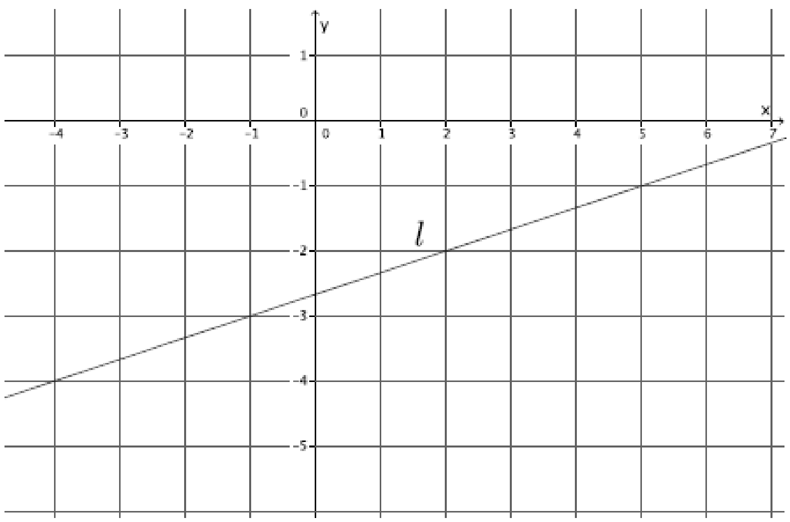


Let l be given in the coordinate plane. What linear equation is the graph of line l?	
Write the equation of the line.	
Concept development - discussion	
What equation can we use to find the slope of a line if we have two sets of integer coordinates?	
What is point-slope form of a linear equation?	

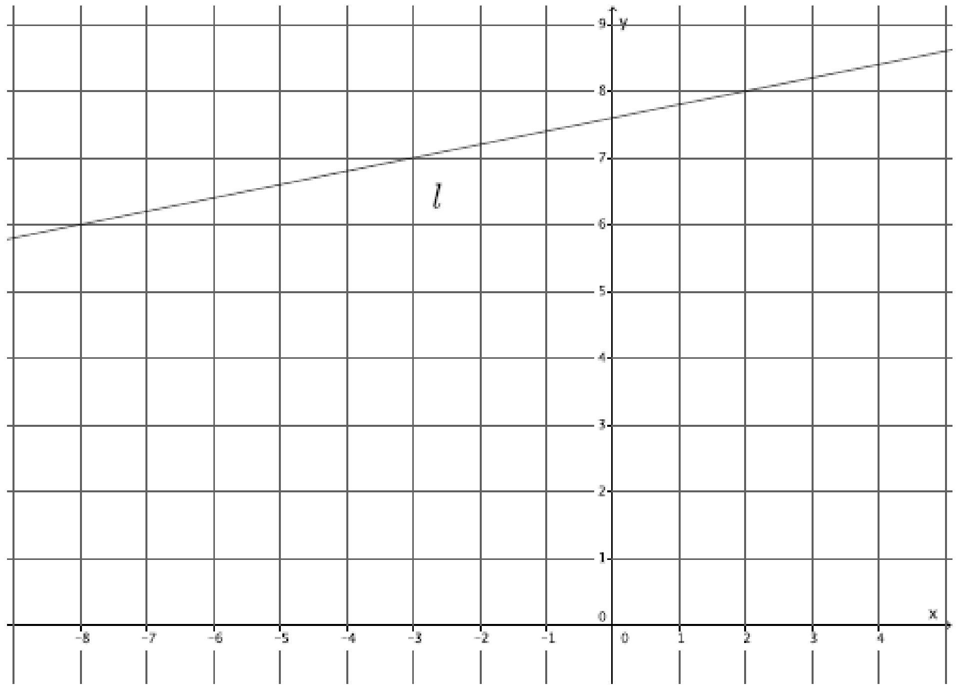
What is slope-intercept form of a line?	
What information must you have to write the equation of a line?	

Exercises 1-7 – complete independently

1. Write the equation for line l shown in the figure.

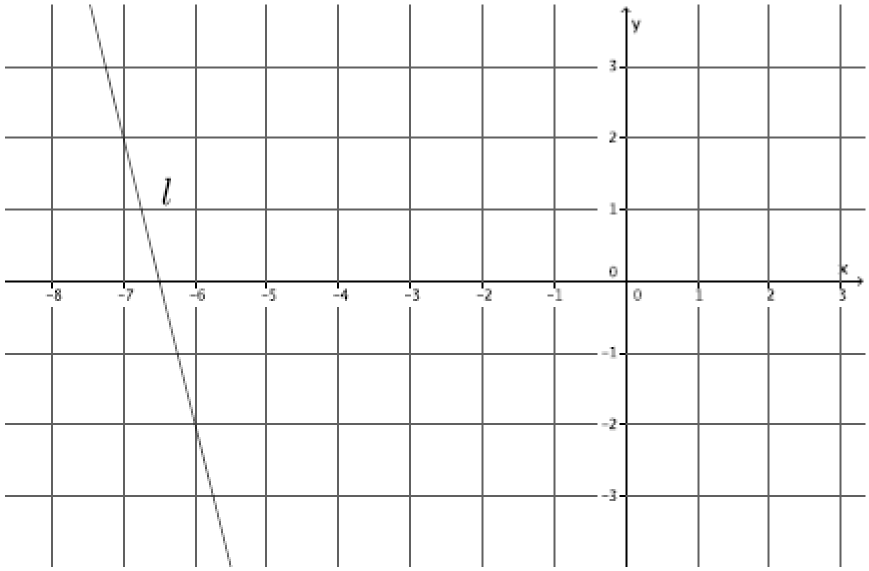


2. Write the equation for line l shown in the figure.



3. Determine the equation of the line that goes through points (-4, 5) and (2, 3).

4. Write the equation for line l shown in the figure.

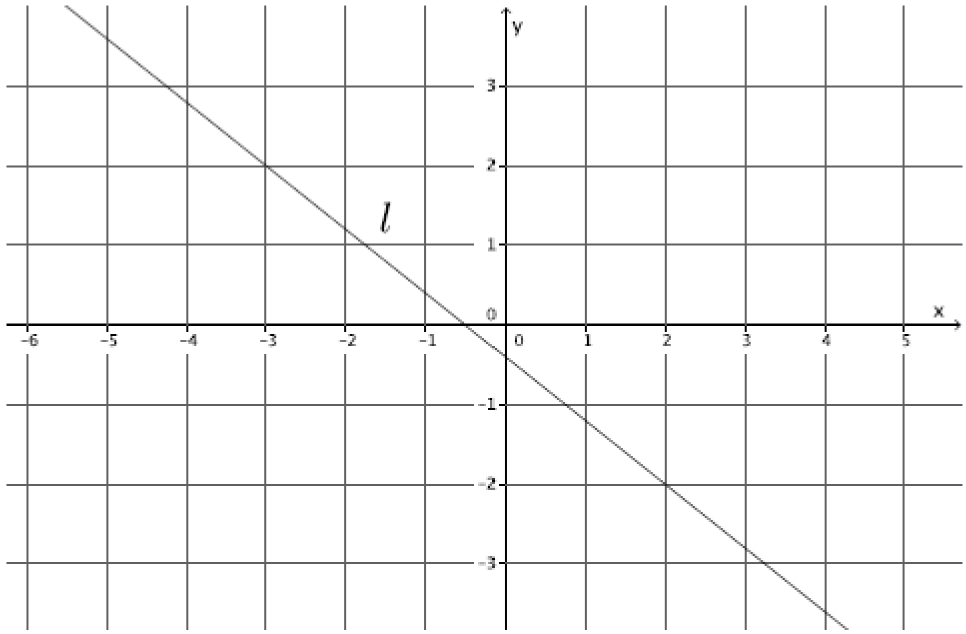


5. A line goes through the point (8, 3) and has slope m = 4. Write the equation of the line.

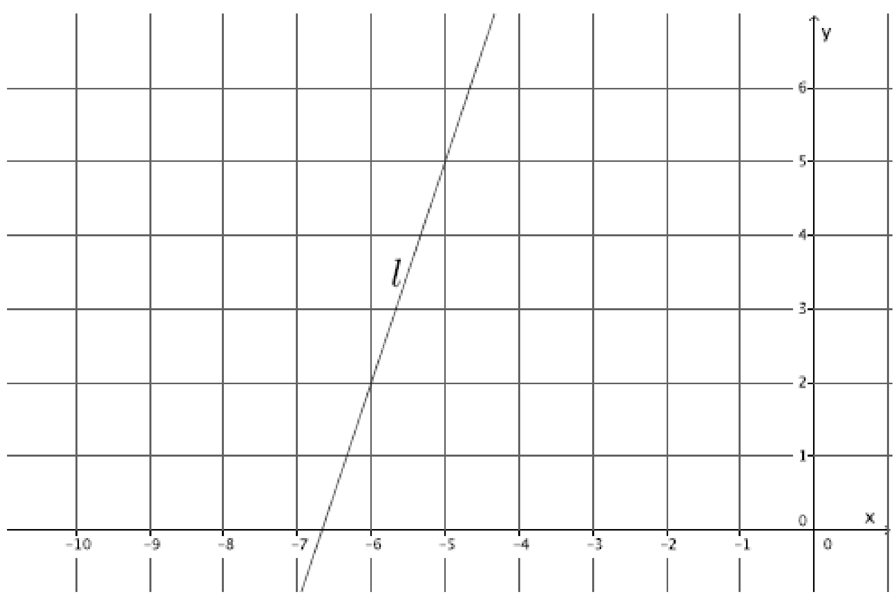
Summary

Independent practice

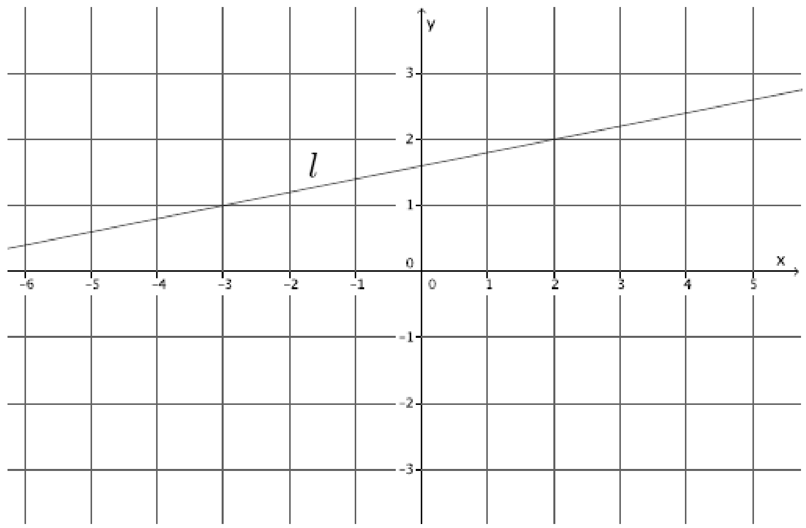
1. Write the equation for the line l shown in the figure.



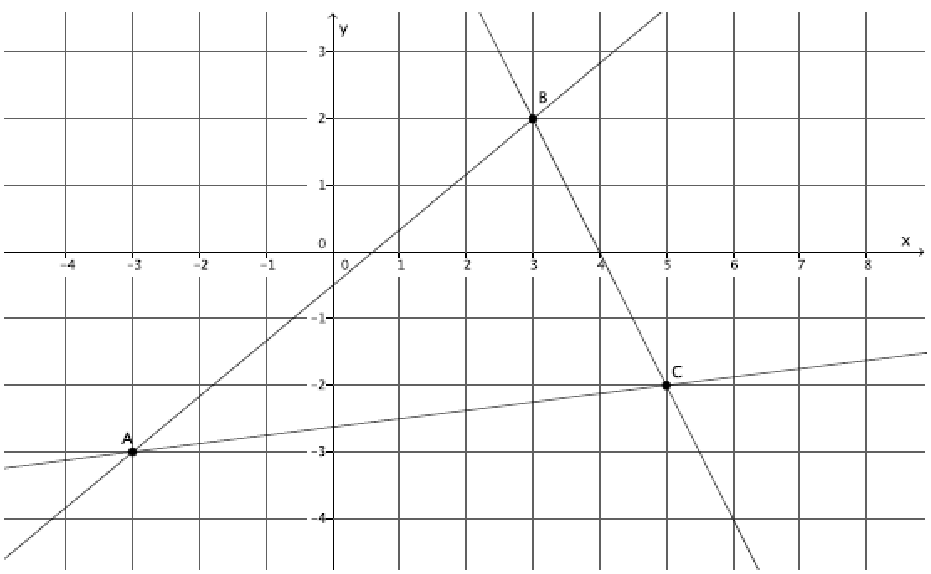
2. Write the equation for the line l shown in the figure.



3. Write the equation for the line l shown in the figure.



4. Triangle ABC is made up of line segments formed from the intersection of lines L_{AB}, L_{BC}, and L_{AC}. Write the equations that represent the lines that make up the triangle.



5. Write the equation for the line that goes through point (-10, 8) with slope m = 6.

6. Write the equation for the line that goes through point (12, 15) with slope m = -2.

7. Write the equation for the line that goes through point (1, 1) with slope m = -9

8. Determine the equation of the line that goes through points (1, 1) and (3, 7)

Lesson 22: Constant Rates Revisited

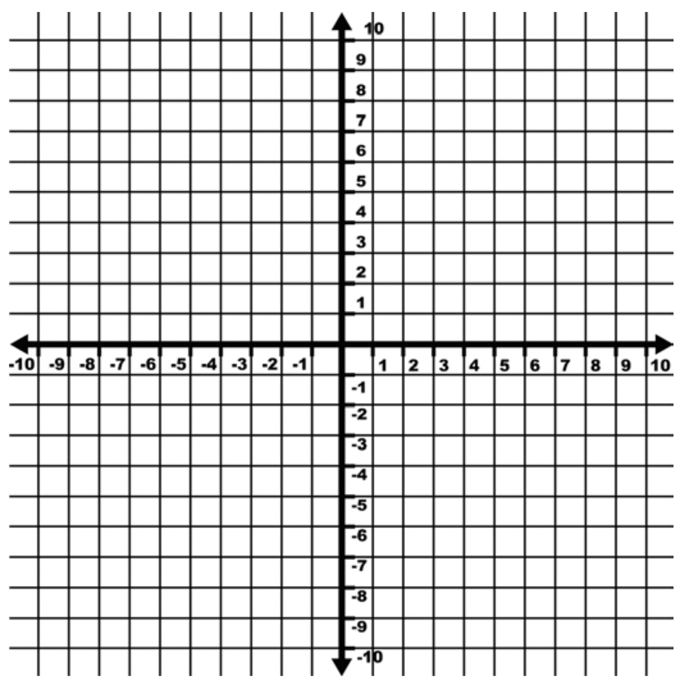
Essential Questions:

Example 1

What is our definition of constant rate?

Erika set her stopwatch to zero and switched it on at the beginning of her walk. She walks at a constant speed of 3 miles per hour. Express this as an equation, table of values, and a graph.

x	y



Example 2

A faucet leaks at a constant rate of 7 gallons per hour. Suppose y gallons leak in x hours from 6:00 a.m.

Express the situation as a linear equation in two variables.

Another faucet leaks at a constant rate and the table below shows the number of gallons, y, that leak in x hour for four selected hours.

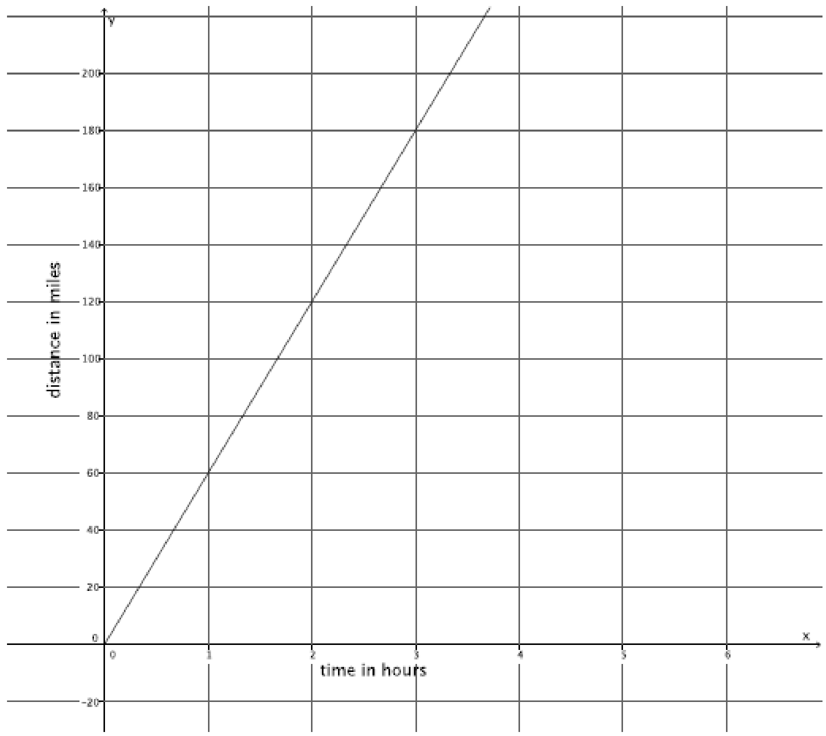
x	y
2	13
4	26
7	45.5
10	65

How can we determine the rate at which this faucet leaks?

Which faucet has a worse leak? That is, which faucet leaks more water over a given time interval?

Example 3

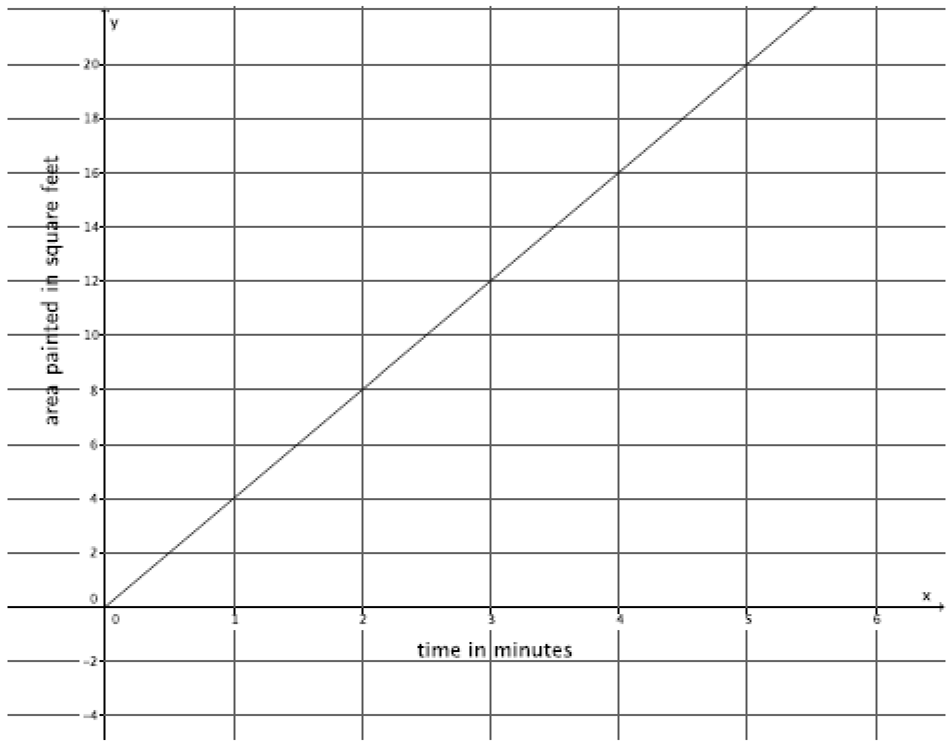
The graph below represents the constant rate at which train A travels.



What is the constant rate of travel for Train A?	
Train B travels at a constant rate. The train travels at an average rate of 95 miles every one and a half hours. Write an equation that represents the constant rate of Train B.	
Which train is traveling at a greater speed? Explain.	
Why do you think the strategy of comparing each rate of change allows us to determine which train is traveling at a greater speed?	

Example 4

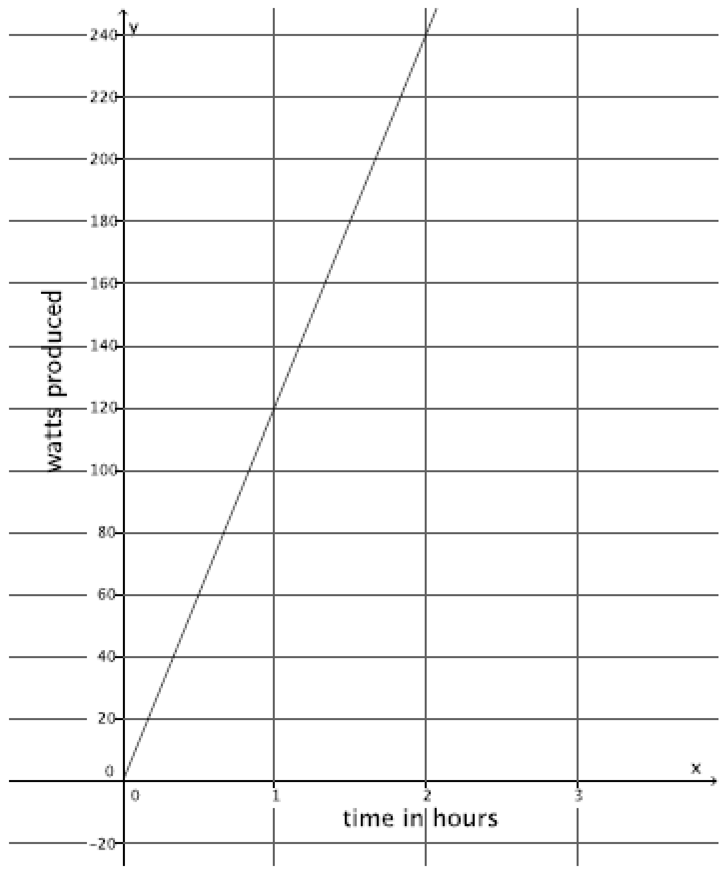
The graph below represents the constant rate at which Kristina can paint.



Kristina's sister, Tracee paints at an average rate of 45 square feet in 12 minutes. Assuming Tracee paints at a constant rate, determine which sister paints faster.	
How does the slope provide the information we need to answer the question about which sister paints faster?	

Example 5

The graph below represents the constant rate of watts of energy produced from a single solar panel produced by Company A



Company B offers a solar panel that produces energy at an average rate of 325 watts in 2.6 hours.

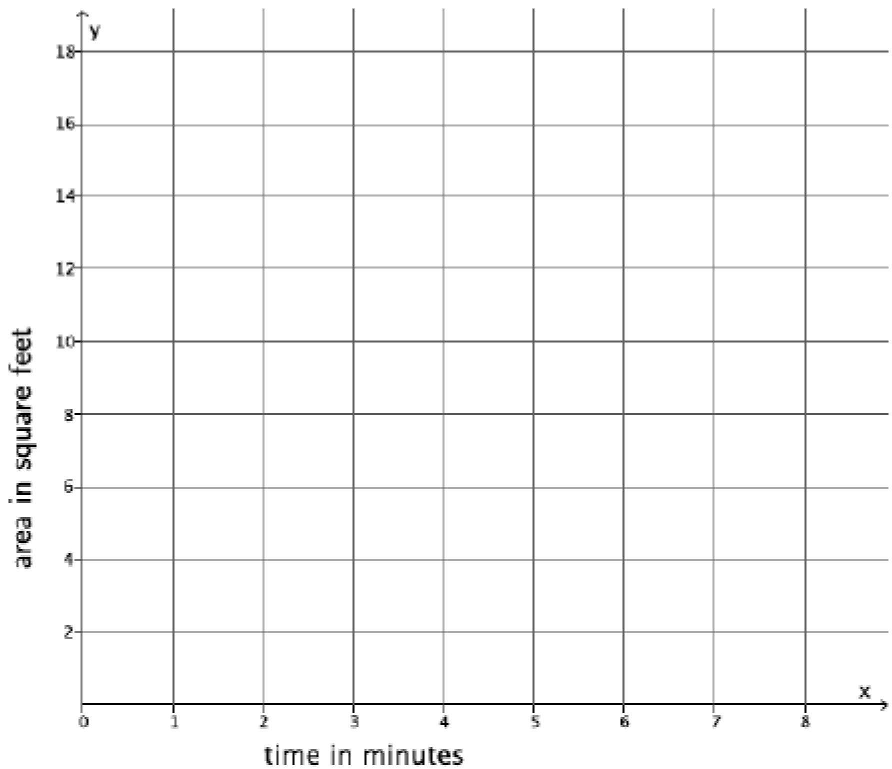
Assuming solar panels produce energy at a constant rate, determine which company produces more efficient solar panels (solar panels that produce more energy per hour).

Exercises 1-5 (work in pairs or small groups to complete)

1. Peter paints a wall at a constant rate of 2 square feet per minute. Assume he paints an area y, in square feet, after x minutes.

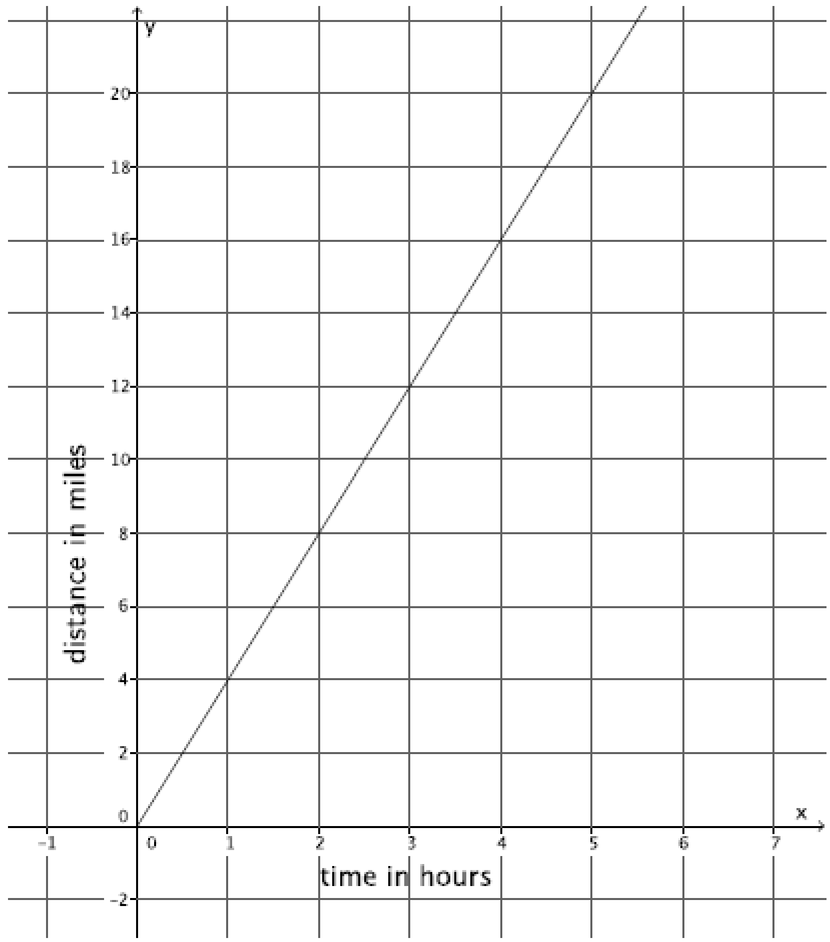
a. Express this situation as a linear equation in two variables.

b. Sketch the graph of the linear equation.



c. Using the graph or the equation, determine the total area he paints after 8 minutes, $1\frac{1}{2}$ hours, and 2 hours. Note that the units are in minutes and hours.

2. The figure below represents Nathan's constant rate of walking.



a. Nicole just finished a 5 mile walkathon. It took her 1.4 hours. Assume she walks at a constant rate. Let y represent the distance Nicole walks in x hours. Describe Nicole's walking at a constant rate as a linear equation in two variables.

b. Who walks at a greater speed? Explain.

3.

a. Susan can type 4 pages of text in 10 minutes. Assuming the types at a constant rate, write the linear equation that represents the situation.

b. The table of values below represents the number of pages that Anne can type, y, in a few selected x minutes. Assume she types at a constant rate.

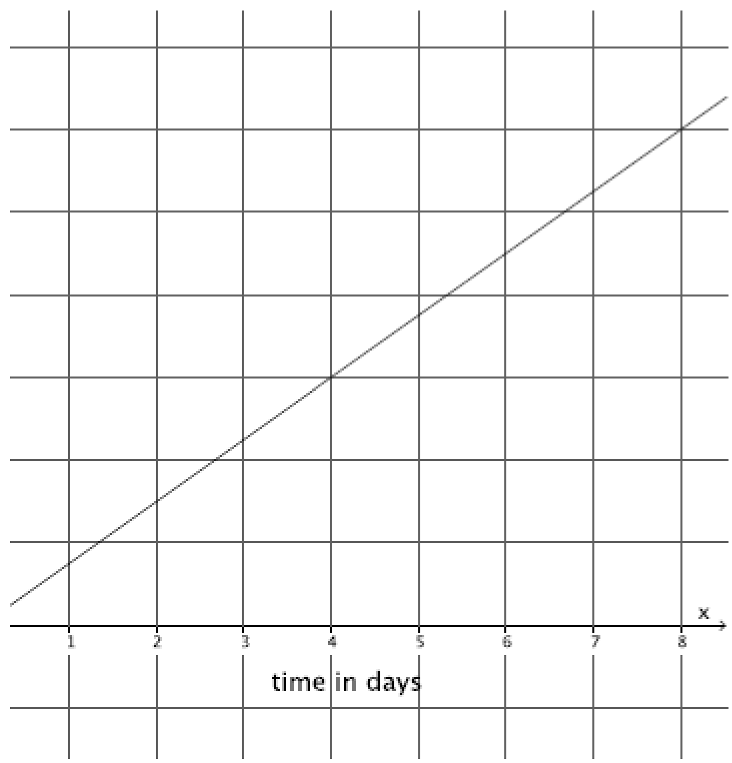
x	y
3	2
5	$\frac{10}{3}$
8	$\frac{16}{3}$
10	$\frac{20}{3}$

Who types faster? Explain.

4.

a. Phil can build 3 birdhouses in 5 days. Assuming he builds birdhouses at a constant rate, write the linear equation that represents the situation.

b. The figure represents Karl's constant rate of building the same kind of birdhouses.



Who builds birdhouses faster?

Explain

5. Explain your general strategy for comparing proportional relationships.

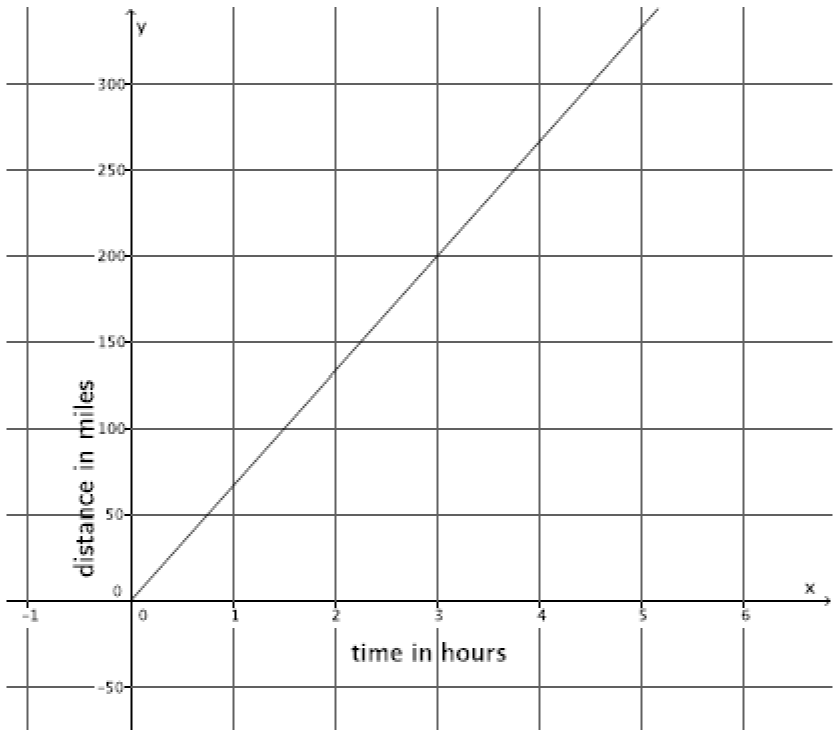
Summary Lesson 22

Independent practice

1.

a. Train A can travel a distance of 500 miles in 8 hours. Assuming the train travels at a constant rate, write the linear equation that represents the situation.

b. The figure represents the constant rate of travel for train B.



Which train is faster? Explain.

2

a. Natalie can paint 40 square feet in 9 minutes. Assuming she paints at a constant rate, write the linear equation that represents the situation.

b. The table of values represents the area painted by Steven for a few selected time intervals.

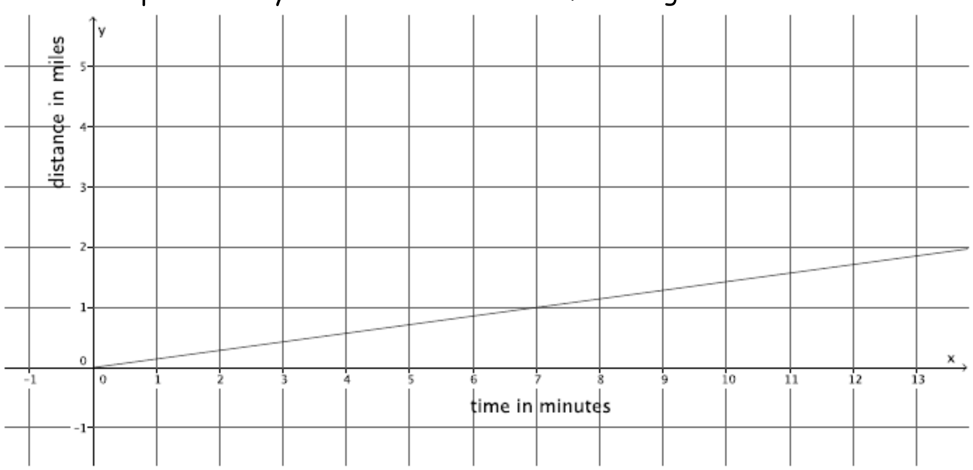
Assume Steven is painting at a constant rate.

Who paints faster? Explain.

Minutes (x)	Area Painted (y)
3	10
5	$\frac{50}{3}$
6	20
8	$\frac{80}{3}$

3 a. Bianca can run 5 miles in 41 minutes. Assuming she runs at a constant rate, write a linear equation that represents the situation.

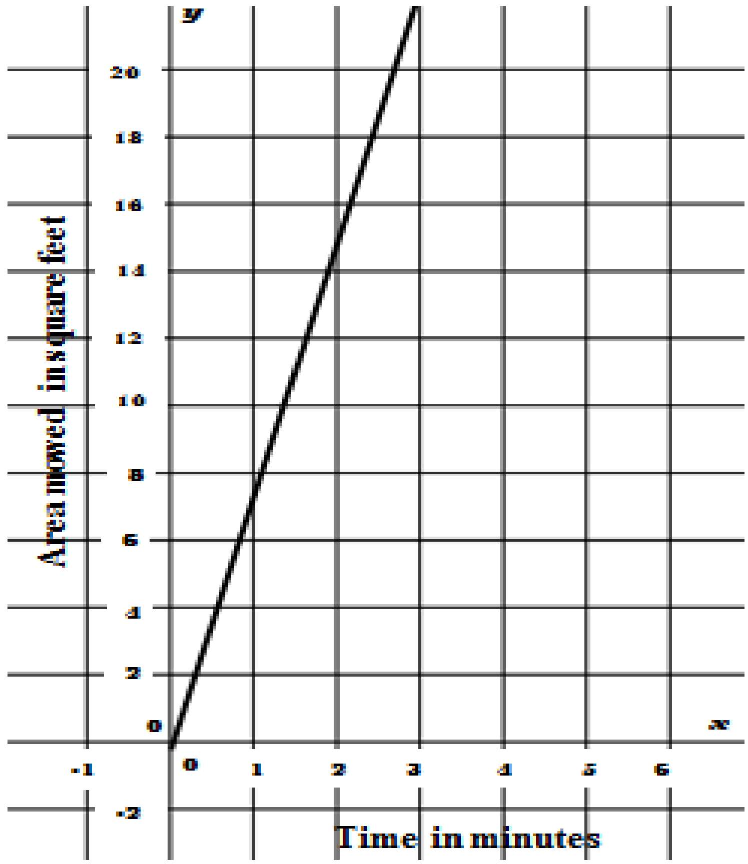
b. The figure below represents Cynthia's constant rate of running.



Who runs faster? Explain.

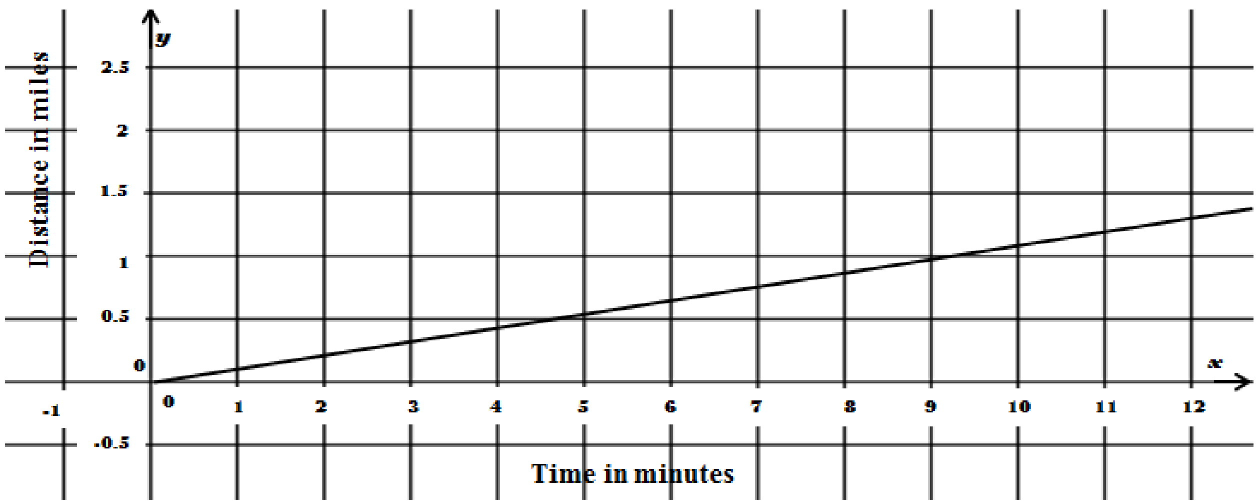
4 a. Geoff can mow an entire lawn of 450 square feet in 30 minutes. Assuming he mows at a constant rate, write the linear equation that represents the situation.

b. The figure represents Mark's constant rate of mowing a lawn.



Who mows faster? Explain.

5 a. Juan can walk to school, a distance of 0.75 miles, in 8 minutes. Assuming he walks at a constant rate, write the linear equation that represents the situation.



b. The figure represents Mark's constant rate of walking.

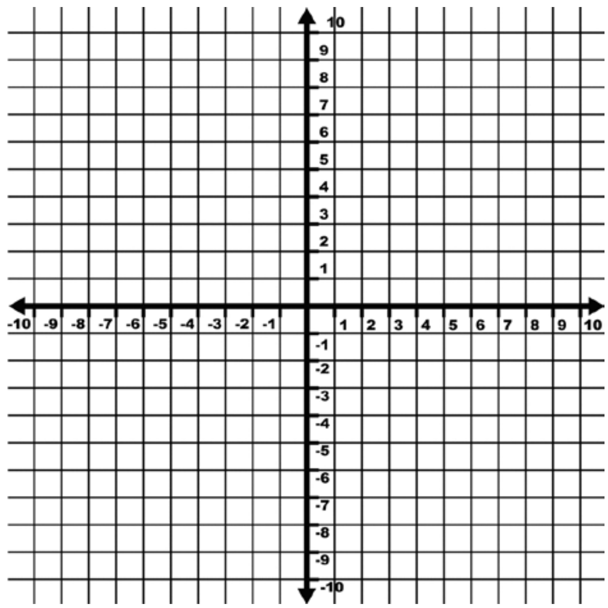
Who mows faster? Explain.

Lesson 23: The Defining Equation of a Line

Essential Questions:

Exploratory Challenge

1. Sketch the graph of the equation 9x + 3y = 18 using intercepts. Then answer parts (a) – (f) that follow.



a. Sketch the graph of the equation y = -3x + 6 on the same coordinate plane.

b. What do you notice about the graphs of 9x + 3y = 18 and y = -3x + 6? Why do you think this is so?

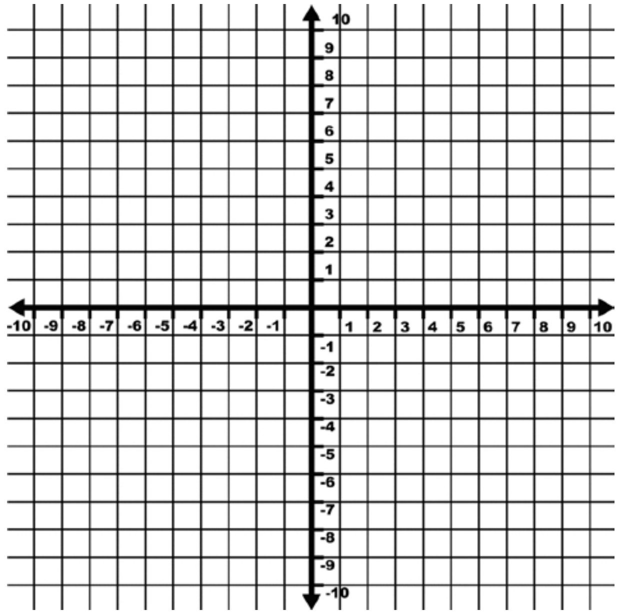
c. Rewrite y = -3x + 6 in standard form.

d. Identify the constants a, b, and c of the equation in standard form from part (c).

e. Identify the constants of the equation 9x + 3y = 18. Note them as a', b' and c'.

f. What do you notice about $\frac{a'}{a}$, $\frac{b'}{b}$, and $\frac{c'}{c}$?

2. Sketch the graph of the equation $y = \frac{1}{2}x + 3$ using the y-intercept and slope. Then answer parts (a) – (f)



a. Sketch the graph of the equation $4x - 8y = -24$ using intercepts on the same coordinate plane.

b. What do you notice about the graphs of $y = \frac{1}{2}x + 3$ and $4x - 8y = -24$? Why do you think this is so?

c. Rewrite $y = \frac{1}{2}x + 3$ in standard form.

d. Identify the constants of the equation $4x - 8y = -24$. Note them as a', b' and c'.

f. What do you notice about $\frac{a'}{a}$, $\frac{b'}{b}$, and $\frac{c'}{c}$?

3. The graphs of the equations $y = \frac{2}{3}x - 4$ and $6x - 9y = 36$ are the same line.

a. rewrite $y = \frac{2}{3}x - 4$ in standard form.

b. Identify the constants a, b, and c of the equation in the standard form from part (a).

c. Identify the constants of the equation $6x - 9y = 36$. Note them as a′, b′ and c′.

d. What do you notice about $\frac{a'}{a}$, $\frac{b'}{b}$, and $\frac{c'}{c}$?

e. You should have noticed that each fraction was equal to the same constant. Multiply that constant by the standard form of the equation from part (a). What do you notice?

Discussion	
What did you notice about the equations you graphed in each of the Exercises 1-3?	
How can we summarize the observations?	
We want to show that (1) is true. We need to show that the graphs of the equations $ax + by = c$ and $a'x + b'y = c'$ are the same. What information are we given that will be useful in showing that the two equations are the same?	

To prove (2), we will assume that $a \neq 0$ and $b \neq 0$, that is, we are not dealing with horizontal or vertical lines.

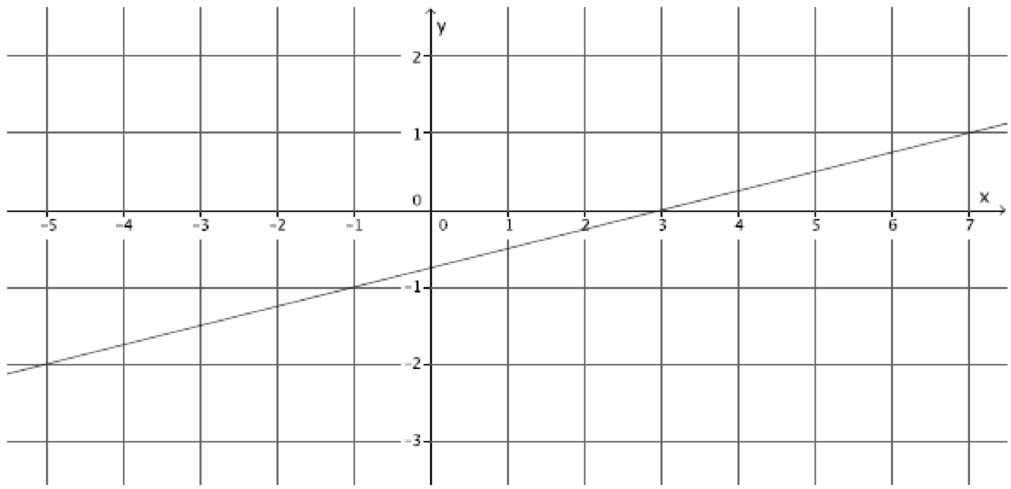
Exercises 4-8

4. Write three equations whose graphs are the same line as the equation $3x + 2y = 7$.

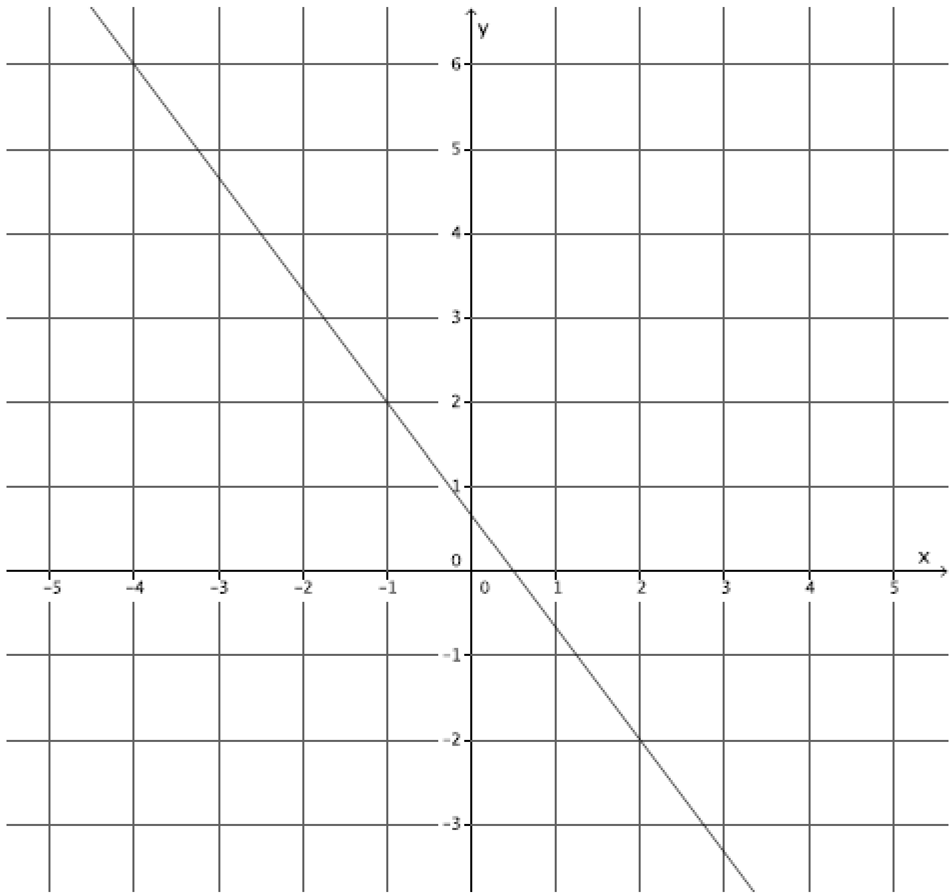
5. Write three equations whose graphs are the same line as the equation $x + 9y = \frac{3}{4}$.

6. Write three equations whose graphs are the same line as the equation $-9x + 5y = -4$.

7. Write at least two equations in the form $ax + by = c$ whose graphs are the line shown below.



8. Write at least two equations in the form $ax + by = c$ whose graphs are the line shown below.



Summary

Independent Practice

1. Do the equations $x + y = -2$ and $3x + 3y = -6$ define the same line? Explain.

2. Do the equations $y = -\frac{5}{4}x + 2$ and $10x + 8y = 16$ define the same line? Explain.

3. Write an equation that would define the same line as $7x - 2y = 5$.

4. Challenge: Show that if the two lines given are $ax + by = c$ and $a'x + b'x = c'$ are the same when b = 0 (vertical lines), then there exists a non-zero number s, so that a' = sa, b' = sb and c' = sc

5. Challenge: Show that if the two lines given are $ax + by = c$ and $a'x + b'x = c'$ are the same when a = 0 (horizontal lines), then there exists a non-zero number s, so that a' = sa, b' = sb and c' = sc

Lesson 24 –Introduction to Simultaneous Equations

Essential Questions:

On Your Own 1-3:

1. Derek scored 30 points in the basketball game he played, and not once did he go to the free throw line. That means that Derek scored two-point shots and three-point shots. List as many combinations of two- and three-pointers as you can that would total 30 points. Number of Two-Pointers Number of Three-Pointers Write an equation to describe the data.

Number of Two-Pointers	Number of Three-Pointers

Write an equation to describe the data.

2. Derek tells you that the number of two-point shots that he made is five more than the number of three-point shots. How many combinations can you come up with that fit this scenario? (Don't worry about the total number of points.) N

Number of Two-Pointers	Number of Three-Pointers

Write an equation to describe the data.

3. Which pair of numbers from your table in Exercise 2 would show Derek's Actual score of 30 points?

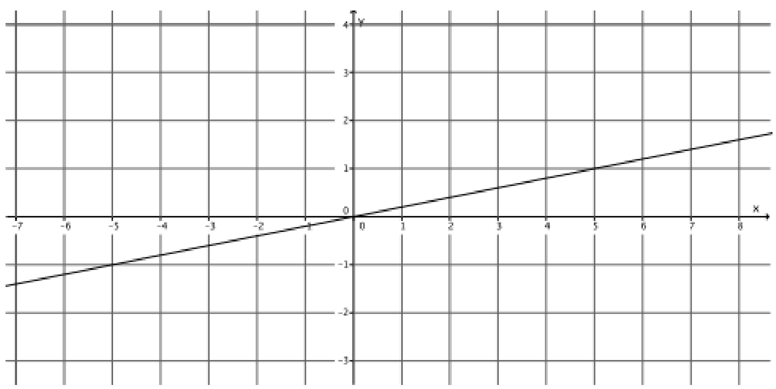
Discussion:

Example 1:
Pia types at a constant rate of 3 pages every 15 minutes. Suppose she types y pages in x minutes. Pia's constant rate can be expressed as the linear equation $y = \frac{1}{5}x$.

Number of Minutes (x)	Pages Typed (y)
0	0
5	1
10	2
15	3
20	4
25	5

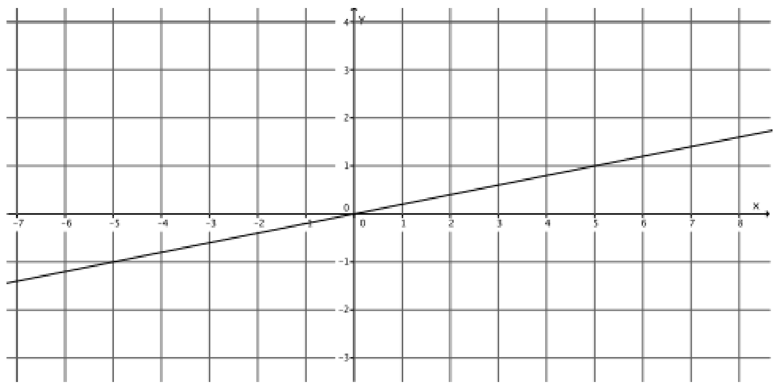
The following is the graph of $y = \frac{1}{5}x$.

Pia typically begins work at 8: 00 a.m. every day. On our graph, her start time is reflected as the origin of the graph (0, 0), that is, zero minutes worked and zero pages typed. For some reason, she started working 5 minutes earlier today.

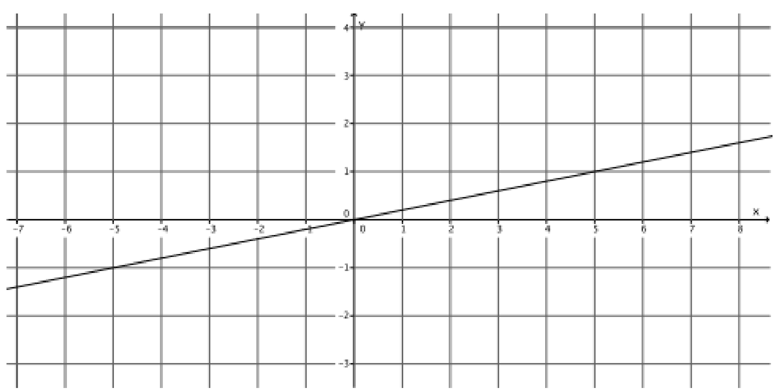


How can we reflect the extra 5 minutes she worked on our graph?

If we translate the graph of $y = \frac{1}{5}x$ to the right 5 units to reflect the additional 5 minutes of work, then we have the following graph. Does a translation of 5 units to the right reflect her working an additional 5 minutes?



Let's see what happens when we translate 5 units to the left. Does a translation of 5 units to the left reflect her working an additional 5 minutes?



If Pia started work 20 minutes early, what equation would represent the number of pages she could type in x minutes?

Example 2
Sandy and Charlie walk at constant speeds. Sandy walks from their school to the train station in 15 minutes, and Charlie walks the same distance in 10 minutes. Charlie starts 4 minutes after Sandy left the school. Can Charlie catch up to Sandy? The distance between the school and the station is 2 miles.

What is Sandy's average speed in 15 minutes? Explain.

What is Charlie's average speed in 10 minutes? Explain.

Let's put some information about Charlie's walk in a table:

Number of Minutes (x)	Miles Walked (y)
0	
5	
10	
15	
20	
25	

At x minutes, Sandy has walked 4 minutes longer than Charlie. Then the distance that Sandy walked in $x + 4$ minutes is y miles. Then the linear equation that represents Sandy's motion is

Let's put some information about Sandy's walk in a table:

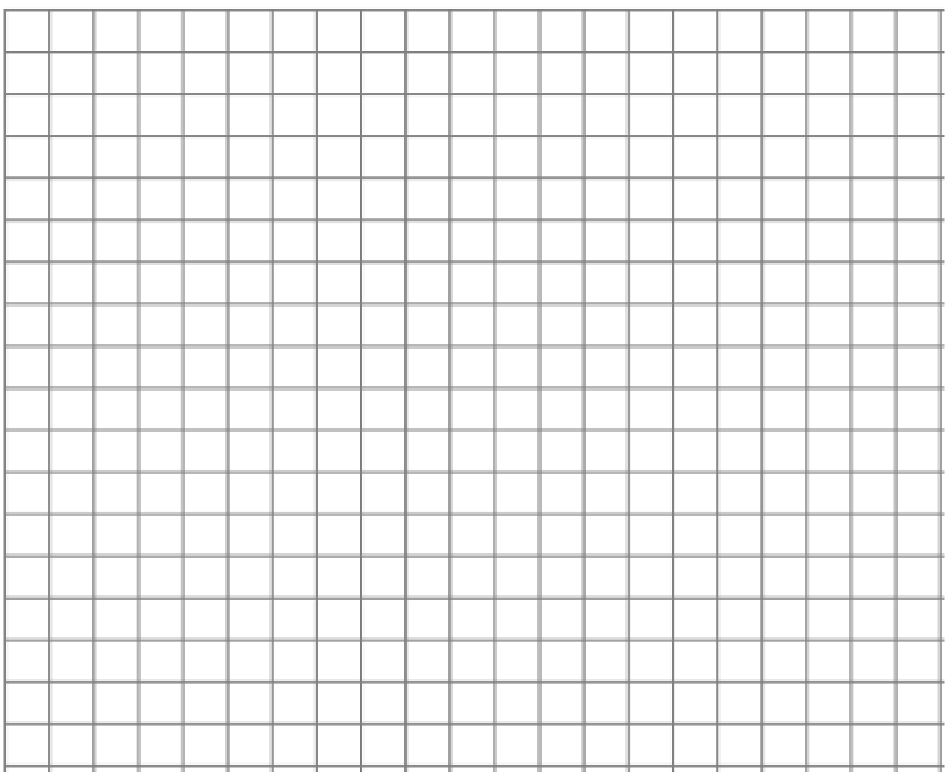
Number of Minutes (x)	Miles Walked (y)
0	
5	
10	
15	
20	
25	

Now let's sketch the graphs of each linear equation on a coordinate plane.

Recall the original question that was asked: Can Charlie catch up to Sandy? (Keep in mind that the train station is 2 miles from the school.)	
At approximately what point do the graphs of the lines intersect?	

Example 3: Randi and Craig ride their bikes at constant speeds. It takes Randi 25 minutes to bike 4 miles. Craig can bike 4 miles in 32 minutes. If Randi gives Craig a 20 minute head start, about how long will it take Randi to catch up to Craig?	Randi's average: Craig's average:

Now we can sketch the graph of the system of equations on a coordinate plane.



Now, answer the question: About how long will it take Randi to catch up to Craig? We can give two answers: one in terms of time and the other in terms of distance. What are those answers?	
At approximately what point do the graphs of the lines intersect?	

On your own

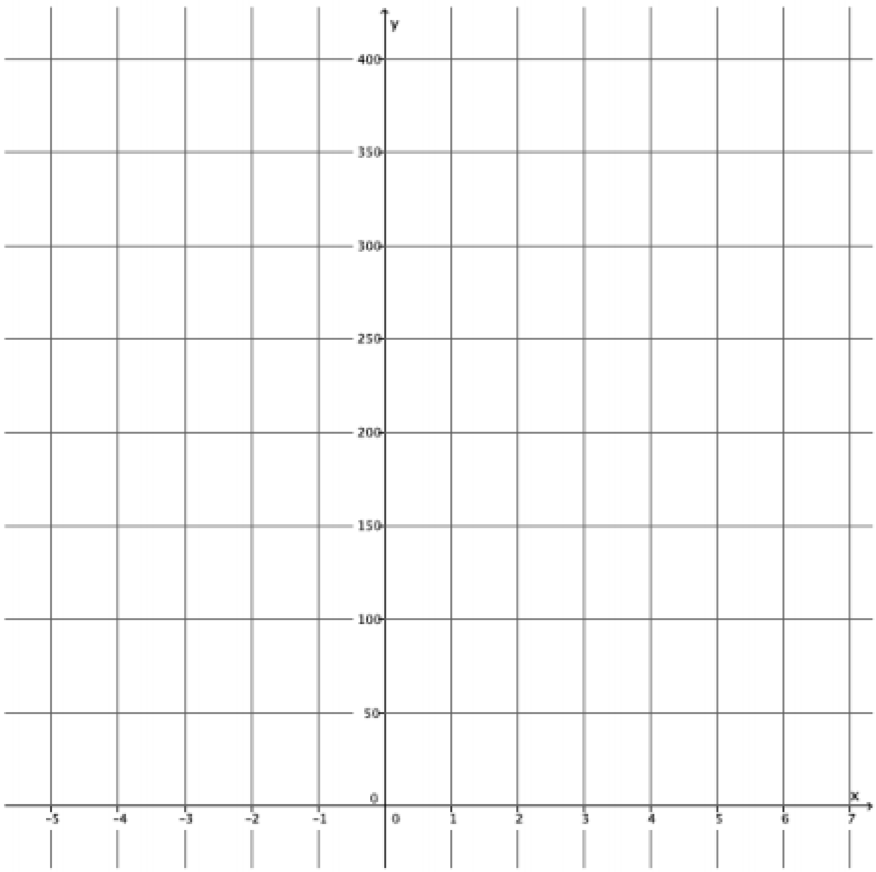
Exercise 4

Efrain and Fernie are on a road trip. Each of them drives at a constant speed. Efrain is a safe driver and travels 45 miles per hour for the entire trip. Fernie is not such a safe driver. He drives 70 miles per hour throughout the trip. Fernie and Efrain left from the same location, but Efrain left at 8: 00 a.m., and Fernie left at 11: 00 a.m. Assuming they take the same route, will Fernie ever catch up to Efrain? If so, approximately when?

a. Write the linear equation that represents Efrain's constant speed. Make sure to include in your equation the extra time that Efrain was able to travel.	
b. Write the linear equation that represents Fernie's constant speed.	

c. Write the system of linear equations that represents this situation.	

d. Sketch the graphs of the two linear equations.



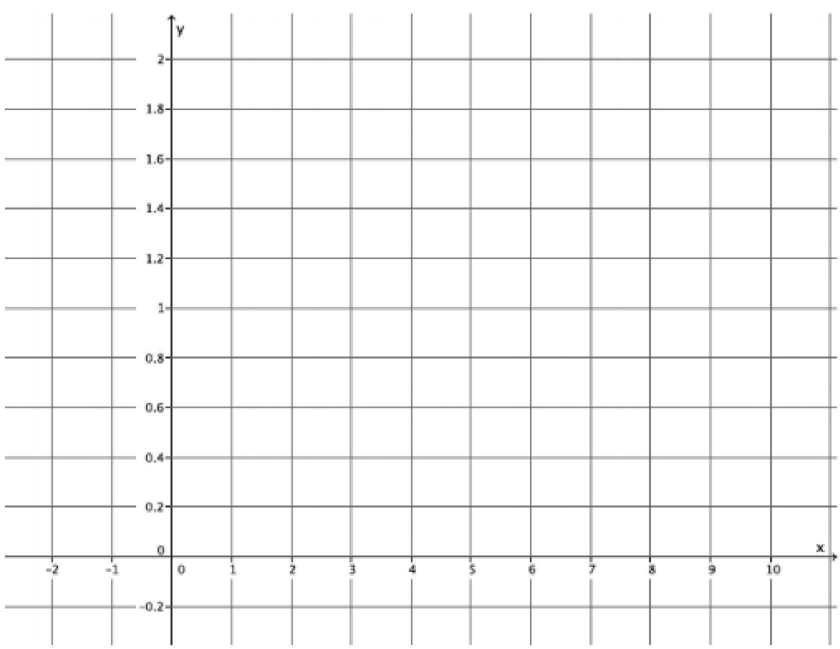
e. Will Fernie ever catch up to Efrain? If so, approximately when?	
f. At approximately what point do the graphs of the lines intersect?	

Exercise 5

Jessica and Karl run at constant speeds. Jessica can run 3 miles in 15 minutes. Karl can run 2 miles in 8 minutes. They decide to race each other. As soon as the race begins, Karl realizes that he did not tie his shoes properly and takes 1 minute to fix them.

a. Write the linear equation that represents Jessica's constant speed. Make sure to include in your equation the extra time that Jessica was able to run.	
b. Write the linear equation that represents Karl's constant speed.	
c. Write the system of linear equations that represents this situation.	

d. Sketch the graphs of the two linear equations.



e. Use the graph to answer the questions below. If Jessica and Karl raced for 2 miles, who would win? Explain.	
f. If the winner of the race was the person who got to a distance of $\frac{1}{2}$ mile first, who would the winner be? Explain.	
g. At approximately what point would Jessica and Karl be tied? Explain.	

Lesson 24 Summary:

Lesson 24 – Independent Practice

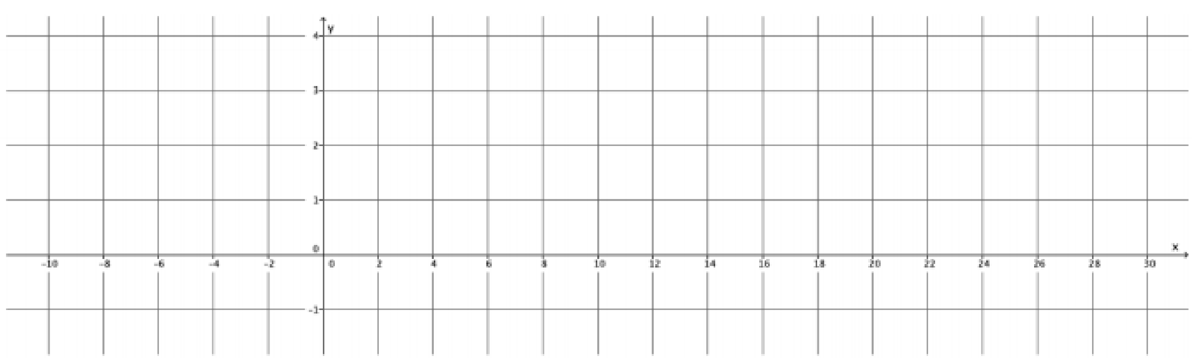
1. Jeremy and Gerardo run at constant speeds. Jeremy can run 1 mile in 8 minutes and Gerardo can run 3 miles in 33 minutes. Jeremy started running 10 minutes after Gerardo. Assuming they run the same path, when will Jeremy catch up to Gerardo?

a. Write the linear equation that represents Jeremy's constant speed.

b. Write the linear equation that represents Gerardo's constant speed. Make sure to include in your equation the extra time that Gerardo was able to run.

c. Write the system of linear equations that represents this situation.

d. Sketch the graphs of the two equations.

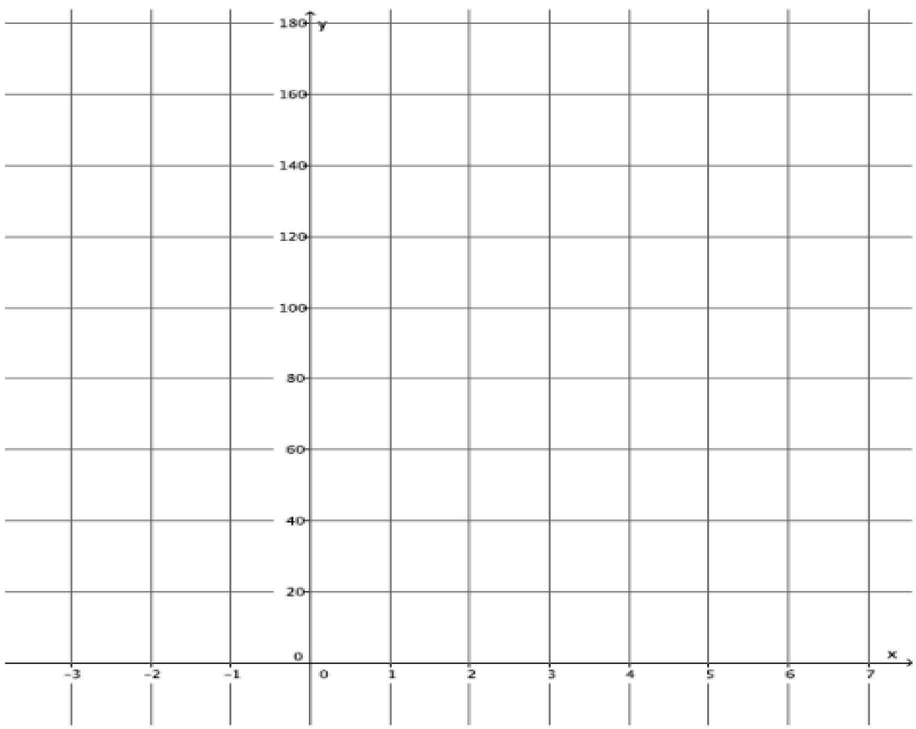


e. Will Jeremy ever catch up to Gerardo? If so, approximately when?

f. At approximately what point do the graphs of the lines intersect?

2. Two cars drive from town A to town B at constant speeds. The blue car travels 25 miles per hour and the red car travels 60 miles per hour. The blue car leaves at 9: 30 a.m., and the red car leaves at noon. The distance between the two towns is 150 miles.

a. Who will get there first? Write and graph the system of linear equations that represents this situation.



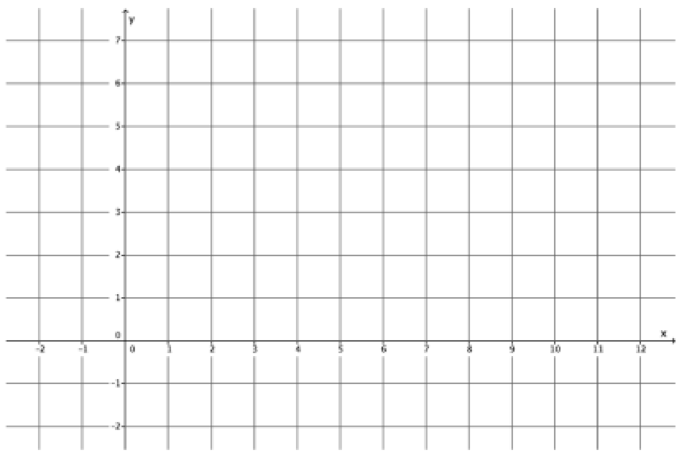
b. At approximately what point do the graphs of the lines intersect?

Lesson 25 – Geometric Interpretation of the Solutions of a Linear System

Essential Questions:

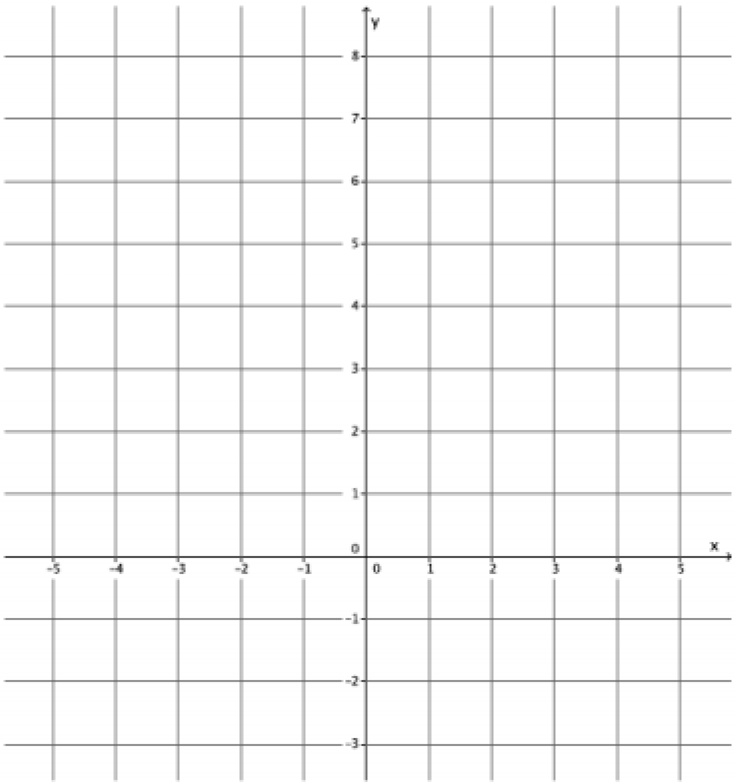
On Your Own:

1. Sketch the graph of the linear system on a coordinate plane: $\begin{cases} 2y + x = 12 \\ y = \frac{5}{6}x - 2 \end{cases}$.	



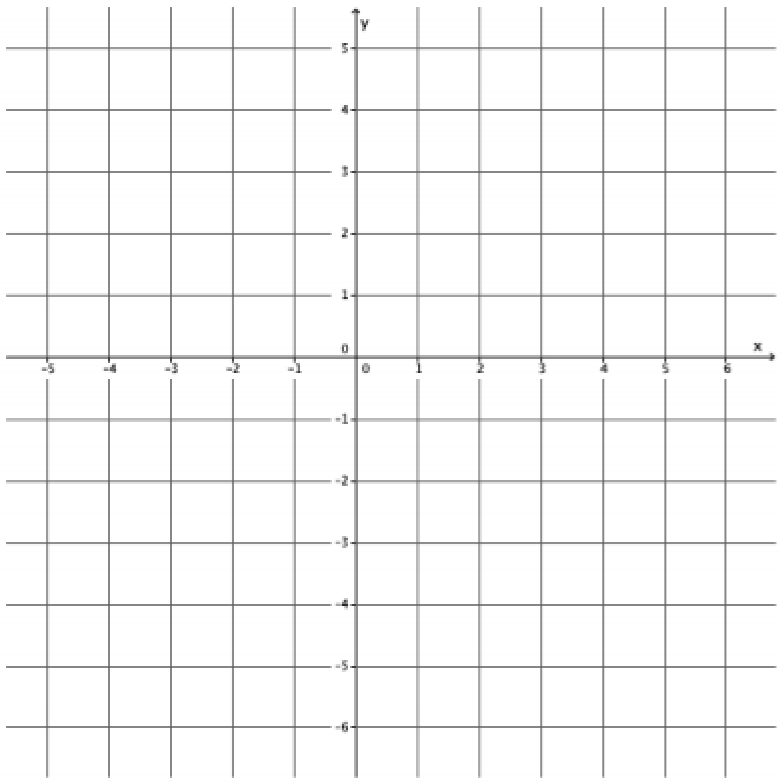
a. Name the ordered pair where the graphs of the two linear equations intersect.	
b. Verify that the ordered pair named in part (a) is a solution to $2y + x = 12$.	
c. Verify that the ordered pair named in part (a) is a solution to $y = \frac{5}{6}x - 2$	
d. Could the point (4, 4) be a solution to the system of linear equations? That is, would (4, 4) make both equations true? Why or why not?	

2. Sketch the graph of the linear system on a coordinate plane: $\begin{cases} x + y = -2 \\ y = 4x + 3 \end{cases}$



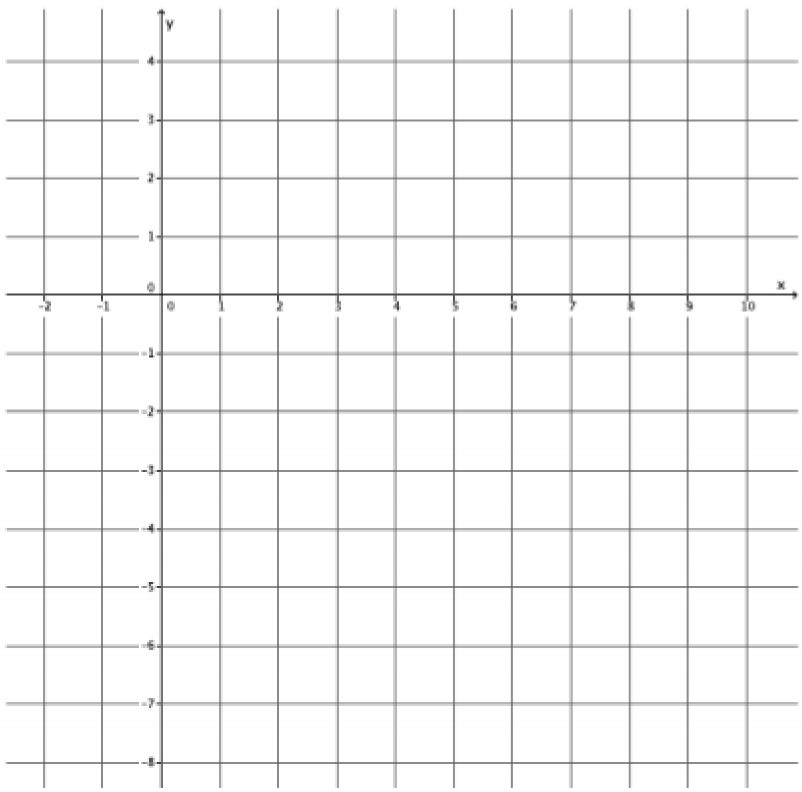
a. Name the ordered pair where the graphs of the two linear equations intersect.	
b. Verify that the ordered pair named in part (a) is a solution to $x + y = -2$.	
c. Verify that the ordered pair named in part (a) is a solution to $y = 4x + 3$	
d. Could the point (−4, 2) be a solution to the system of linear equations? That is, would (−4, 2) make both equations true? Why or why not?	

3. Sketch the graph of the linear system on a coordinate plane: $\begin{cases} 3x + y = -3 \\ -2x + y = 2 \end{cases}$



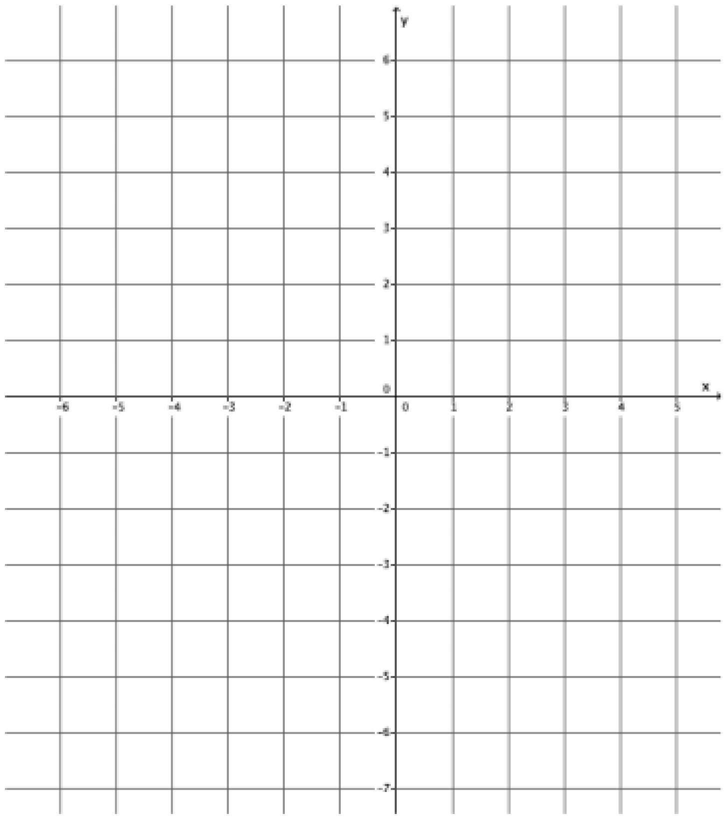
a. Name the ordered pair where the graphs of the two linear equations intersect.	
b. Verify that the ordered pair named in part (a) is a solution to $3x + y = -3$.	
c. Verify that the ordered pair named in part (a) is a solution to $-2x + y = 2$.	
d. Could the point (1, 4) be a solution to the system of linear equations? That is, would (1, 4) make both equations true? Why or why not?	

4. Sketch the graph of the linear system on a coordinate plane: $\begin{cases} 2x - 3y = 18 \\ 2x + y = 2 \end{cases}$



a. Name the ordered pair where the graphs of the two linear equations intersect.	
b. Verify that the ordered pair named in part (a) is a solution to $2x - 3y = 18$.	
c. Verify that the ordered pair named in part (a) is a solution to $2x + y = 2$.	
d. Could the point (3, –1) be a solution to the system of linear equations? That is, would (3,–1) make both equations true? Why or why not?	

5. Sketch the graph of the linear system on a coordinate plane: $\begin{cases} y - x = 3 \\ y = -4x - 2 \end{cases}$



a. Name the ordered pair where the graphs of the two linear equations intersect.	
b. Verify that the ordered pair named in part (a) is a solution to $y - x = 3$.	
c. Verify that the ordered pair named in part (a) is a solution to $y = -4x - 2$.	
d. Could the point (–2, 6) be a solution to the system of linear equations? That is, would (–2, 6) make both equations true? Why or why not?	

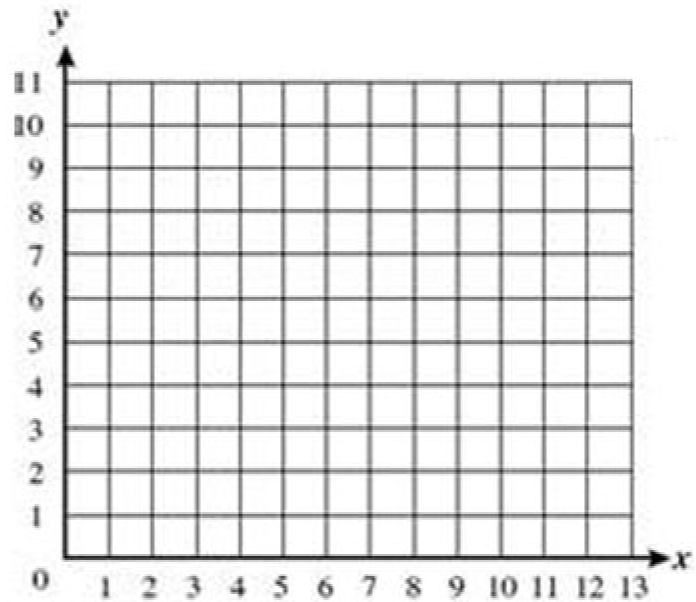
Example 6

6. Write two different systems of equations with $(1, -2)$ as the solution.

Lesson 25 Summary:

Lesson 25 – Independent Practice

1. Sketch the graphs of the linear system on a coordinate plane: $\begin{cases} y = \frac{1}{3}x + 1 \\ y = -3x + 11 \end{cases}$

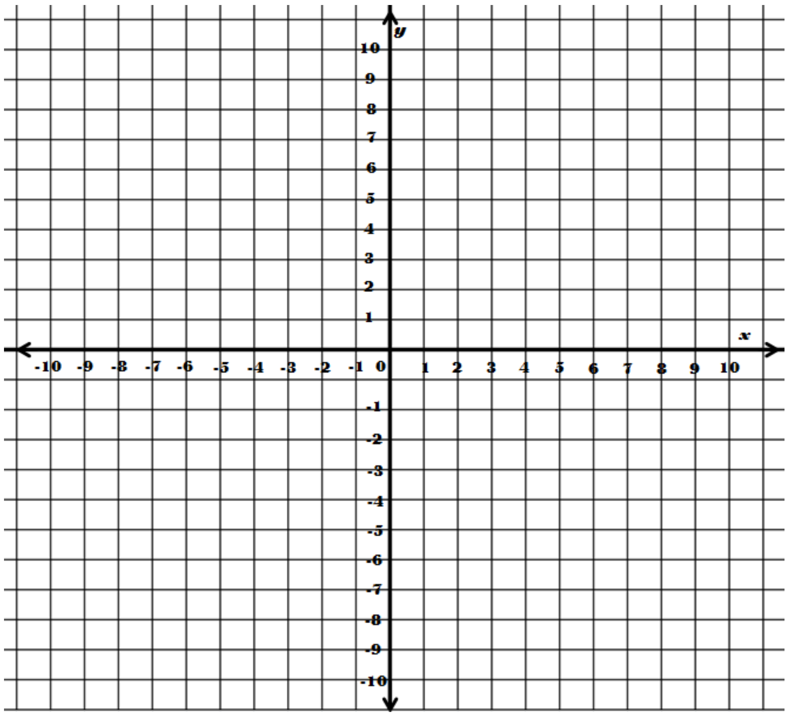


a. Name the ordered pair where the graphs of the two linear equations intersect.

b. Verify that the ordered pair named in part (a) is a solution to $y = \frac{1}{3}x + 1$.

c. Verify that the ordered pair named in part (a) is a solution to $y = -3x + 11$.

2. Sketch the graphs of the linear system on a coordinate plane: $\begin{cases} y = \frac{1}{2}x + 4 \\ x + 4y = 4 \end{cases}$

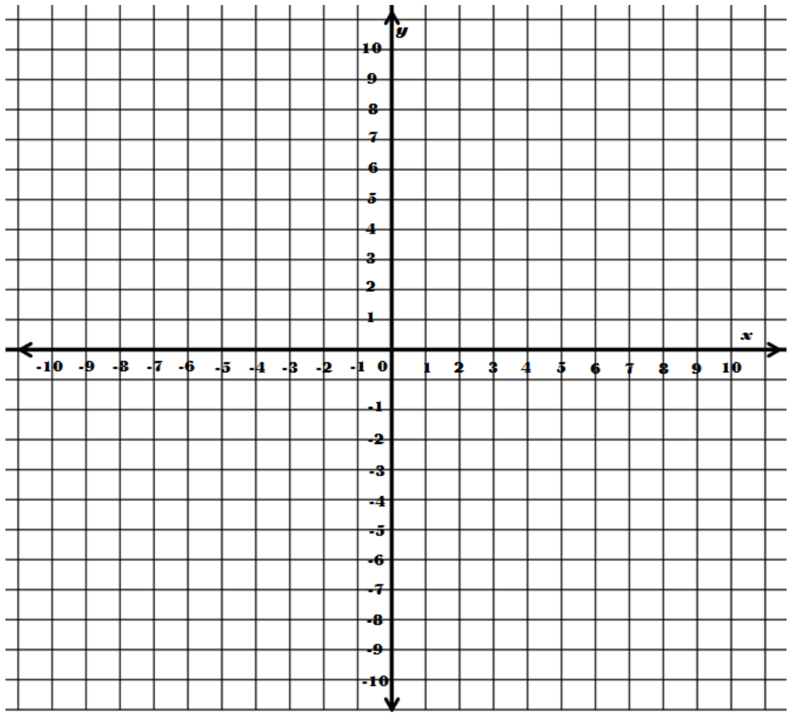


a. Name the ordered pair where the graphs of the two linear equations intersect.

b. Verify that the ordered pair named in part (a) is a solution to $y = \frac{1}{2}x + 4$.

c. Verify that the ordered pair named in part (a) is a solution to $x + 4y = 4$

3. Sketch the graphs of the linear system on a coordinate plane: $\begin{cases} y = 2 \\ x + 2y = 10 \end{cases}$

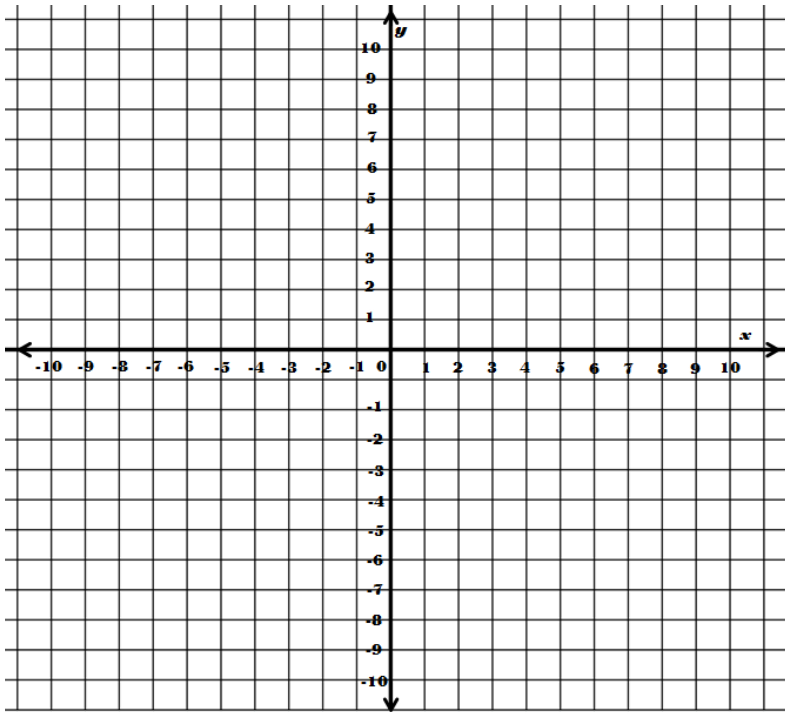


a. Name the ordered pair where the graphs of the two linear equations intersect.

b. Verify that the ordered pair named in part (a) is a solution to y = 2.

c. Verify that the ordered pair named in part (a) is a solution to x + 2y = 10.

4. Sketch the graphs of the linear system on a coordinate plane: $\begin{cases} -2x + 3y = 18 \\ \ \ 2x + 3y = 6 \end{cases}$

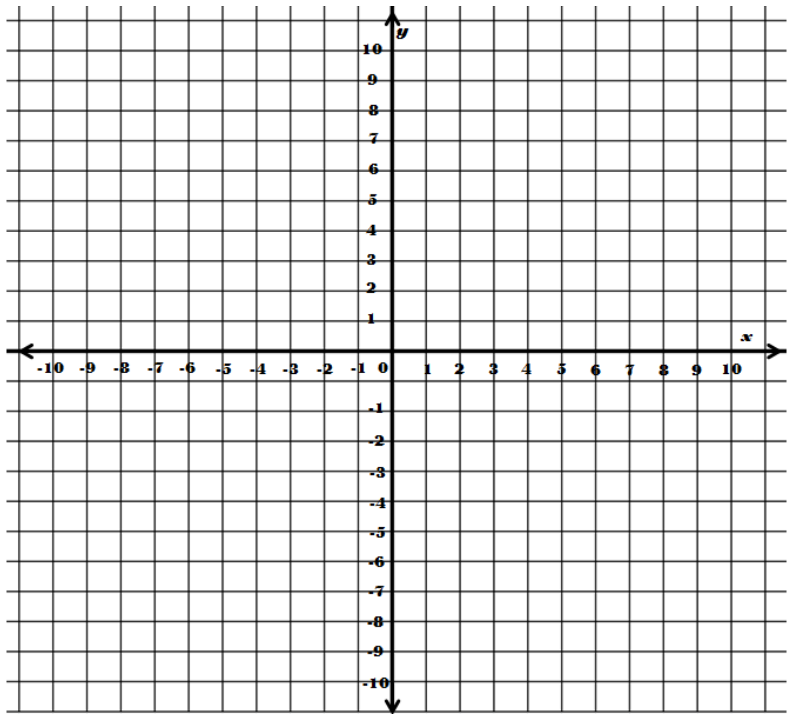


a. Name the ordered pair where the graphs of the two linear equations intersect.

b. Verify that the ordered pair named in part (a) is a solution to $-2x + 3y = 18$.

c. Verify that the ordered pair named in part (a) is a solution to $2x + 3y = 6$

5. Sketch the graphs of the linear system on a coordinate plane $\begin{cases} x + 2y = 2 \\ y = \frac{2}{3}x - 6 \end{cases}$



a. Name the ordered pair where the graphs of the two linear equations intersect.

b. Verify that the ordered pair named in part (a) is a solution to $x + 2y = 2$

c. Verify that the ordered pair named in part (a) is a solution to $y = \frac{2}{3}x - 6$

6. Without sketching the graph, name the ordered pair where the graphs of the two linear equations intersect.

$$\begin{cases} x = 2 \\ y = -3 \end{cases}$$

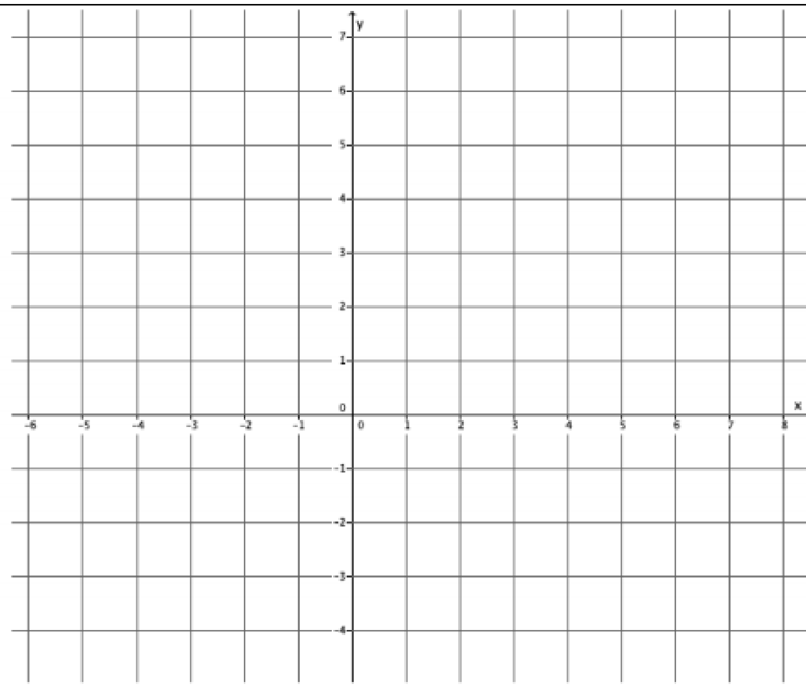
Lesson 26 – Characterization of Parallel Lines

Essential Questions:

On your own 1-3:

1. Sketch the graphs of the system:

$$\begin{cases} y = \frac{2}{3}x + 4 \\ y = \frac{4}{6}x - 2 \end{cases}$$

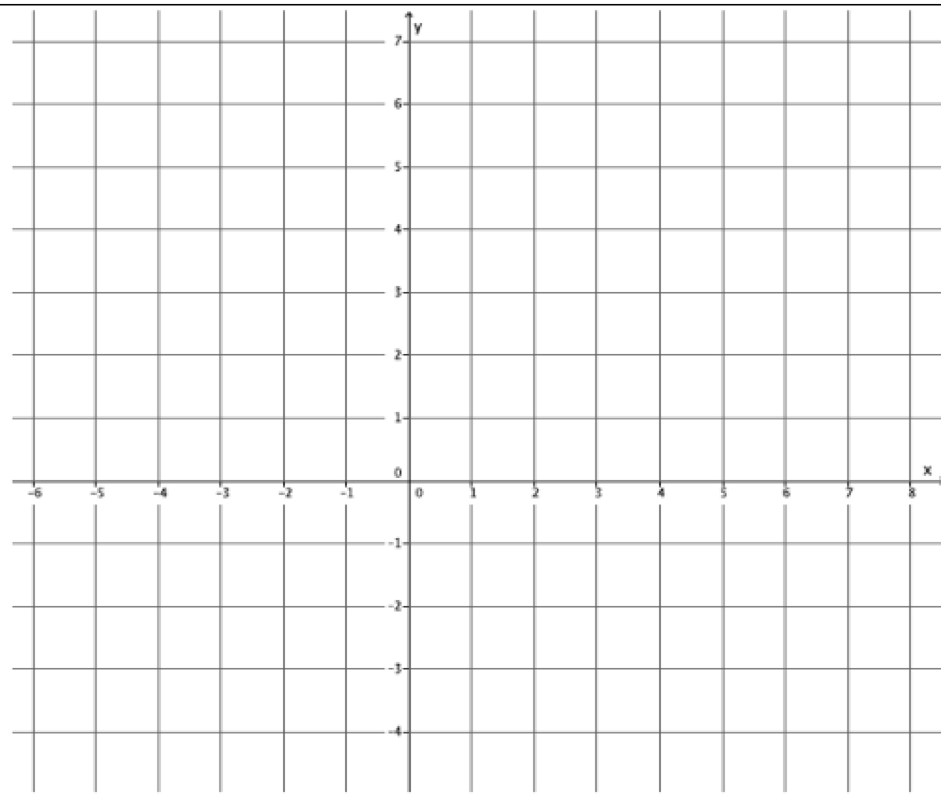


a. Identify the slope of each equation. What do you notice?

b. Identify the y-intercept of each equation. Are the y-intercepts the same or different?

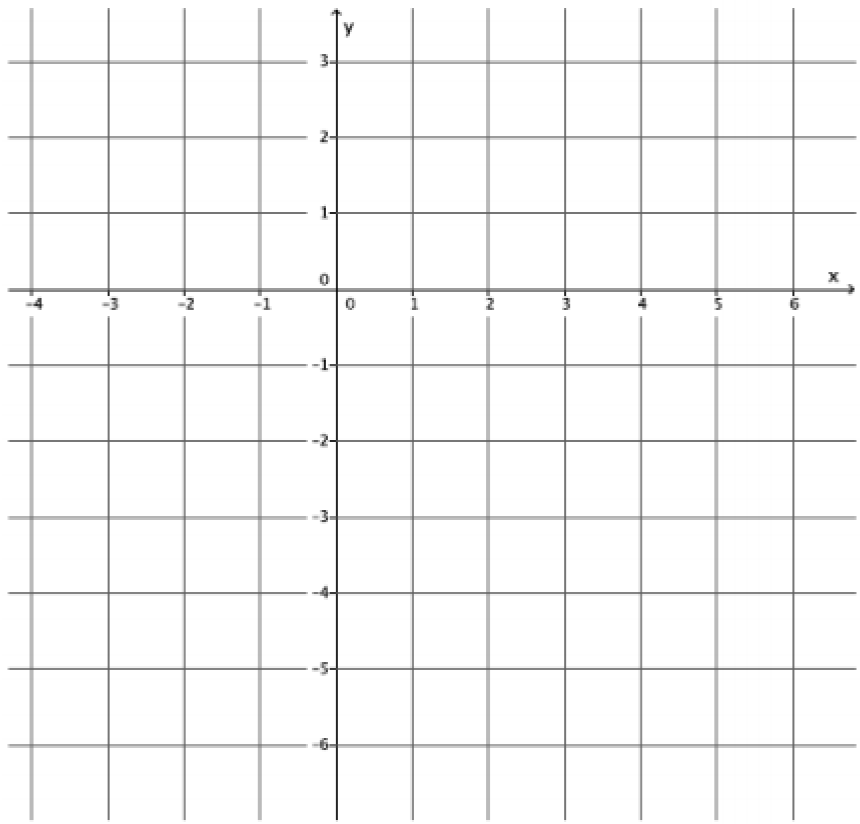
2. Sketch the graphs of the system:

$$\begin{cases} y = -\frac{5}{4}x + 7 \\ y = -\frac{5}{4}x + 2 \end{cases}$$



a. Identify the slope of each equation. What do you notice?

b. Identify the y -intercept of each equation. Are the y -intercepts the same or different?

3. Sketch the graphs of the system: $\begin{cases} y = 2x - 5 \\ y = 2x - 1 \end{cases}$	
a. Identify the slope of each equation. What do you notice?	
b. Identify the y-intercept of each equation. Are the y-intercepts the same or different?	

Discussion: What did you notice about each of the systems you graphed in On your own 1-3?	
What did you notice about the slopes in each system?	
What did you notice about the y-intercepts of the equations in each system?	
If the equations had the same y-intercepts and the same slope, what would we know about the graphs of the lines?	
Write a summary of the conclusions you have reached by completing On Your own 1-3.	
Suppose you have a pair of parallel lines on a coordinate plane. In how many places will those lines intersect?	
Suppose you are given a system of linear equations whose graphs are parallel lines. How many solutions will the system have?	

A system can easily be recognized as having no solution when it is in the form of $\begin{cases} x = 2 \\ x = -7 \end{cases}$ or $\begin{cases} y = 6 \\ y = 15 \end{cases}$ Why is that so?	
On Your Own: 4-10	
4. Write a system of equations that has no solution.	
5. Write a system of equations that has (2, 1) as a solution.	
6. How can you tell if a system of equations has a solution or not?	

7. Does the system of linear equations shown below have a solution? Explain. $\begin{cases} 6x - 2y = 5 \\ 4x - 3y = 5 \end{cases}$	
8. Does the system of linear equations shown below have a solution? Explain. $\begin{cases} -2x + 8y = 14 \\ x = 4y + 1 \end{cases}$	
9. Does the system of linear equations shown below have a solution? Explain. $\begin{cases} 12x + 3y = -2 \\ 4x + y = 7 \end{cases}$	
10. Genny babysits for two different families. One family pays her \$6 each hour and a bonus of \$20 at the end of the night. The other family pays her \$3 every half hour and a bonus of \$25 dollars at the end of the night. Write and solve the system of equations that represents this situation. At what number of hours do the two families pay the same for babysitting services from Genny?	

Lesson 26 Summary:

Lesson 26 Independent Practice

Answer Problems 1–5 without graphing the equations.

1. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} 2x + 5y = 9 \\ -4x - 10y = 4 \end{cases}$$

2. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} \frac{3}{4}x - 3 = y \\ 4x - 3y = 5 \end{cases}$$

3. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} x + 7y = 8 \\ 7x - y = -2 \end{cases}$$

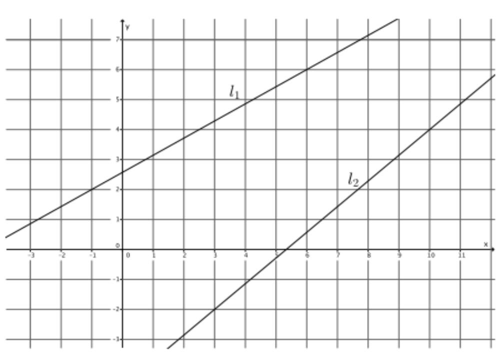
4. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} y = 5x + 12 \\ 10x - 2y = 1 \end{cases}$$

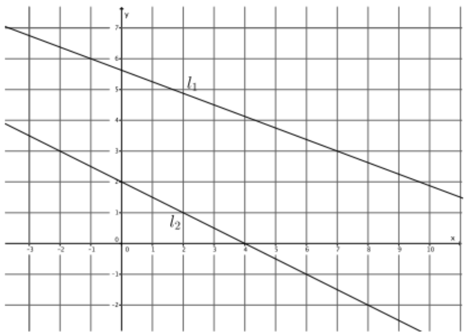
5. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} y = \frac{5}{3}x + 15 \\ 5x - 3y = 6 \end{cases}$$

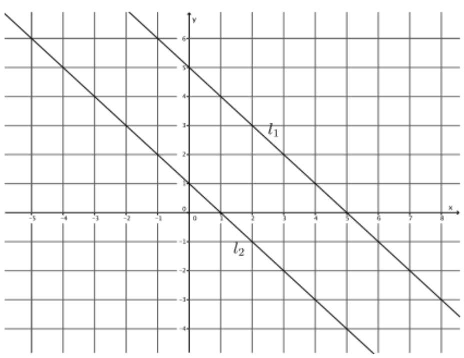
6. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



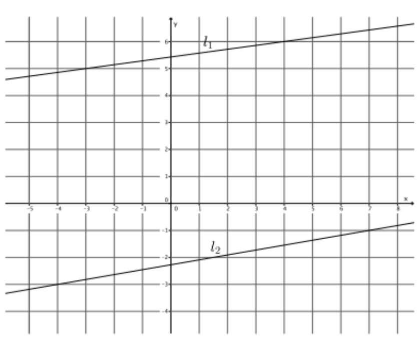
7. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



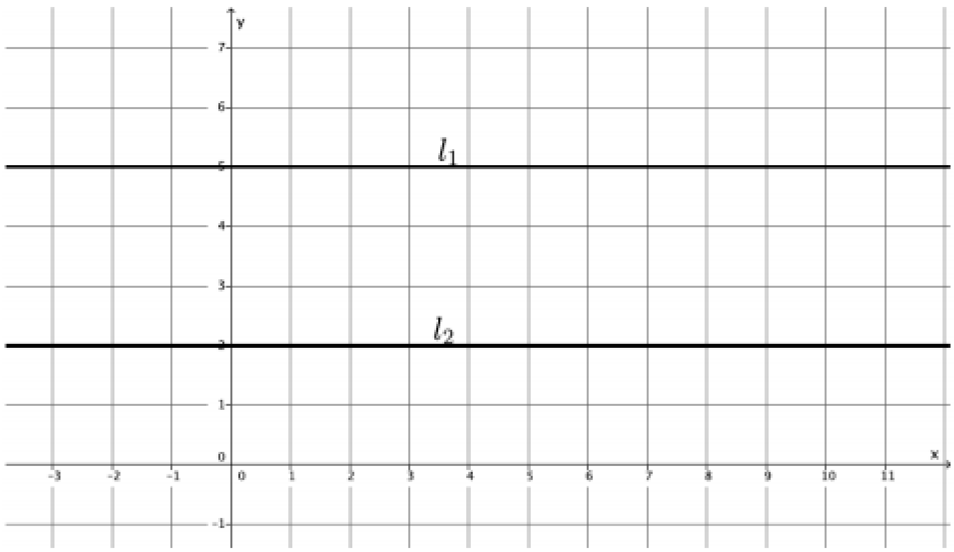
8. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



9. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



10. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



Lesson 27: Nature of Solutions of System of Linear Equations

Essential Questions:

Own your own 1-3:

Determine the nature of the solution to each system of linear equations.

1. $\begin{cases} 3x + 4y = 5 \\ y = -\frac{3}{4}x + 1 \end{cases}$

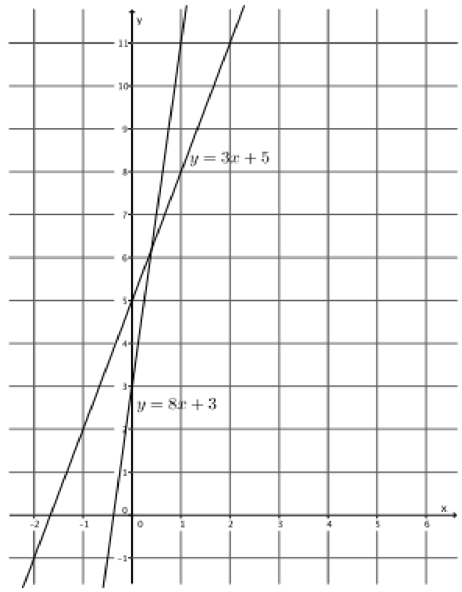
2. $\begin{cases} 7x + 2y = -4 \\ x - y = 5 \end{cases}$

3. $\begin{cases} 9x + 6y = 3 \\ 3x + 2y = 1 \end{cases}$

Discussion

Summarize the nature of the solutions for each of the On your own 1-3.	
Let's look number 3. What do you noticed, if anything, about the equations in number 3	
Consider the following system of linear equations below. $\begin{cases} 3x & +2y & =5 \\ 6x & +4y & =10 \end{cases}$ What do you notice about the constants a, b, and c of the first equation, compared to the constants a', b', and c' of the second equation? What does this mean about the graphs of the equations in the system?	

Example 1:

The following figure contains the graphs of the system $\begin{cases} y = 3x + 5 \\ y = 8x + 3 \end{cases}$	
Estimate the solution based on the graph.	
When two linear expressions are equal to the same number, then the expressions are equal to each other. Look at the system of equations given in this example: $\begin{cases} y = 3x + 5 \\ y = 8x + 3 \end{cases}$	
Does the solution look like this is correct?	

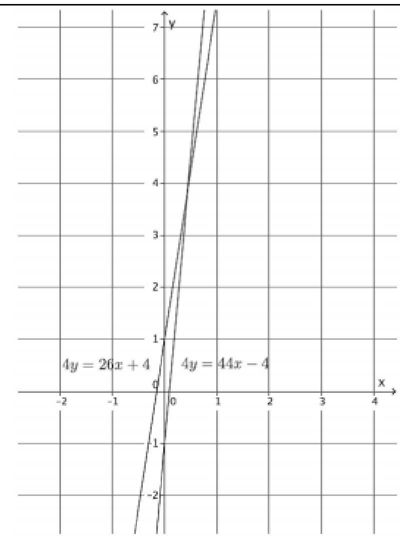
Example 2:

Does the system $\begin{cases} y = 7x - 2 \\ 2y - 4x = 10 \end{cases}$ have a solution?	
Solve using substitution	

Example 3:

Does the system $\begin{cases} 4y = 26x + 4 \\ y = 11x - 1 \end{cases}$ have a solution?	
Solve this system using substitution.	

Compare the solution we got algebraically to the graph of the system of linear equations:



On Your Own: 4-7

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

4. $\begin{cases} 3x + 3y = -21 \\ x + y = -7 \end{cases}$

5. $\begin{cases} y = \frac{3}{2}x - 1 \\ 3y = x + 2 \end{cases}$

6. $\begin{cases} x = 12y - 4 \\ x = 9y + 7 \end{cases}$

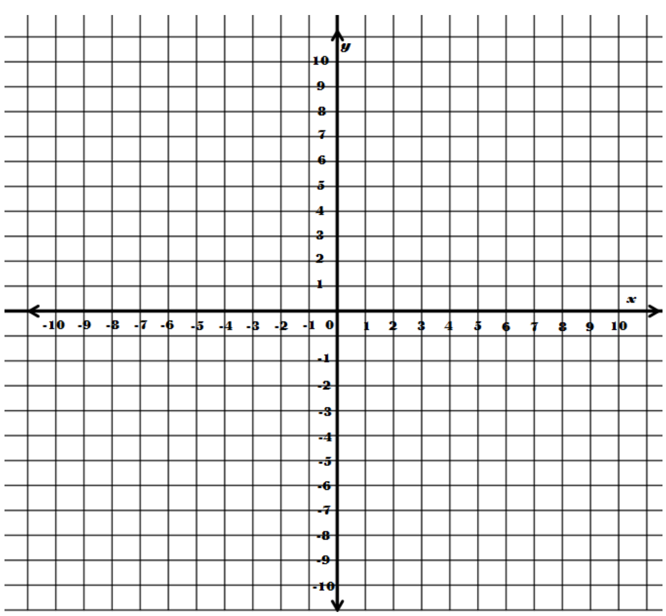
7. Write a system of equations with (4, -5) as its solution.

Lesson 27 Summary:

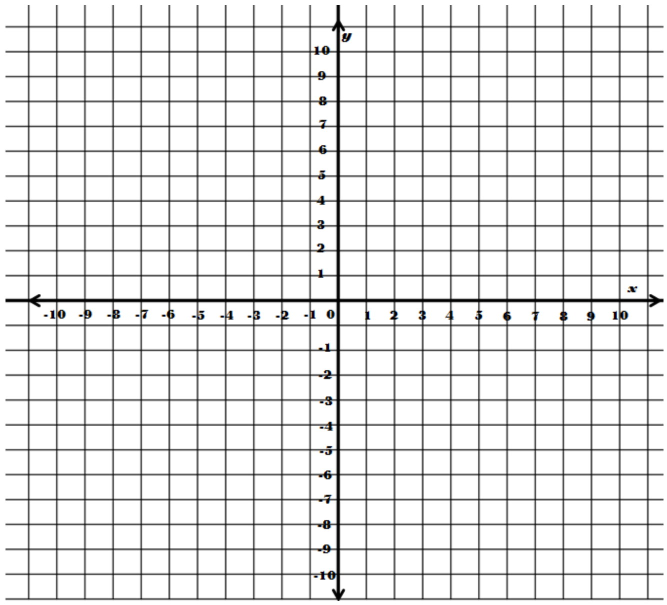
Lesson 27 Independent Practice

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

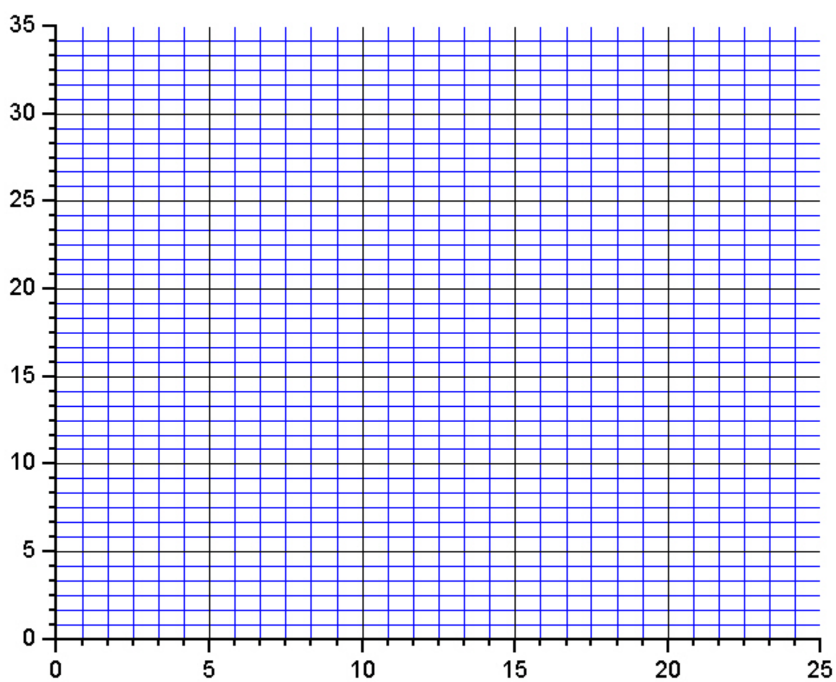
1. $\begin{cases} y = \frac{3}{7}x - 8 \\ 3x - 7y = 1 \end{cases}$



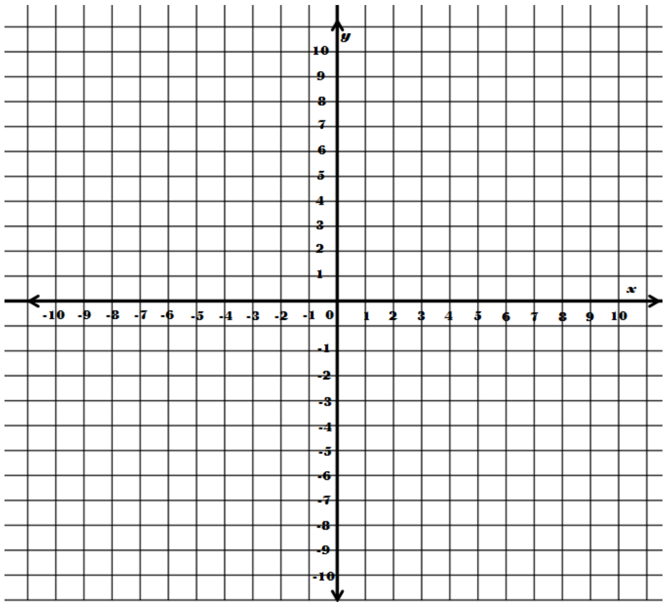
2. $\begin{cases} 2x - 5 = y \\ -3x - 1 = 2y \end{cases}$



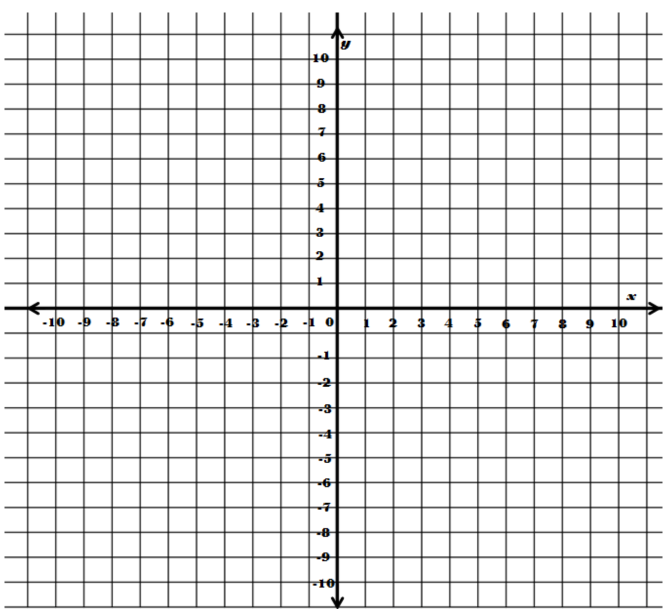
3. $\begin{cases} x = 6y + 7 \\ x = 10y + 2 \end{cases}$



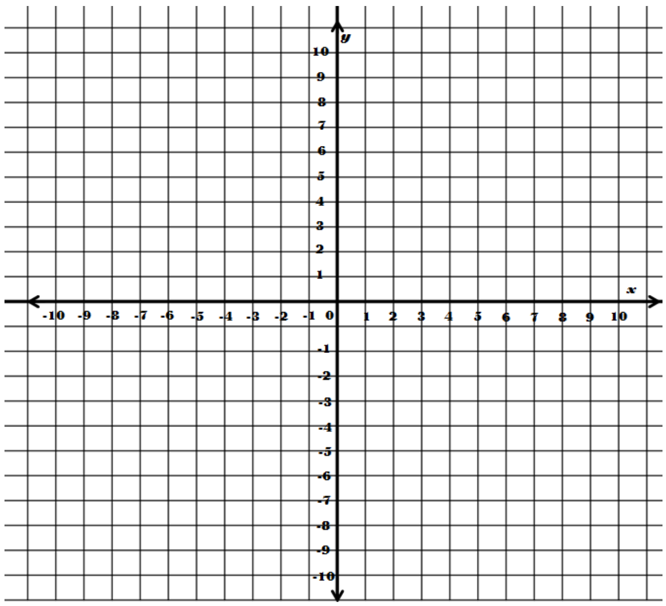
4. $\begin{cases} 5y = \frac{15}{4}x + 25 \\ y = \frac{3}{4}x + 5 \end{cases}$



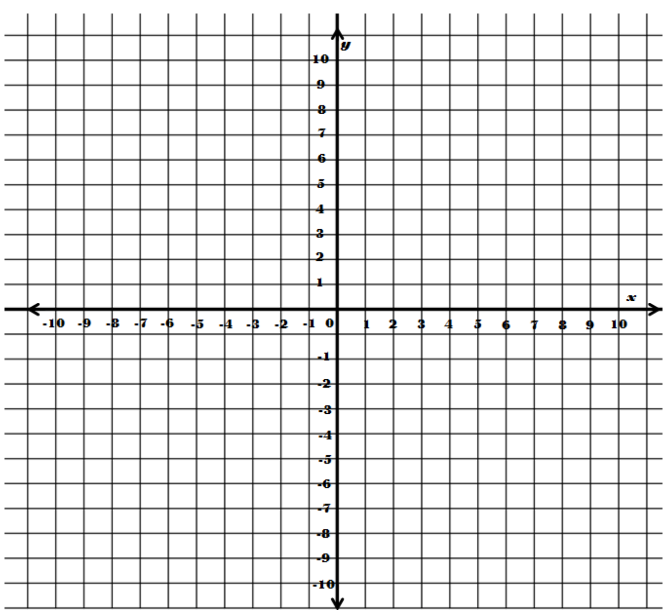
5. $\begin{cases} x + 9 = y \\ x = 4y - 6 \end{cases}$



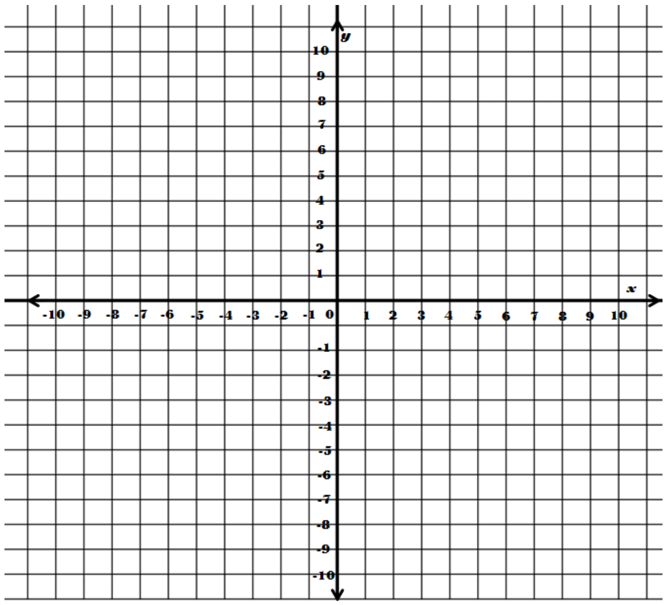
6. $\begin{cases} 3y = 5x - 15 \\ 3y = 13x - 2 \end{cases}$



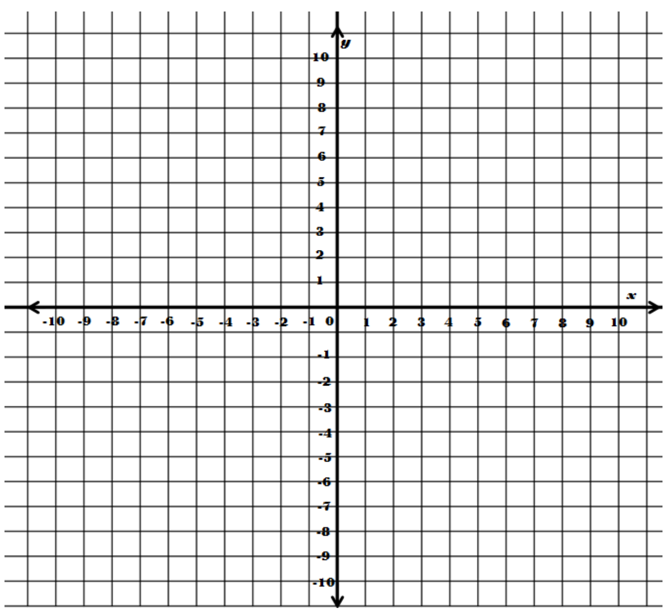
7. $\begin{cases} 6x - 7y = \frac{1}{2} \\ 12x - 14y = 1 \end{cases}$



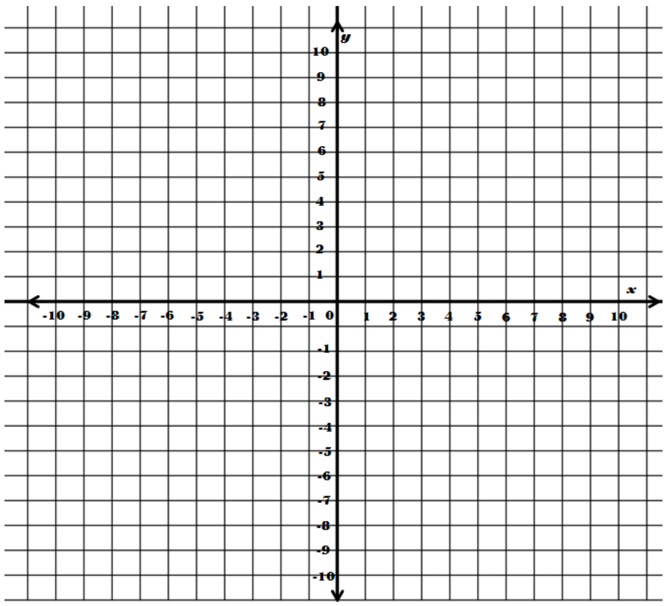
8. $\begin{cases} 5x - 2y = 6 \\ -10x + 4y = -14 \end{cases}$



9. $\begin{cases} y = \frac{3}{2}x - 6 \\ 2y = 7 - 4x \end{cases}$



10. $\begin{cases} 7x - 10 = y \\ y = 5x + 12 \end{cases}$



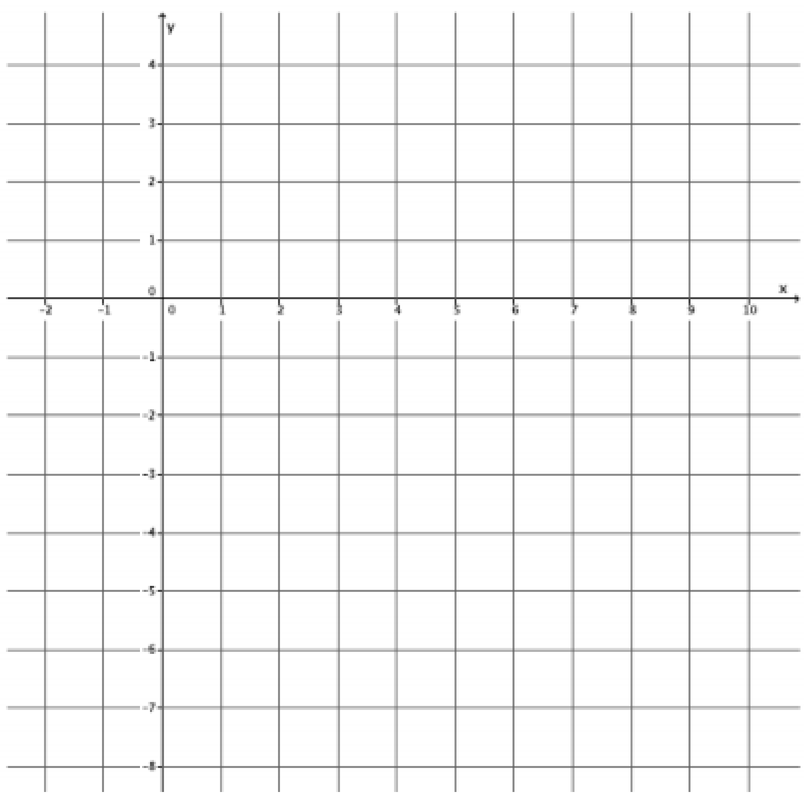
11. Write a system of linear equations with (-3, 9) as its solution.

Lesson 28 – Another Computational Method of solving a Linear System

Essential Questions:

Discussion:

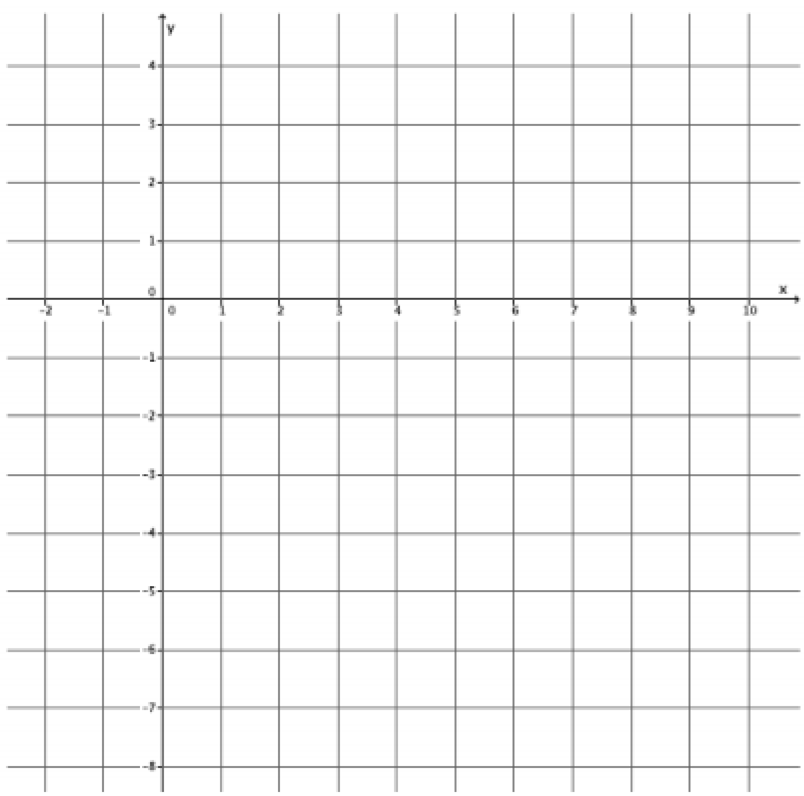
Describe how you would solve this system algebraically: $\begin{cases} y = 3x + 5 \\ y = 8x + 3 \end{cases}$	
Describe how you would solve this system algebraically: $\begin{cases} y = 7x - 2 \\ 2y - 4x = 10 \end{cases}$	
Describe how you would solve this system algebraically: $\begin{cases} x = 6y + 7 \\ x = 10y + 2 \end{cases}$	

Example 1: Use what you noticed about adding equivalent expressions to solve the following system by elimination. $\begin{cases} 6x - 5y = 21 \\ 2x + 5y = -5 \end{cases}$	
We can verify our solution by sketching the graphs of the system.	

Example 2
Solve the following system by elimination.

$$\begin{cases} -2x + 7y = 5 \\ 4x - 2y = 14 \end{cases}$$

We can verify our solution by sketching the graphs of the system.



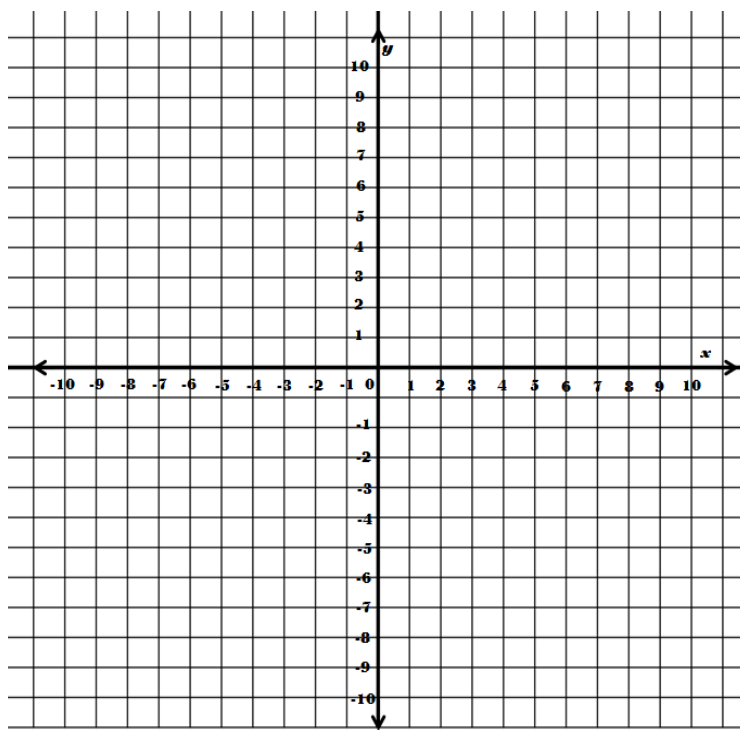
Example 3
Solve the following system by elimination.

$$\begin{cases} 7x - 5y = -2 \\ 3x - 3y = 7 \end{cases}$$

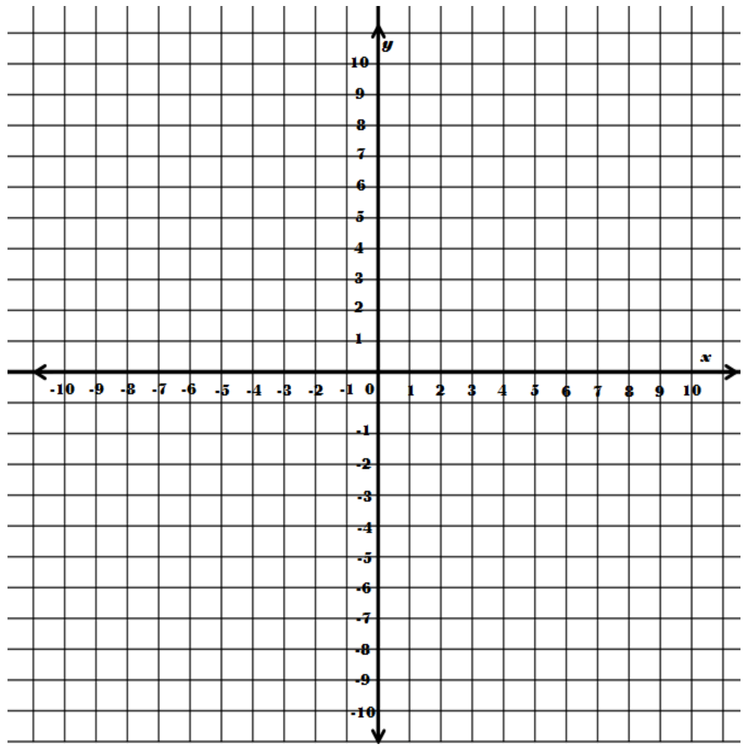
Own Your Own 1-3:

Each of the following systems has a solution. Determine the solution to the system by eliminating one of the variables. Verify the solution using the graph of the system.

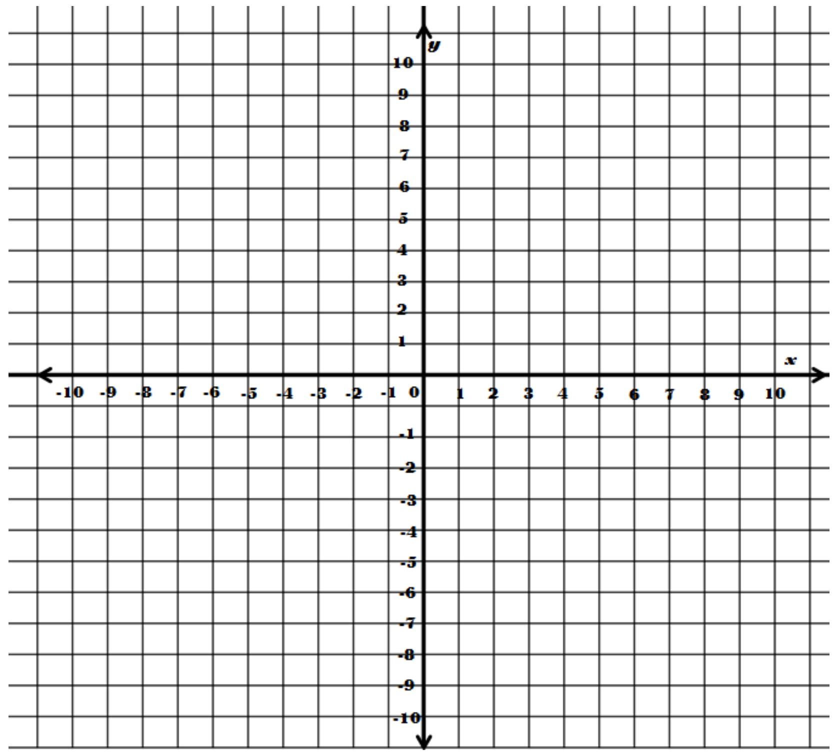
1. $\begin{cases} 6x - 7y = -10 \\ 3x + 7y = -8 \end{cases}$



2. $\begin{cases} x - 4y = 7 \\ 5x + 9y = 6 \end{cases}$



3. $\begin{cases} 2x - 3y = -5 \\ 3x + 5y = 1 \end{cases}$



Discussion:

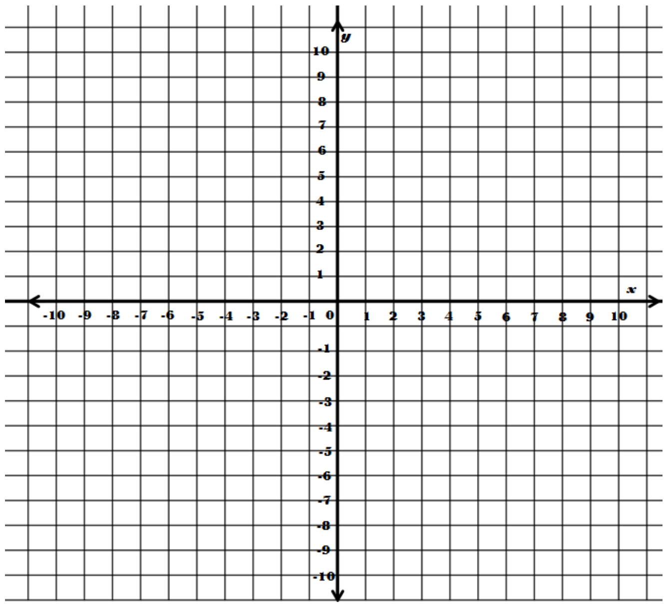
Which method do you think would be most efficient for the following system? Explain. $\begin{cases} y = 5x - 19 \\ 3x + 11 = y \end{cases}$	
What method would you use for the following system? Explain. $\begin{cases} 2x - 9y = 7 \\ x + 9y = 5 \end{cases}$	
What method would you use for the following system? Explain. $\begin{cases} 4x - 3y = -8 \\ x + 7y = 4 \end{cases}$	
What method would you use for the following system? Explain. $\begin{cases} x + y = -3 \\ 6x + 6y = 6 \end{cases}$	

Lesson 28 Summary:

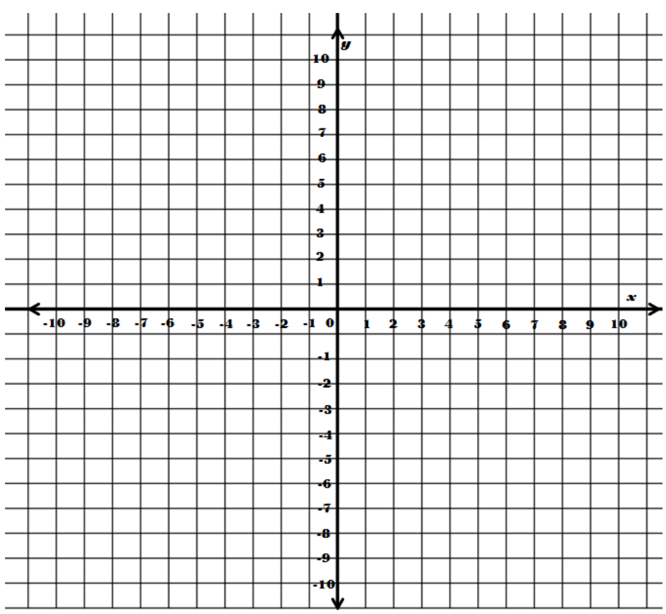
Lesson 28 – Independent Practice

Determine the solution, if it exists, for each system of linear equations. Verify your solution on the coordinate plane.

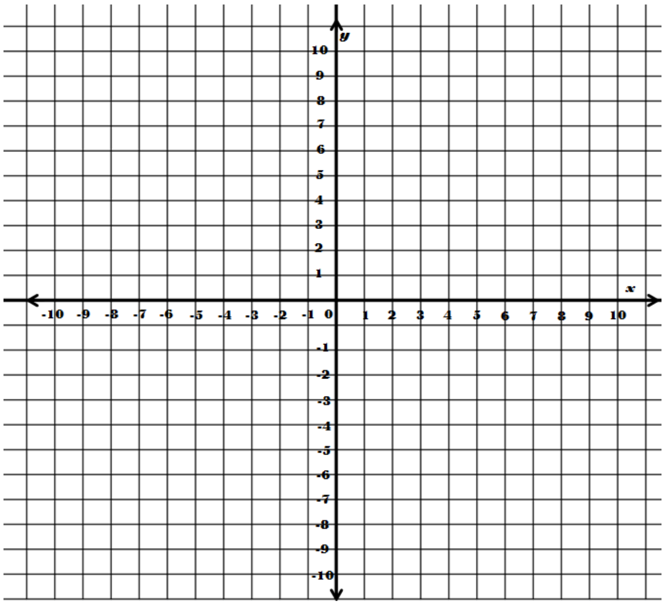
1. $\begin{cases} \frac{1}{2}x + 5 = y \\ 2x + y = 1 \end{cases}$



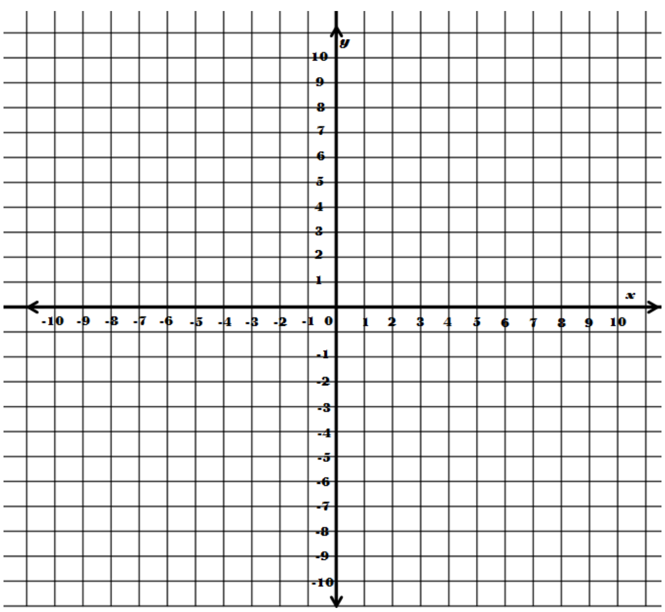
2. $\begin{cases} 9x + 2y = 9 \\ -3x + y = 2 \end{cases}$



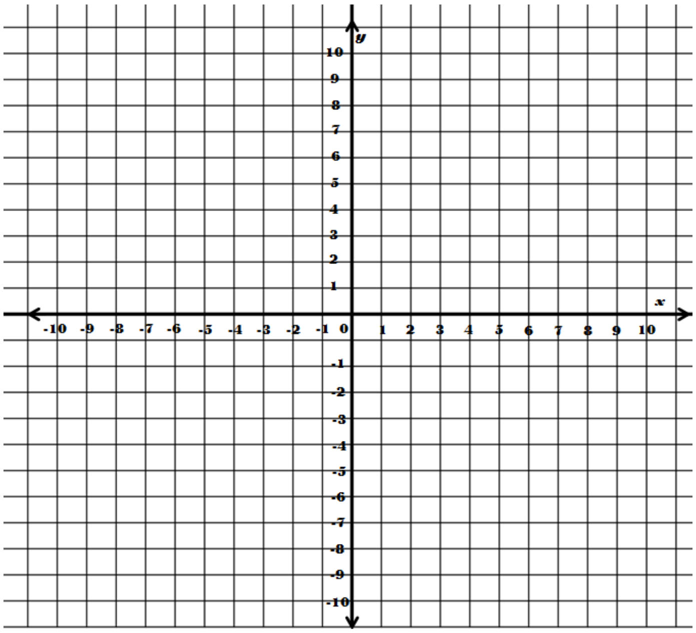
3. $\begin{cases} y = 2x - 2 \\ 2y = 4x - 4 \end{cases}$



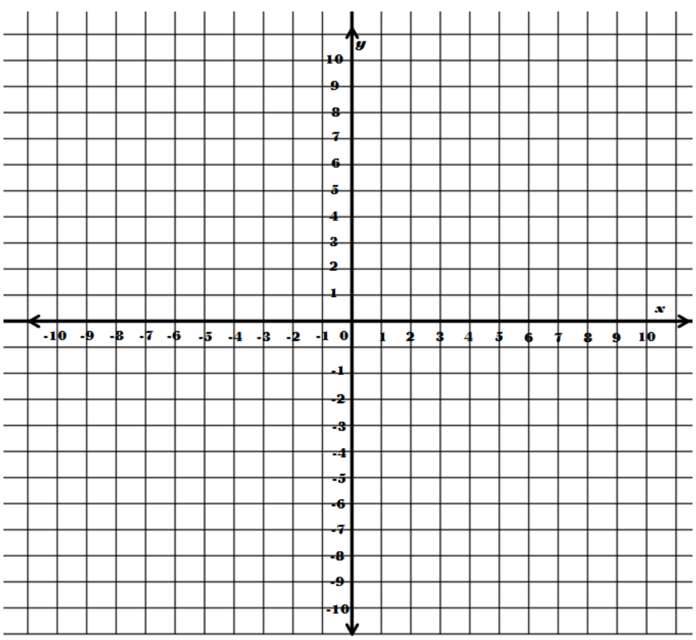
4. $\begin{cases} 8x + 5y = 19 \\ -8x + y = -1 \end{cases}$



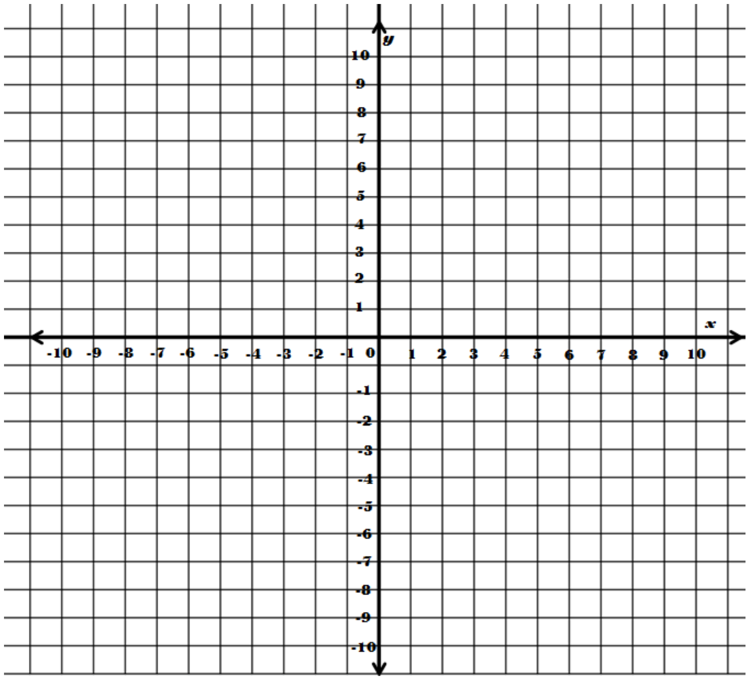
5. $\begin{cases} x + 3 = y \\ 3x + 4y = 7 \end{cases}$



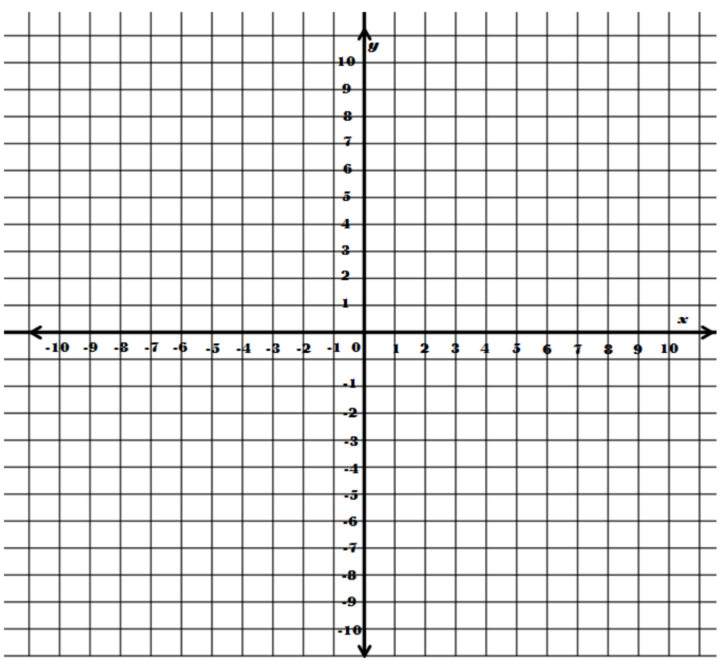
6. $\begin{cases} y = 3x + 2 \\ 4y = 12 + 12x \end{cases}$



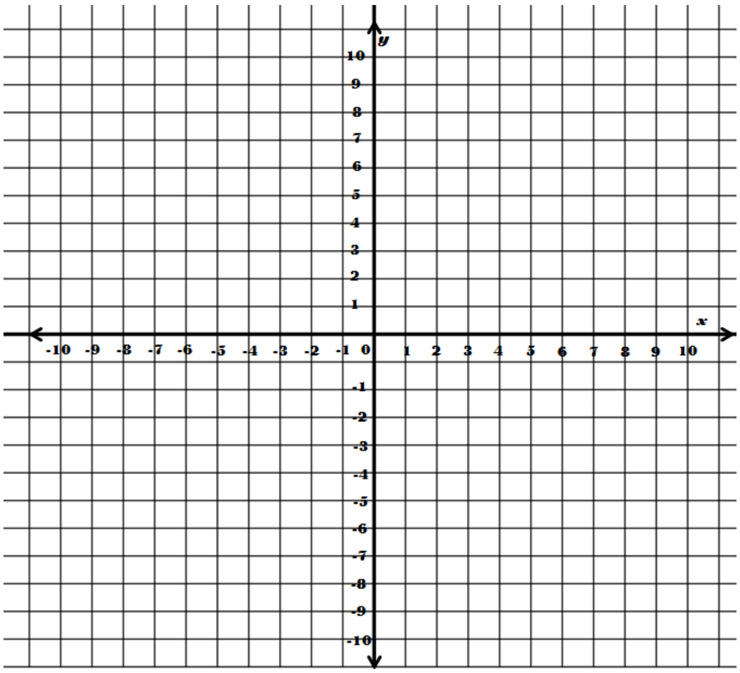
7. $\begin{cases} 4x - 3y = 16 \\ -2x + 4y = -2 \end{cases}$



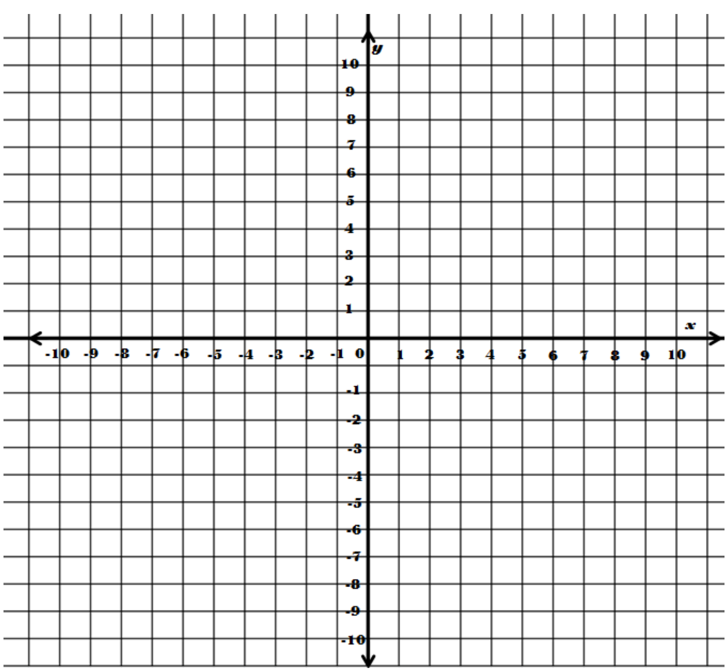
8. $\begin{cases} 2x + 2y = 4 \\ 12 - 3x = 3y \end{cases}$



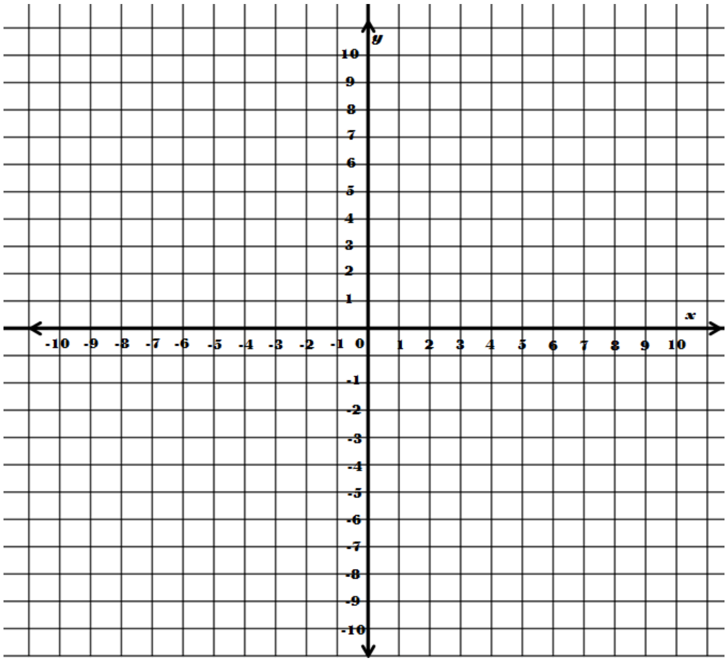
9. $\begin{cases} y = -2x + 6 \\ 3y = x - 3 \end{cases}$



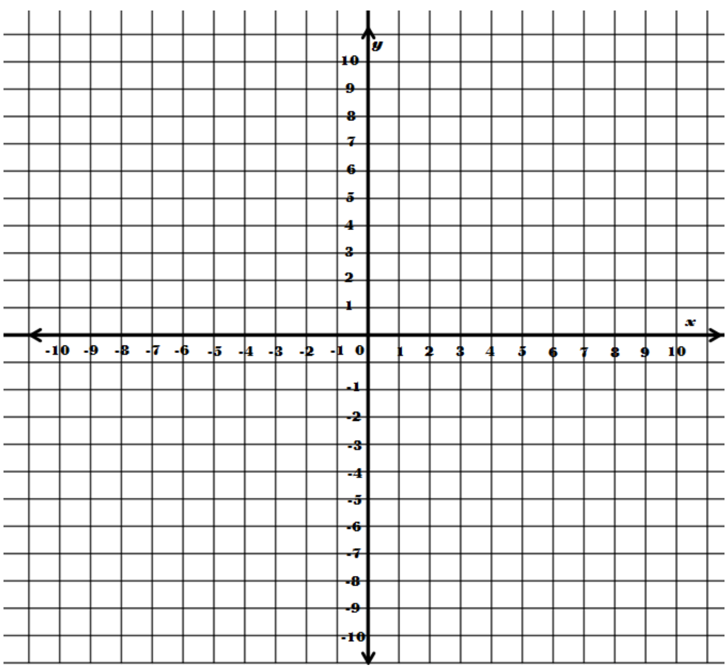
10. $\begin{cases} y = 5x - 1 \\ 10x = 2y + 2 \end{cases}$



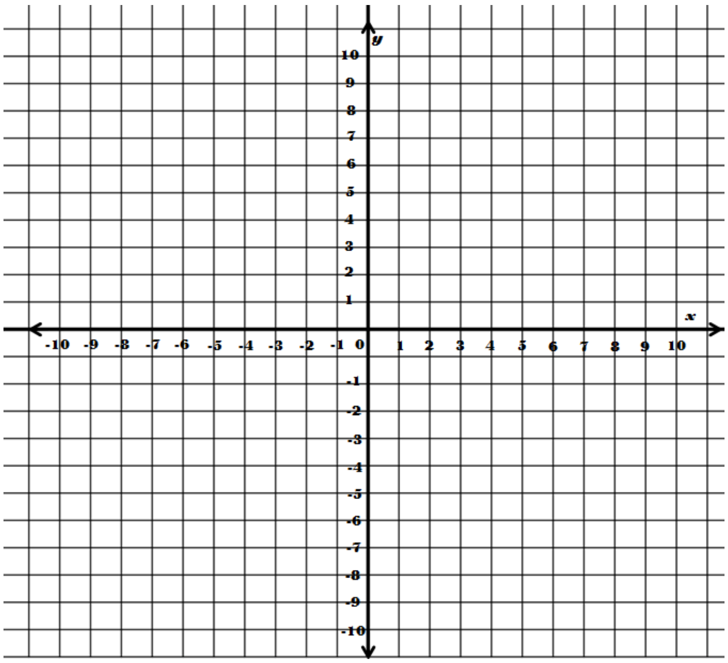
11. $\begin{cases} 3x - 5y = 17 \\ 6x + 5y = 10 \end{cases}$



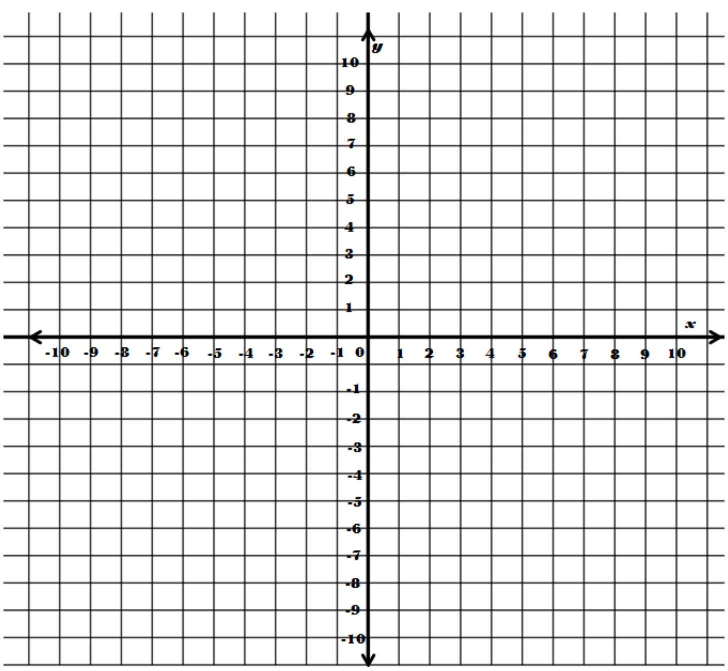
12. $\begin{cases} y = \frac{4}{3}x - 9 \\ y = x + 3 \end{cases}$



13. $\begin{cases} 4x - 7y = 11 \\ x + 2y = 10 \end{cases}$



14. $\begin{cases} 21x + 14y = 7 \\ 12x + 8y = 16 \end{cases}$



Lesson 29 – Word Problems

Essential Questions:

On your own 1-3:

Example 1: The sum of two numbers is 361 and the difference between the two numbers is 173. What are the two numbers?	
Solve the system:	
Does it matter which number is x and which number is y?	

Example 2 There are 356 eighth-grade students at Euclid's Middle School. Thirty-four more than four times the number of girls is equal to half the number of boys. How many boys are in eighth grade at Euclid's Middle School? How many girls?	
Solve the system:	
How can we be sure we are correct?	

Example 3: A family member has some five-dollar bills and one-dollar bills in her wallet. Altogether she has 18 bills and a total of \$62. How many of each bill does she have?	
Solve the system:	
What does the solution mean in context?	

Example 4 A friend bought 2 boxes of pencils and 8 notebooks for school, and it cost him $11. He went back to the store the same day to buy school supplies for his younger brother. He spent $11.25 on 3 boxes of pencils and 5 notebooks. How much would 7 notebooks cost?	
Solve the system:	
What does the solution mean in context?	

Own Your Own:

1. A farm raises cows and chickens. The farmer has a total of 42 animals. One day he counts the legs of all of his animals and realizes he has a total of 114. How many cows does the farmer have? How many chickens?	
2. The length of a rectangle is 4 times the width. The perimeter of the rectangle is 45 inches. What is the area of the rectangle?	

3. The sum of the measures of angles x and y is 127°. If the measure of $\angle x$ is 34° more than half the measure of $\angle y$, what is the measure of each angle?	

Lesson 28 Summary:

Lesson 29 Independent Practice

1. Two numbers have a sum of 1,212 and a difference of 518. What are the two numbers?

2. The sum of the ages of two brothers is 46. The younger brother is 10 more than a third of the older brother's age. How old is the younger brother?

3. One angle measures 54 more degrees than 3 times another angle. The angles are supplementary. What are their measures?

4. Some friends went to the local movie theater and bought four buckets of large popcorn and six boxes of candy. The total for the snacks was \$46.50. The last time you were at the theater, you bought a large popcorn and a box of candy and the total was \$9.75. How much would 2 large buckets of popcorn and 3 boxes of candy cost?

5. You have 59 total coins for a total of $12.05. You only have quarters and dimes. How many of each coin do you have?

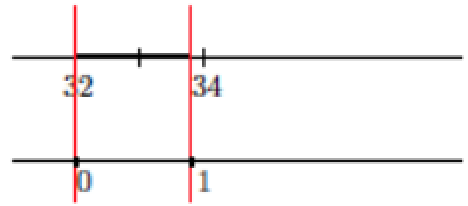
6. A piece of string is 112 inches long. Isabel wants to cut it into 2 pieces so that one piece is three times as long as the other. How long is each piece?

Lesson 30: Conversion Between Celsius and Fahrenheit

Essential Questions:

Own your own:

Mathematical Modeling Exercise (1) If t is a number, what is the degree in Fahrenheit that corresponds to t°C? (2) If t is a number, what is the degree in Fahrenheit that corresponds to $(-t)$°C?	There are two methods for measuring temperature: (a) Fahrenheit, which assigns the number 32 to the temperature of water freezing and the number 212 to the temperature of water boiling; and (b) Celsius, which assigns the numbers 0 and 100, respectively, to the same temperatures. These numbers will be denoted by 32°F, 212°F, 0°C, 100°C, respectively. Our goal is to address the following two questions:
Instead of trying to answer these questions directly, let's try something simpler. With this in mind, can we find out what degree in Fahrenheit corresponds to 1°C? Explain. We can use the following diagram (double number line) to organize our thinking.	
Where on the diagram would 1°C be located? Be specific.	
How would we determine the precise number in Fahrenheit that cooresponds to 1°C?	

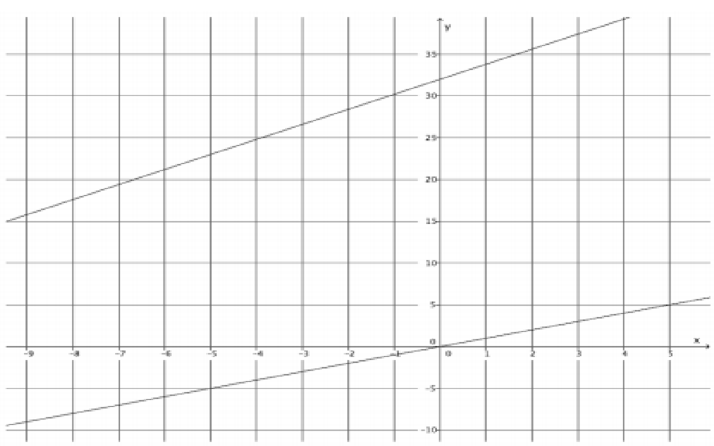
Based on your computation, what number falls at the intersection of the Fahrenheit number line and the red line that corresponds to 1°C? Explain.	

On Your Own:

1. How many degrees Fahrenheit is 25°C?	
2. How many degrees Fahrenheit is 42°C?	
3. How many degrees Fahrenheit is 94°C?	
4. How many degrees Fahrenheit is 63°C?	
5. How many degrees Fahrenheit is t °C?	

Discussion Now that Question (1) has been answered, let's begin thinking about Question (2). Where on the number line would we find a negative Celsius temperature? How can we determine the Fahrenheit temperature that corresponds to $-1°C$?	
How many degrees Fahrenheit corresponds to $(-15)°C$?	
How many degrees Fahrenheit corresponds to $(-36)°C$?	
How many degrees Fahrenheit corresponds to $(-t)°C$?	

On a coordinate plane, if we let x be the given temperature and y be the temperature in Celsius, then we have the equation $y = x$. That is, there is no change in temperature. But, when we let x be the given temperature and y be the temperature in Fahrenheit, we have the equation $y = 32 + 1.8x$.



Will these lines intersect? Explain?	

What will that point of intersection represent?	
Solve the system of equations algebraically to determine at what number $t°C = t°F$.	

Lesson 30 Summary:

Lesson 30 Independent Practice

1. Does the equation, t°C = (32 + 1.8t)°F, work for any rational number t? Check that it does with $t = 8\frac{2}{3}$ and $t = -8\frac{2}{3}$.

2. Knowing that $t\ °C = (32 + \frac{9}{5}t)$ °F for any rational t, show that for any rational number d, $d°F = \left(\frac{5}{8}(d - 32)\right)°\mathrm{C}$.

3. Drake was trying to write an equation to help him predict the cost of his monthly phone bill. He is charged $35 just for having a phone, and his only additional expense comes from the number of texts that he sends. He is charged $0.05 for each text. Help Drake out by completing parts (a)–(f).

a. How much was his phone bill in July when he sent 750 texts?

b. How much was his phone bill in August when he sent 823 texts?

c. How much was his phone bill in September when he sent 579 texts?

d. Let y represent the total cost of Drake's phone bill. Write an equation that represents the total cost of his phone bill in October if he sends t texts.

e. Another phone plan charges $20 for having a phone and $0.10 per text. Let y represent the total cost of the phone bill for sending t texts. Write an equation to represent his total bill.

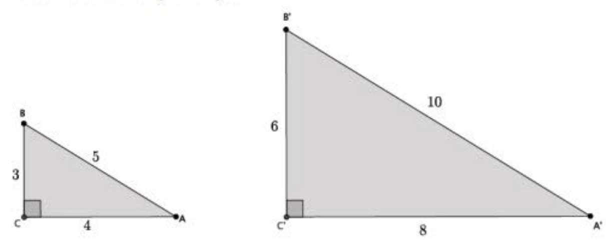
f. Write your equations in parts (d) and (e) as a system of linear equations and solve. Interpret the meaning of the solution in terms of the phone bill.

Lesson 31: Pythagorean Theorem

Essential Questions:

Discussion:
Pythagorean Triples:

Shown are the two right triangles.
Discuss with your partners how the method for finding Pythagorean triples can be explained mathematically.



On Your own:

1. Identify two Pythagorean triples using the known triple 3, 4, 5 (other than 6, 8, 10).	
2. Identify two Pythagorean triples using the known triple 5, 12, 13.	
3. Identify two triples using either 3, 4, 5 or 5, 12, 13.	

Discussion:
Pythagorean triples can also be explained algebraically. For example, assume a, b, c represent a Pythagorean triple. Let m be a positive integer. Then by the Pythagorean theorem, $a^2 + b^2 = c^2$:

On Your Own:

Use the system $\begin{cases} x+y=\frac{t}{s} \\ x-y=\frac{s}{t} \end{cases}$ to find Pythagorean triples for the given values of s and t. Recall that the solution in the form $\left(\frac{c}{b},\frac{a}{b}\right)$ is the triple a, b, c.	
4. s = 4, t = 5	
5. s = 7, t = 10	

6. $s = 1$, $t = 4$	
7. Use a calculator to verify that you found a Pythagorean triple in each of the Exercises 4–6. Show your work below.	For example 4: For example 5: For example 6:

Discussion

Solve the system generally.

$$\begin{cases} x + y = \dfrac{t}{s} \\ x - y = \dfrac{s}{t} \end{cases}$$

Which method should we use to solve this system? Explain.

Lesson 31 Summary:

Lesson 31 Independent Practice

1. Explain in terms of similar triangles why it is that when you multiply the known Pythagorean triple 3, 4, 5 by 12, it generates a Pythagorean triple.

2. Identify three Pythagorean triples using the known triple 8, 15, 17.

3. Identify three triples (numbers that satisfy $a^2 + b^2 = c^2$, but a, b, c are not whole numbers) using the triple 8, 15, 17.

Use the system $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$ to find Pythagorean triples for the given values of s and t. Recall that the solution in the form $\left(\frac{c}{b}, \frac{a}{b}\right)$, is the triple a, b, c.

4. $s = 2$, $t = 9$

5. $s = 6$, $t = 7$

6. $s = 3$, $t = 4$

7. Use a calculator to verify that you found a Pythagorean triple in each of the Problems 4–6. Show your work below.

For problem 4:

For problem 5:

For problem 6: