CCSS – 8.G.1, 8.G.2, 8.G.5. 8.G.6, 8.G.7

Track your progress:

Lesson #	Homework	Quiz/Exit slip
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Summative lessons 1-6:____________________

Lesson #	Homework	Quiz/Exit slip
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Summative lessons 7 – 13: ________________________

Module 2 Vocabulary

Word	Definition	Example	Other Hint/Picture/Spanish
Alternate Exterior			
Alternate Interior			
Angle Degree			
Bisect			
Coincide			
Collinear			

Word	Definition	Example	Other Hint/Picture/Spanish
Complementary			
Congruent (Congruence)			
Corresponding Angles			
Dilation			
Image			
In the Plane			

Word	Definition	Example	Other Hint/Picture/Spanish
Intersect			
Line			
Mapping			
Parallel			
Perpendicular			
Preserves Distance			

Word	Definition	Example	Other Hint/Picture/Spanish
Prime			
Quadrilateral			
Ray/Vector			
Reflection			
Rigid Motion			
Rotation			

Word	Definition	Example	Other Hint/Picture/Spanish
Segment			
Supplementary			
Transformation			
Translation			
Vertical			

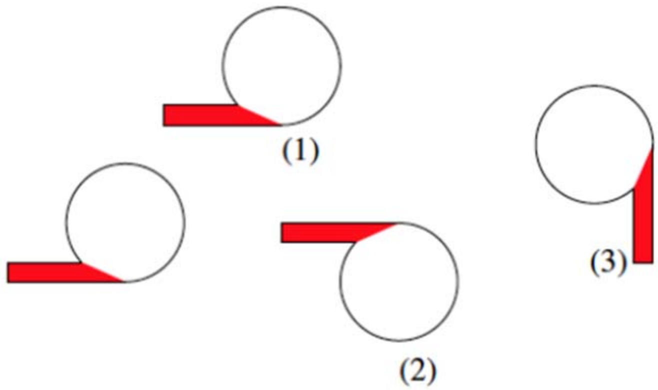
Lesson 1 – Why Move Things Around

Essential Questions:

Given two segments AB and CD, which could be very far apart, how can we find out if they have the same length **without measuring** them individually? Do you think they have the same length? How do you check?	
Given angles $\angle A0\mathrm{B}$ and $\angle A'O'B'$, how can we tell whether they have the same degree without having to measure each angle individually?	
If two lines L and L' are parallel and they are intersected by another line, how can we tell if the angles $\angle a$ and $\angle b$ (as shown) have the same degree when measured?	

On your own - Exploratory Challenge

1. Describe, intuitively, what kind of transformation will be required to move the figure on the left to each of the figures (1)–(3) on the right. To help with this exercise, use a transparency to copy the figure on the left. Note: Begin by moving the left figure to each of the locations in (1), (2), and (3).



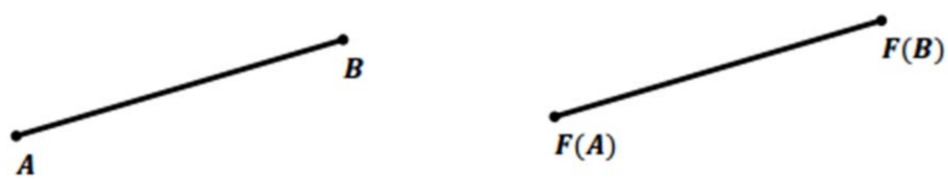
With a Partner or Small Group – Exploratory Challenge

2. Given two segments AB and CD, which could be very far apart, how can we find out if they have the same length without measuring them individually? Do you think they have the same length? How do you check? In other words, why do you think we need to move things around on the plane?

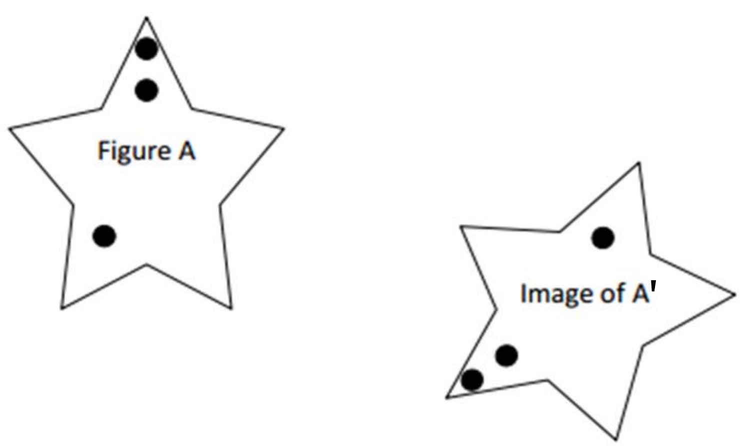
Lesson 1 summary:

Lesson 1 – Independent Practice

1. Using as much of the new vocabulary as you can, try to describe what you see in the diagram below.



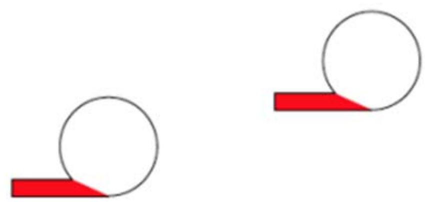
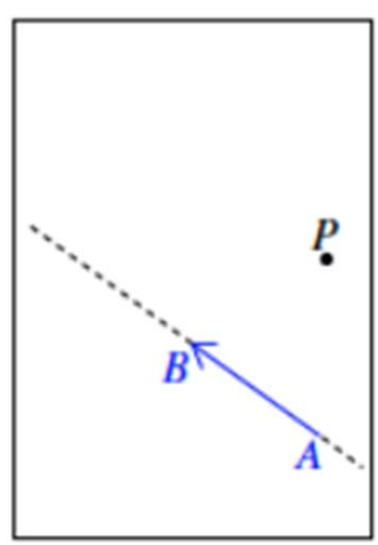
2. Describe, intuitively, what kind of transformation will be required to move Figure A on the left to its image, A' on the right.



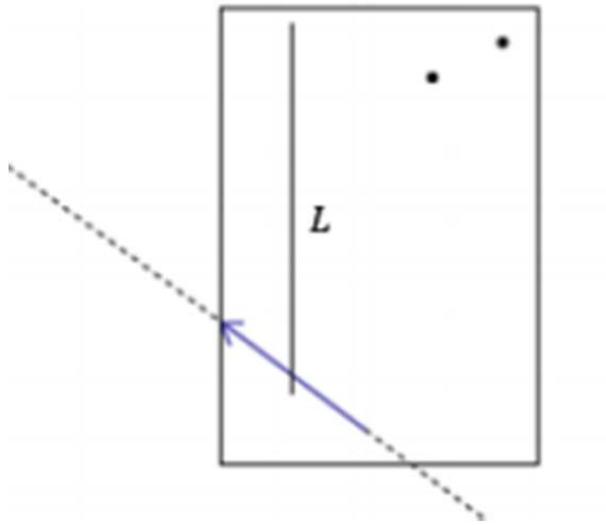
Lesson 2 – Definition of Translation and Three Basic Properties

Essential Questions:

Example 1

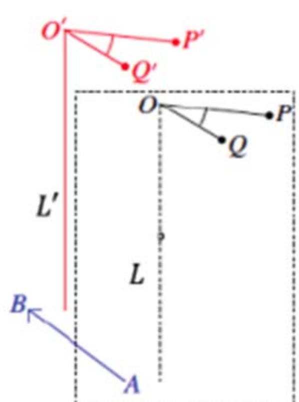
Revisiting our opening discussion question, What is the simplest transformation that would map one of the following figures to the other?	
What is a vector?	
Example 2	

Example 3



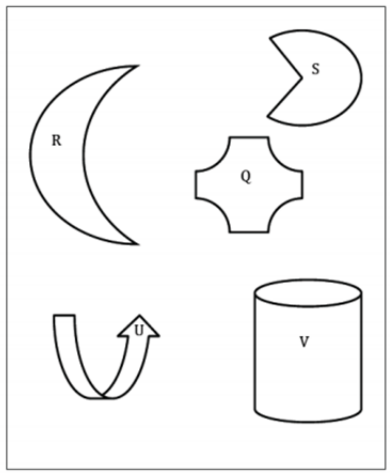
Example 4

What are some observations about the basic properties of translations?



Exercise 1 – work in pairs

Draw at least three different vectors, and show what a translation of the plane along each vector will look like. Describe what happens to the following figures under each translation using appropriate vocabulary and notation as needed.

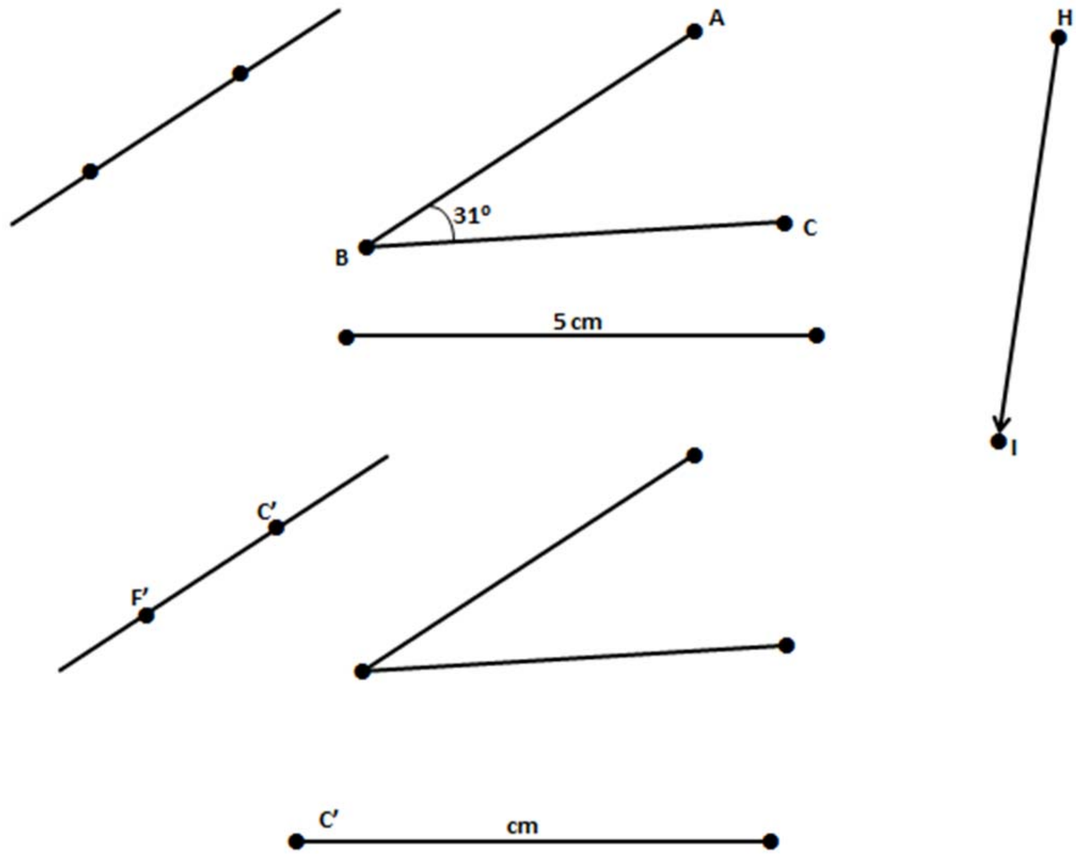


Example 5

The notation to represent the image is…

Exercise 2 – On your own

The diagram below shows figures and their images under a translation along vector $\overrightarrow{HI}$. Use the original figures and the translated images to fill in missing labels for points and measures.

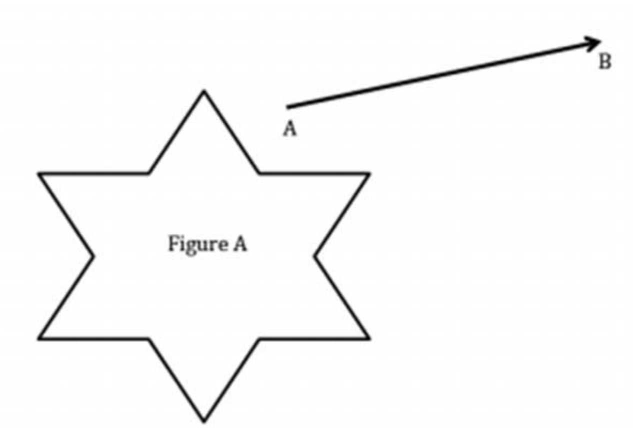


Lesson 2 Summary

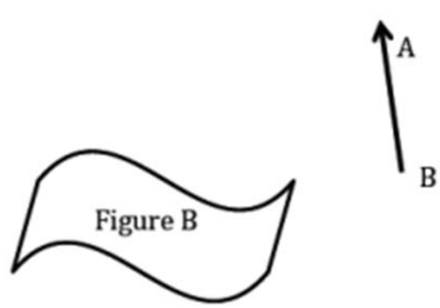
Be sure to include the three basic properties of translations.

Lesson 2 – Independent Practice

1. Translate the plane containing Figure A along AB. Use your transparency to sketch the image of Figure A by this translation. Mark points on Figure A and label the image of Figure A accordingly.



2. Translate the plane containing Figure B along $\overrightarrow{BA}$. Use your transparency to sketch the image of Figure B by this translation. Mark points on Figure B and label the image of Figure B accordingly.



Independent Practice - continued

3. Draw an acute angle (your choice of degree), a segment with length 3 cm, a point, a circle with radius 1 in., and a vector (your choice of length, i.e., starting point and ending point). Label points and measures (measurements do not need to be precise, but your figure must be labeled correctly). Use your transparency to translate all of the figures you have drawn along the vector. Sketch the images of the translated figures and label them.

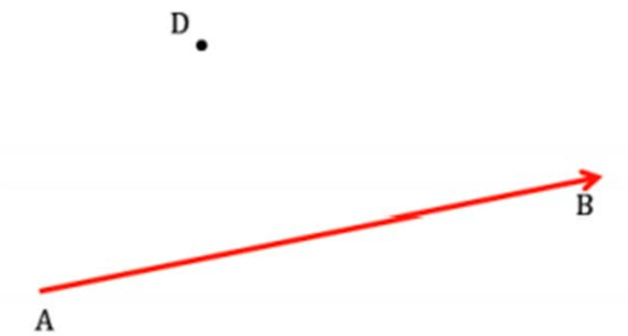
4. What is the length of the translated segment? How does this length compare to the length of the original segment? Explain.

5. What is the length of the radius in the translated circle? How does this radius length compare to the radius of the original circle? Explain.

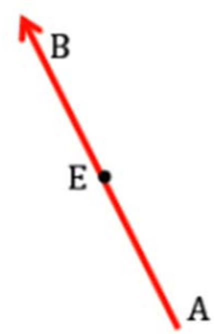
6. What is the degree of the translated angle? How does this degree compare to the degree of the original angle? Explain

Independent Practice - continued

7. Translate point D along vector $\overrightarrow{AB}$ and label the image D'. What do you notice about the line containing vector $\overrightarrow{AB}$ and the line containing points D and D'? (Hint: Will the lines ever intersect?)



8. Translate point E along vector $\overrightarrow{AB}$ and label the image E'. What do you notice about the line containing vector $\overrightarrow{AB}$ and the line containing points E and E'?

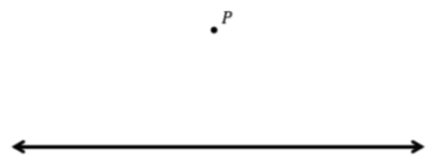


Lesson 3 – Translating Lines

Essential Questions: :

Warm-Up:
Exercise 1 – Work Independently

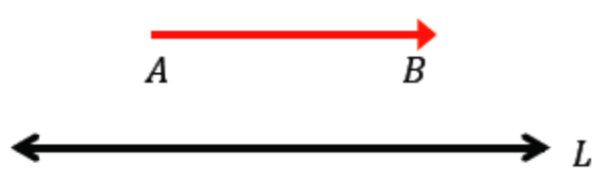
Draw a line passing through point P that is parallel to line L. Draw a second line passing through point P that is parallel to line L, and that is distinct (i.e., different) from the first one. What do you notice?



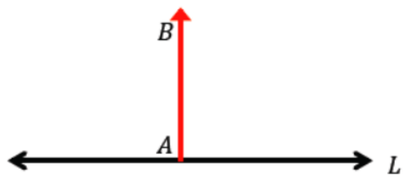
Complete Exercises 2–4 independently

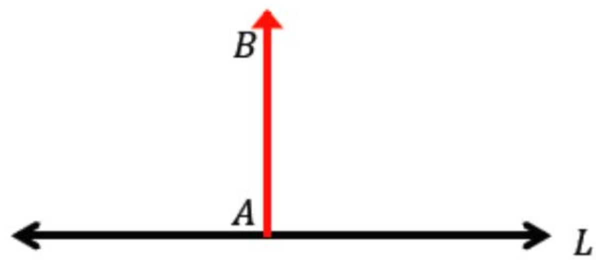
2. Translate line L along the vector $\overrightarrow{AB}$. What do you notice about L and its image L'?

3. Line L is parallel to vector $\overrightarrow{AB}$. Translate line L along vector $\overrightarrow{AB}$. What do you notice about L and its image, L'?



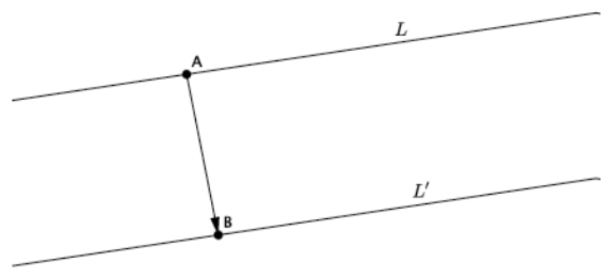
4. Translate line $\boldsymbol{L}$ along the vector $\overrightarrow{AB}$. What do you notice about $\boldsymbol{L}$ and its image, $\boldsymbol{L}'$?



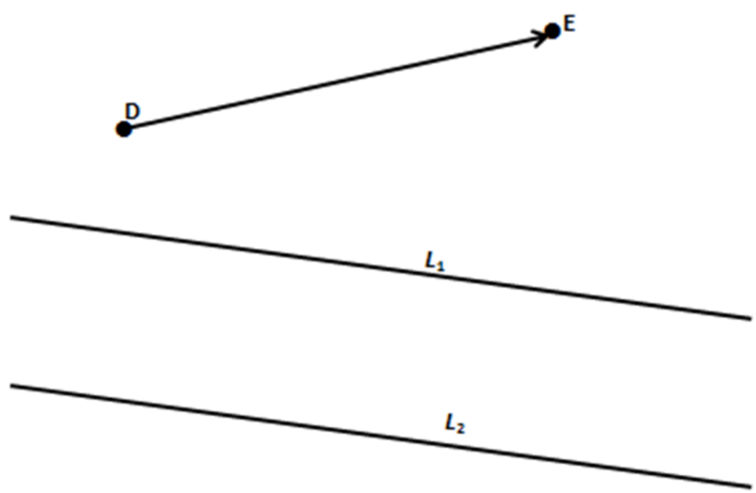


Exercises 5-6 work in pairs or small groups.

5. Line $\boldsymbol{L}$ has been translated along vector $\overrightarrow{AB}$ resulting in $\boldsymbol{L}'$. What do you know about lines $\boldsymbol{L}$ and $\boldsymbol{L}'$?



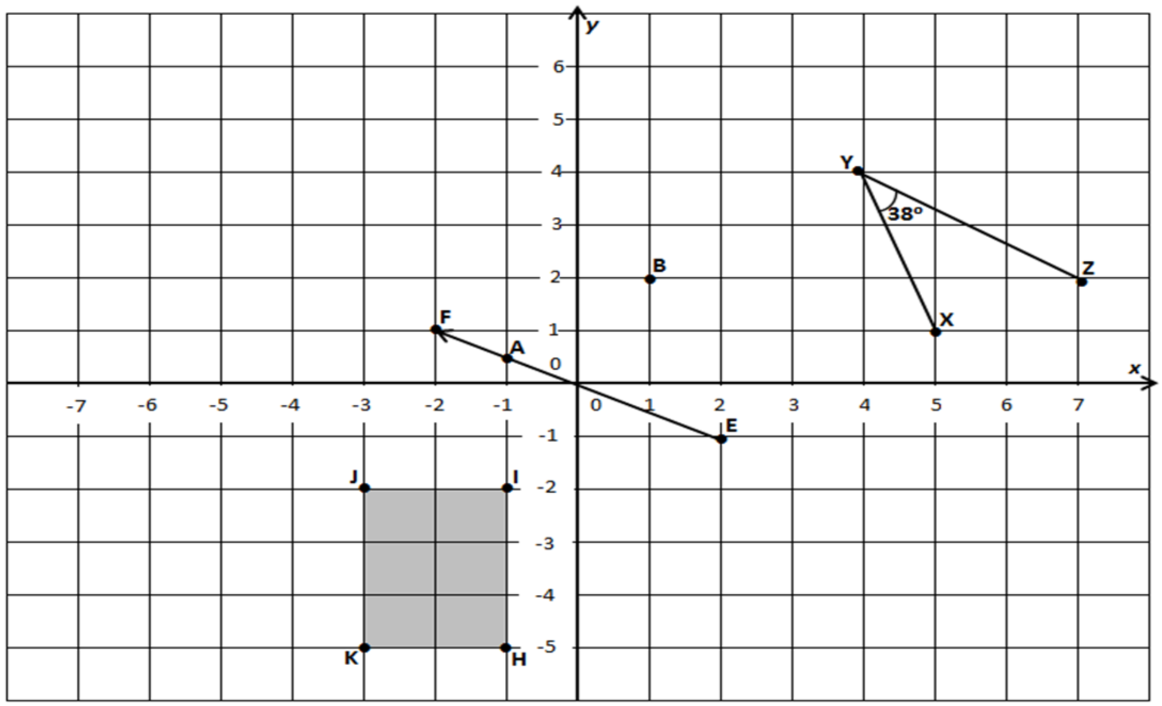
6. Translate $\boldsymbol{L1}$ and $\boldsymbol{L2}$ along vector $\overrightarrow{DE}$. Label the images of the lines. If lines $\boldsymbol{L1}$ and $\boldsymbol{L2}$ are parallel, what do you know about their translated images?



Lesson 3 summary

Lesson 3 – Independent Practice

1. Translate $\angle XYZ$, point A, point B, and rectangle $HIJK$ along vector $\overrightarrow{EF}$. Sketch the images and label all points using prime notation.



2. What is the measure of the translated image of $\angle XYZ$. How do you know?

3. Connect $B$to B'. What do you know about the line formed by B' and the line containing the vector $\overrightarrow{EF}$?

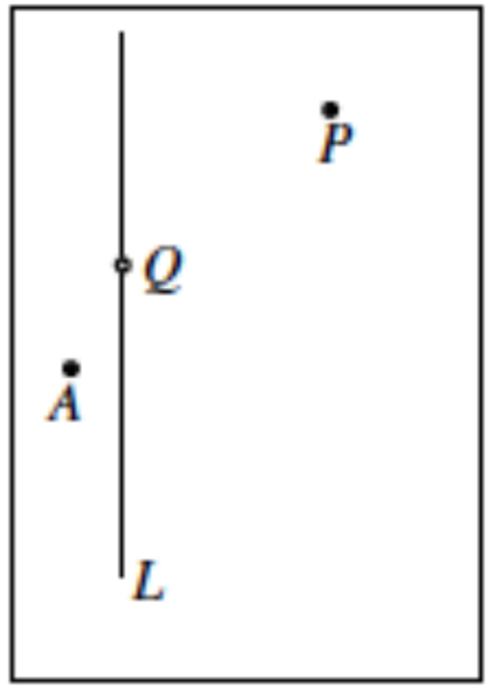
4. Connect A to A'. What do you know about the line formed by A' and the line containing the vector $\overrightarrow{EF}$?

5. Given that figure HIJK is a rectangle, what do you know about lines HI and JK and their translated images? Explain.

Lesson 4: Definition of Reflection and Basic Properties

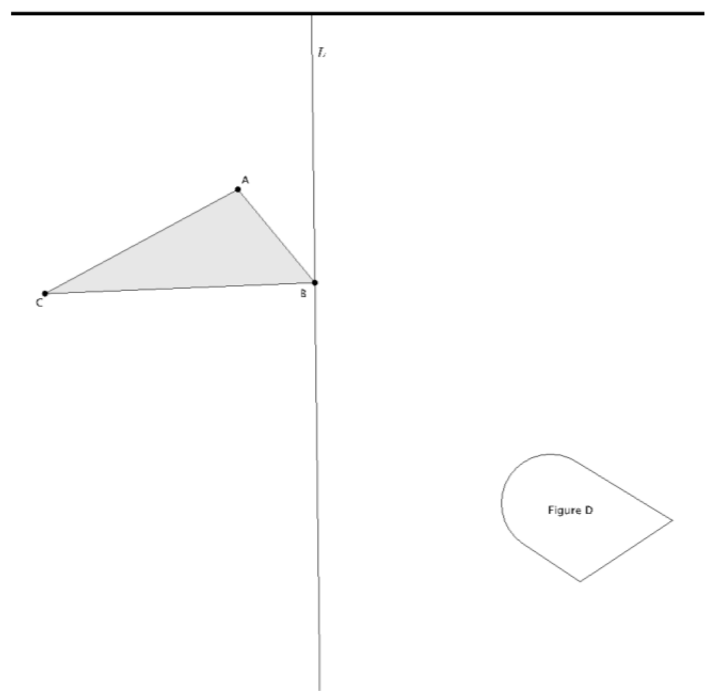
Essential Questions:

Hands-on activity:



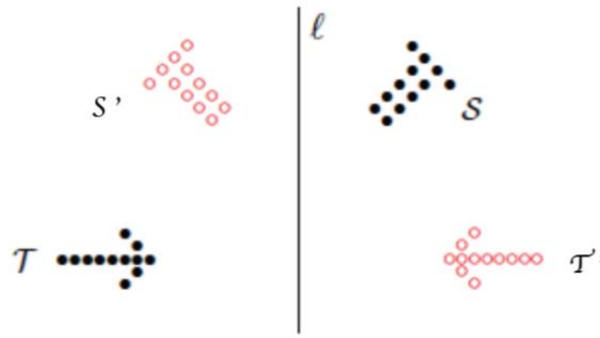
Complete exercise 1 and 2 independently

1. Reflect $\triangle A$BC and Figure D across line L. Label the reflected images.

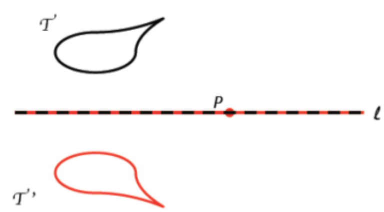


2. Which figure(s) were not moved to a new location on the plane under this transformation?

Example 2

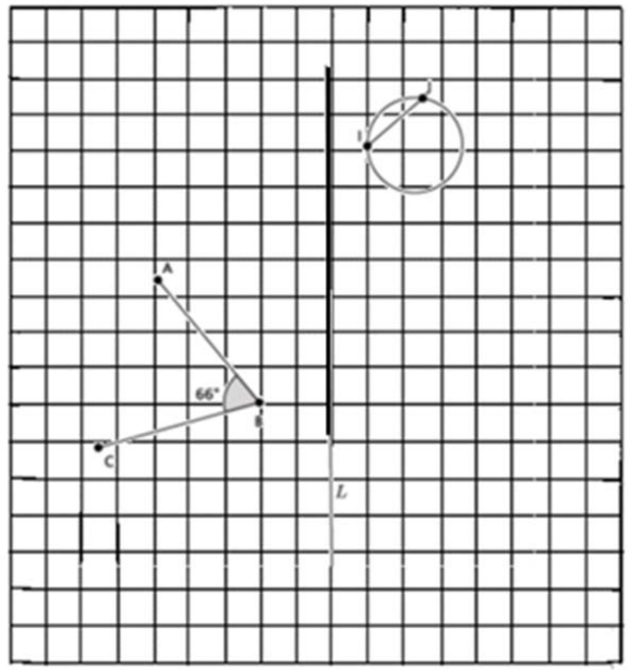


Example 3



Complete exercises 3-5 independently

3. Reflect the images across line L. Label the reflected images.



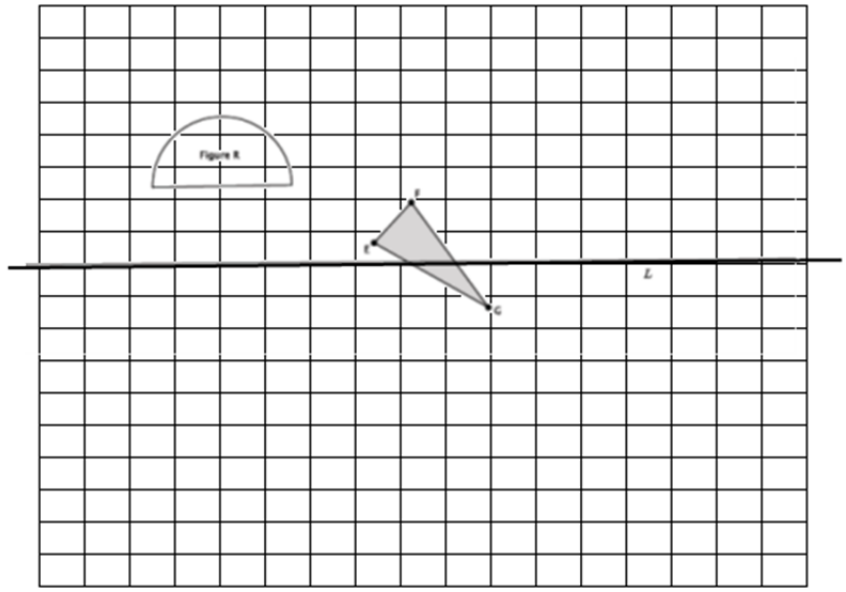
4. Answer the questions about the image above.

a. Use a protractor to measure the reflected $\angle A'B'C'$. What do you notice?

b. Use a ruler to measure the length of IJ and the length of the image of $I'J'$ after the reflection. What do you notice?

5.

Reflect Figure R and ΔEFG across line L. Label the reflected images



Discussion

What are the three basic properties of reflections?

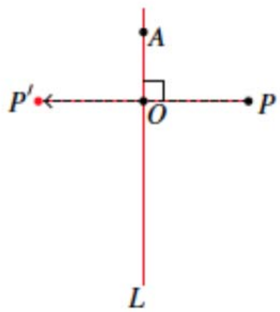
Reflection 1:

Reflection 2:

Reflection 3:

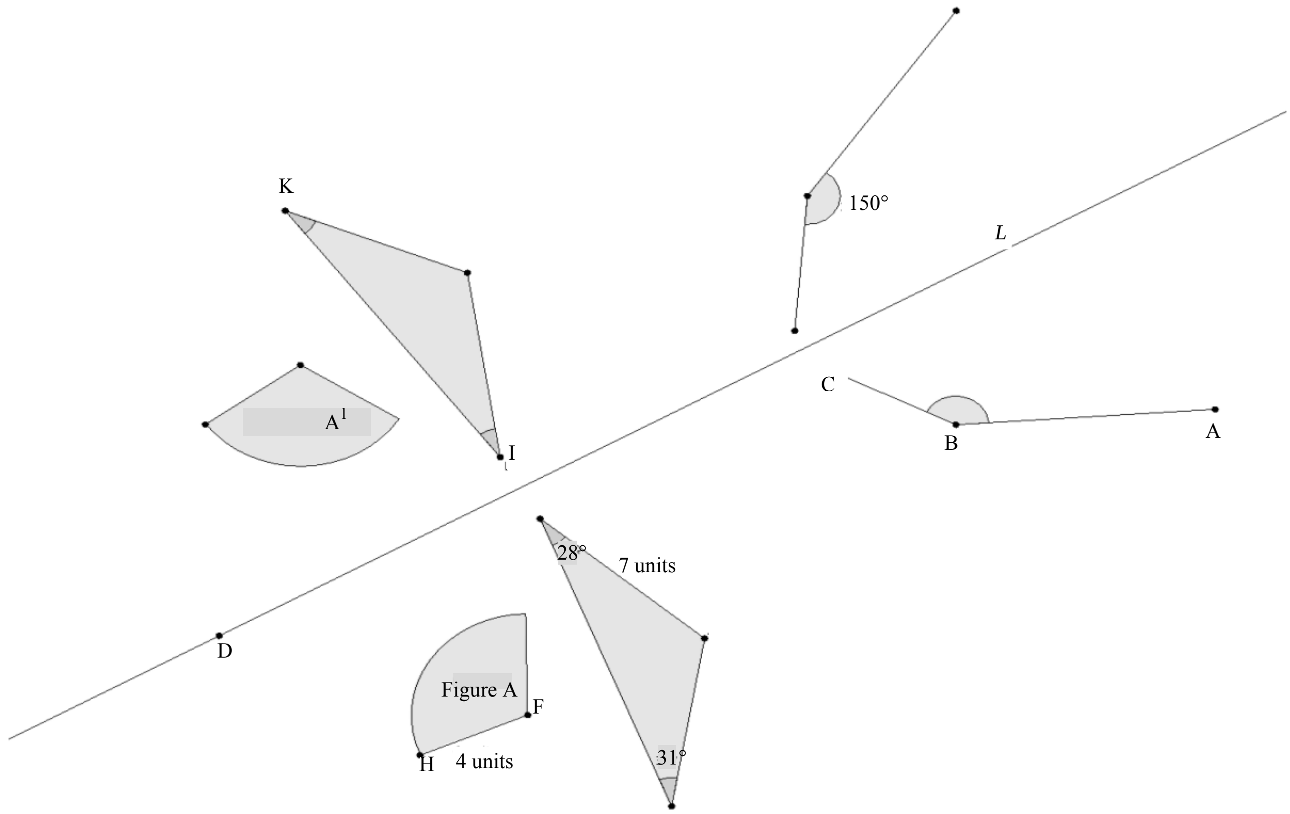
Example 4

A simple consequence of (Reflection 2) is that it gives a more precise description of the position of the reflected image of a point.



Complete exercises 6-9 independently

Use the picture below for exercises 6-9

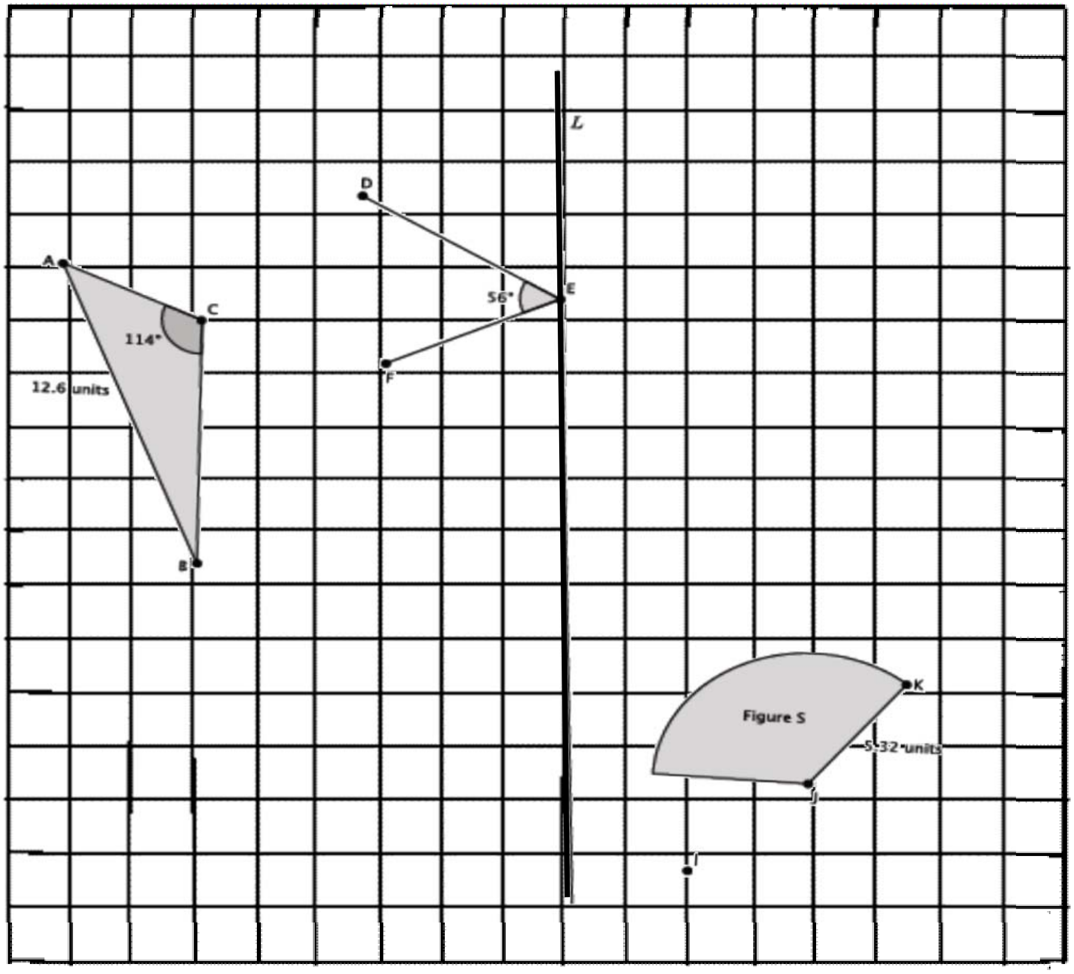


6. Use the picture to label the unnamed points.	
7. What is the measure of $\angle JKI$? $\angle KIJ$? $\angle ABC$? How do you know?	
8. What is the length of segment F'H'? I'J'? How do you know?	
9. What is the location of D'? Explain.	

Lesson 4 Summary

Lesson 4 – Independent practice

1. In the picture below, $\angle DEF=56°$, $\angle ACB=114°$, $AB=12.6$ units, $JK=5.32$ units, point E is on line L, and point I is off of line L. Let there be a reflection across line L. Reflect and label each of the figures, and answer the questions that follow.



2. What is the measure of $\angle D'E'F'$? Explain.

3. What is the length of Reflection $J'K'$? Explain.

4. What is the measure of Reflection $\angle A'C'B'$?

5. What is the length of Reflection $A'B'$)?

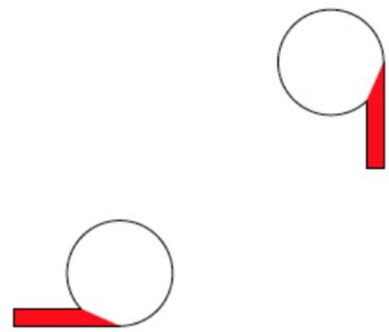
6. Two figures in the picture were not moved under the reflection. Name the two figures and explain why they were not moved.

7. Connect points I and I'. Name the point of intersection of the segment with the line of reflection point Q. What do you know about the lengths of segments IQ and QI'?

Lesson 5: Definition of Rotation and Basic Properties

Essential Questions: **:**

What is the simplest transformation that would map one of the following figures to the other?



Complete exercises 1-4 independently

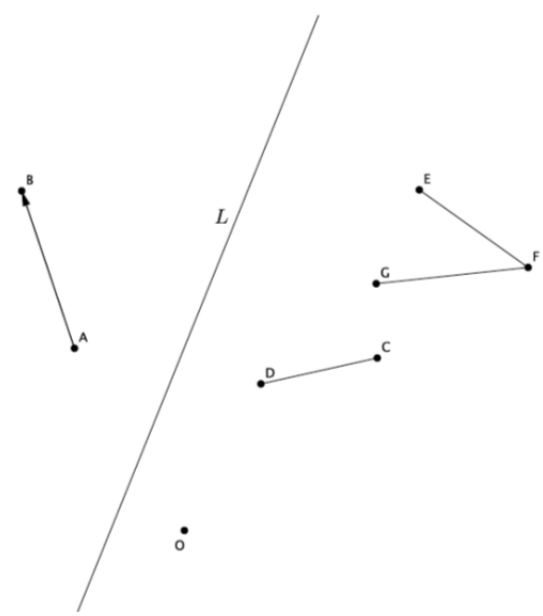
1. Let there be a rotation of d degrees around center O. Let P be a point other than O. Select d so that $d \geq 0$. Find P' (i.e., the rotation of point P) using a transparency.

2. Let there be a rotation of d degrees around center O. Let P be a point other than O. Select d so that $d < 0$. Find P' (i.e., the rotation of point P) using a transparency.

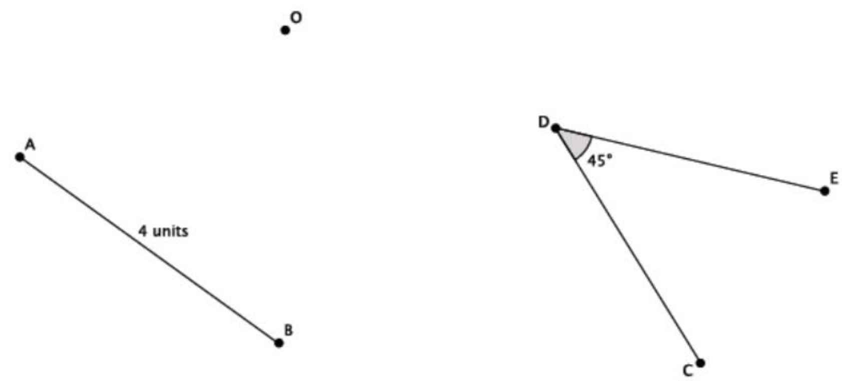
3. Which direction did the point P rotate when d≥0?	
4. Which direction did the point P rotate when d<0?	

Complete exercise 5 and 6 independently

5. Let L be a line, $\overrightarrow{AB}$ be a ray, CD be a segment, and ∠EFG be an angle, as shown. Let there be a rotation of d degrees around point O. Find the images of all figures when d≥0.



6. Let AB be a segment of length 4 units and $\angle\mathrm{CDE}$ be an angle of size 45°. Let there be a rotation by d degrees, where d<0, about O. Find the images of the given figures. Answer the questions that follow.



a. What is the length of the rotated segment $\mathrm{Rotation}(\mathrm{AB})$?

b. What is the degree of the rotated angle $\mathrm{Rotation}(\angle\mathrm{CDE})$?

Concept development

What are the basic properties of Rotations?

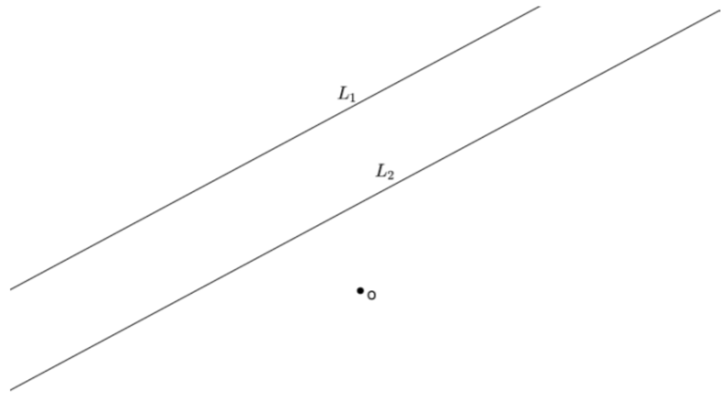
Rotation 1

Rotation 2

Rotation 3

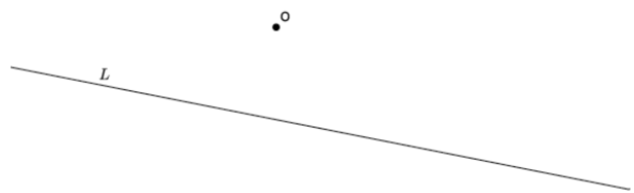
Complete exercises 7 and 8 independently

7. Let $L1$ and $L2$ be parallel lines. Let there be a rotation by d degrees, where $-360<d<360$, about O. Is $(L1)' \parallel (L2)'$?



8. Let L be a line and O be the center of rotation. Let there be a rotation by d degrees, where $d \neq 180$ about O. Are the lines L and L' parallel?

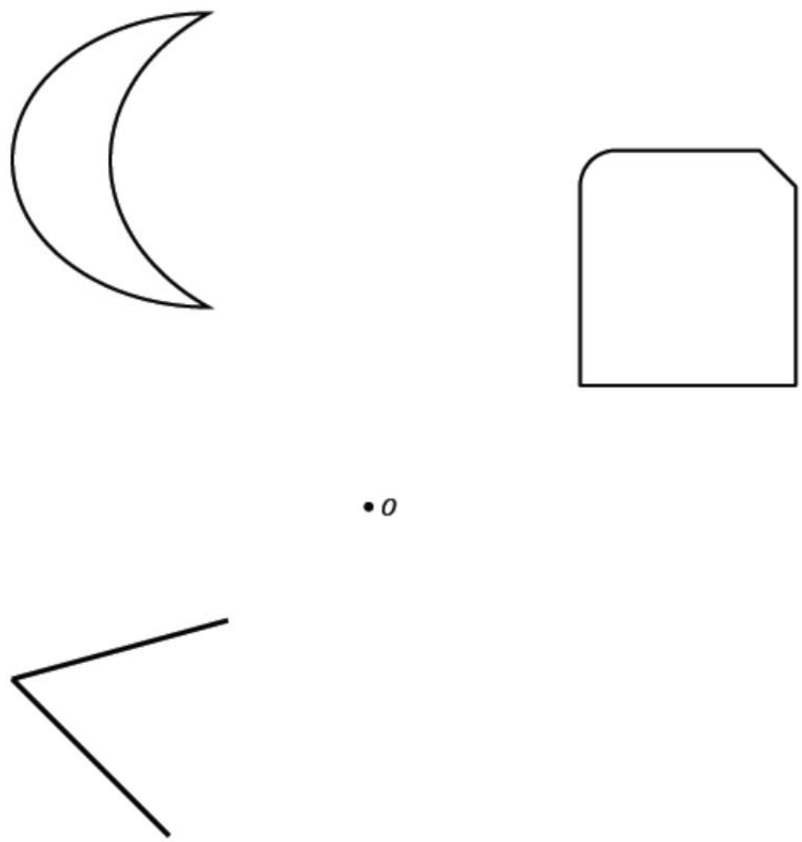
How do you know?



Lesson 5 Summary

Lesson 5 Independent Practice

1. Let there be a rotation by -90^0 around the center O.



2. Explain why a rotation of 90 degrees around any point O never maps a line to a line parallel to itself.

3. A segment length of 94 cm has been rotated d degrees around center O. What is the length of the rotated line segment? How do you know?

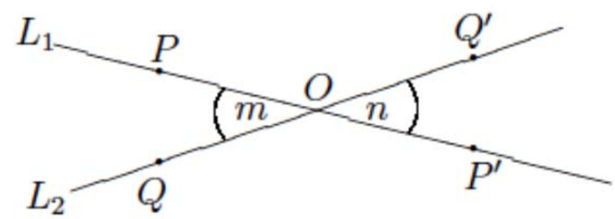
4. An angle size of 124^0 has been rotated d degrees around a center O. What is the size of the rotated angle? How do you know?

Lesson 6: Rotations of 180 Degrees

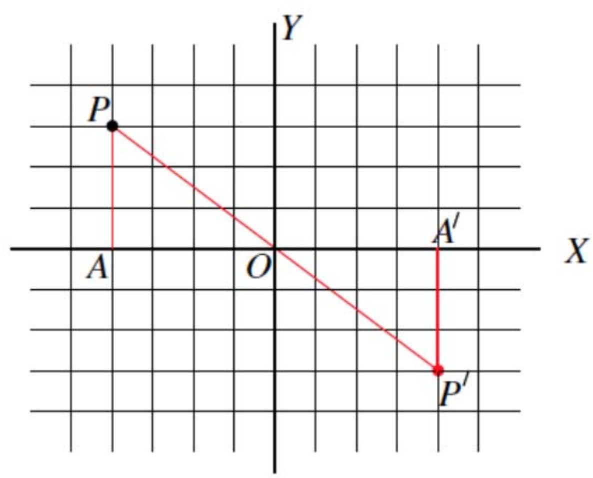
Essential Questions: :

Example 1:

This picture shows a 180 rotation around point O.



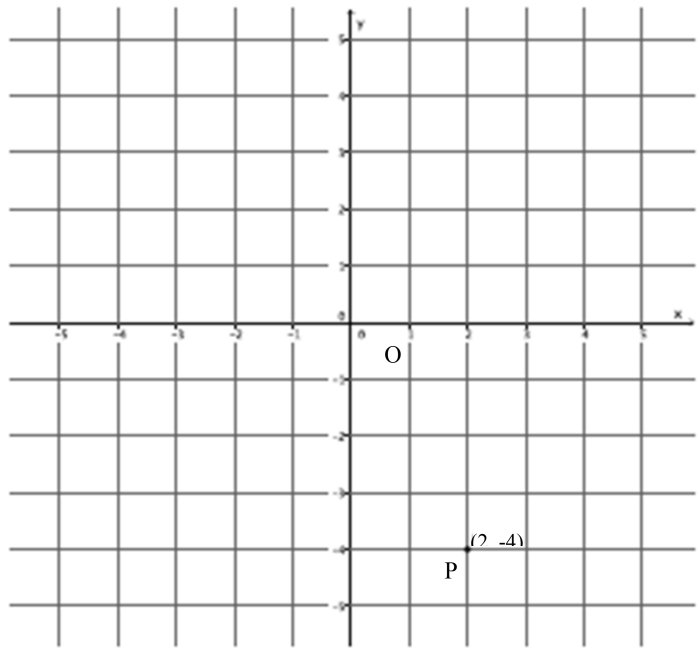
Example 2:
The picture shows what happens when there is a rotation of 180° around center O, the origin of the coordinate plane.



Complete exercise 1 and 2 independently

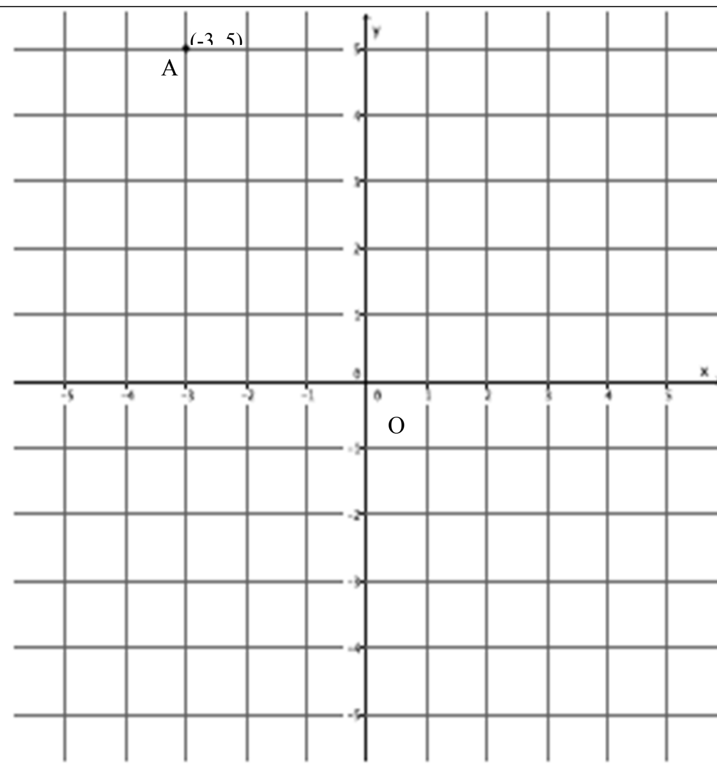
Exercise 1

Using your transparency, rotate the plane 180 degrees, about the origin. Let this rotation be P' What are the coordinates of P'?



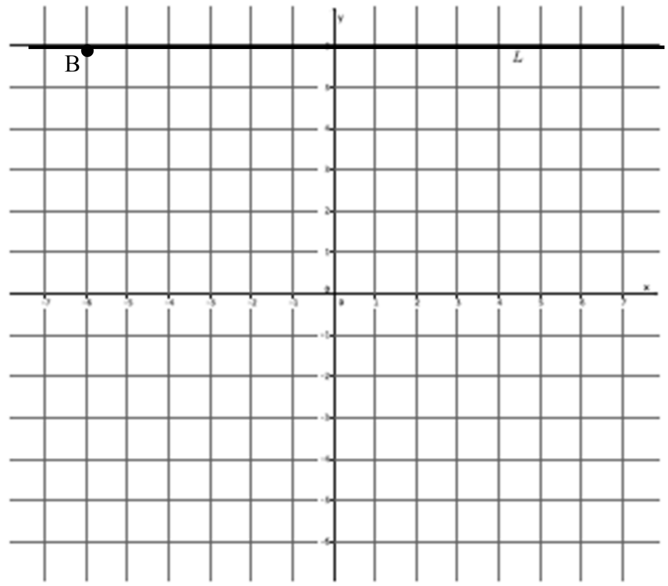
Exercise 2

2. Let A' be the rotation of the plane by 180 degrees, about the origin. **<u>Without</u>** using your transparency, find A'.

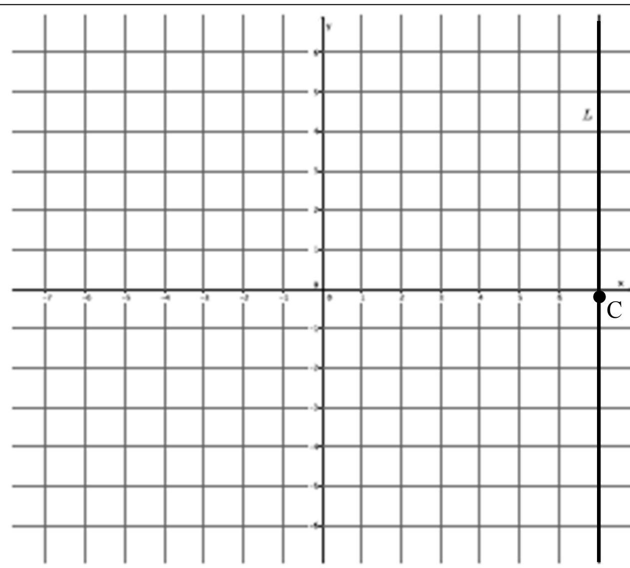


Working in pairs, complete exercise 3 and 4

3. Let B' be the rotation of 180 degrees around the origin. Let L be the line passing through (–6,6) B parallel to the x-axis. Find B' and line L'. Use your transparency if needed.

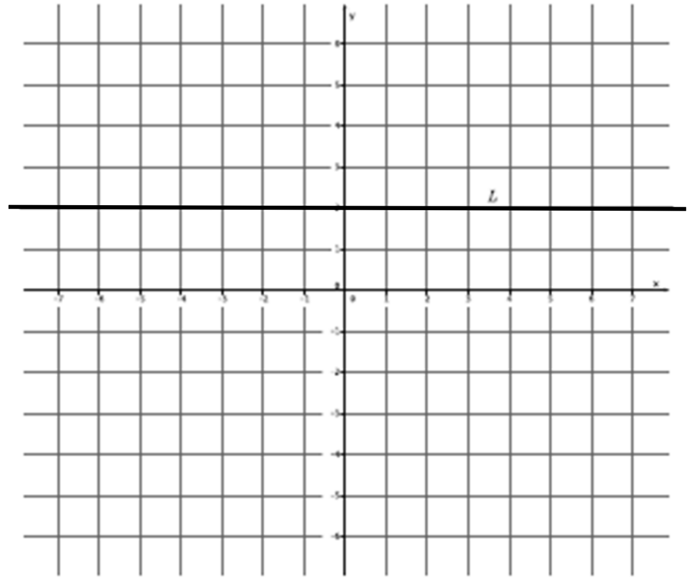


4. Let C' be the rotation of 180 degrees around the origin. Let L be the line passing through (7,0) parallel to the y-axis. Find C' and line L'. Use your transparency if needed.

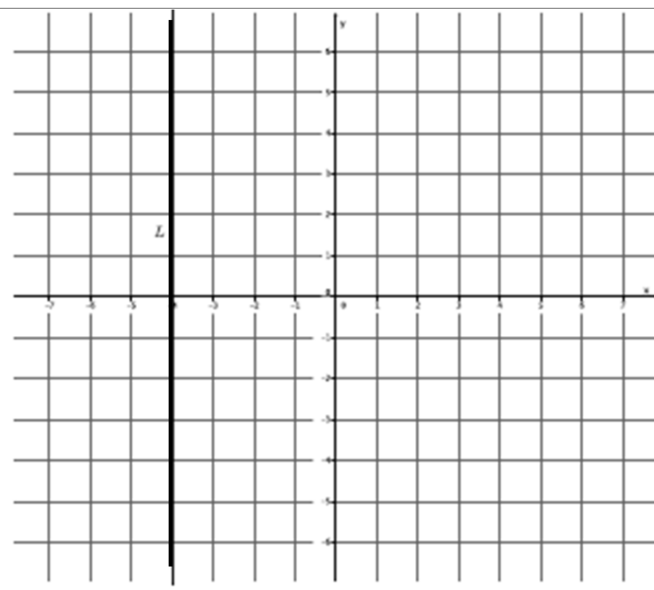


Compete exercises 5-9 independently

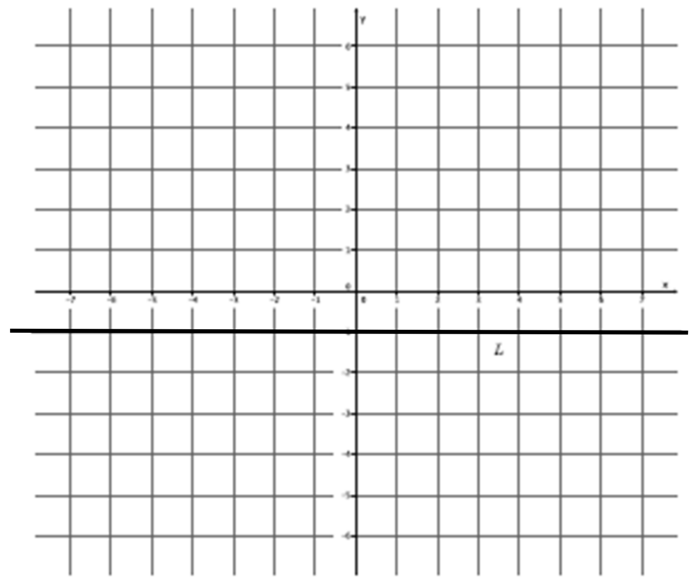
5. Let there be a rotation of 180 degrees around the origin. Let L be the line passing through (0,2) parallel to the x-axis. Is L parallel to L'?



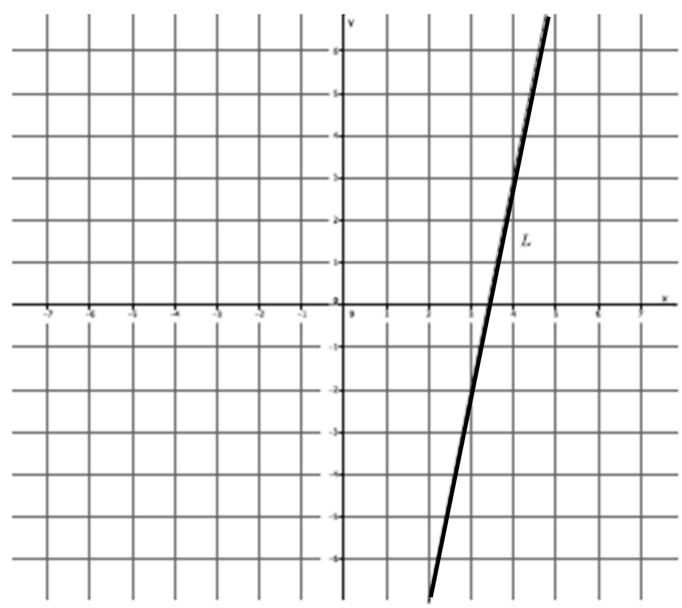
6. Let there be a rotation of 180 degrees around the origin. Let L be the line passing through (-4,0) parallel to the y-axis. Is L parallel to L'?



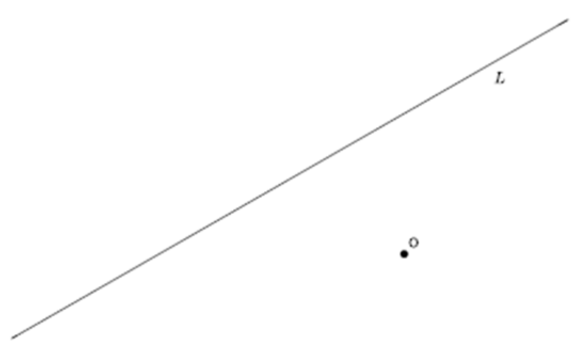
7. Let there be a rotation of 180 degrees around the origin. Let L be the line passing through (0,–1) parallel to the x-axis. Is L parallel to L'?



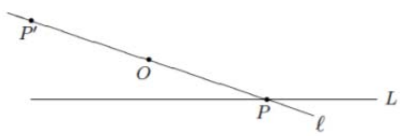
8. Let there be a rotation of 180 degrees around the origin. Is L parallel to L'? Use your transparency if needed.



9. Let $Rotation_O$ be the rotation of 180 degrees around the origin. Is L parallel to $Rotation_O(L)$? Use your transparency if needed.



Example 3
Theorem: Let O be a point not lying on a given line L. Then, the 180-degree rotation around O maps L to a line parallel to L.

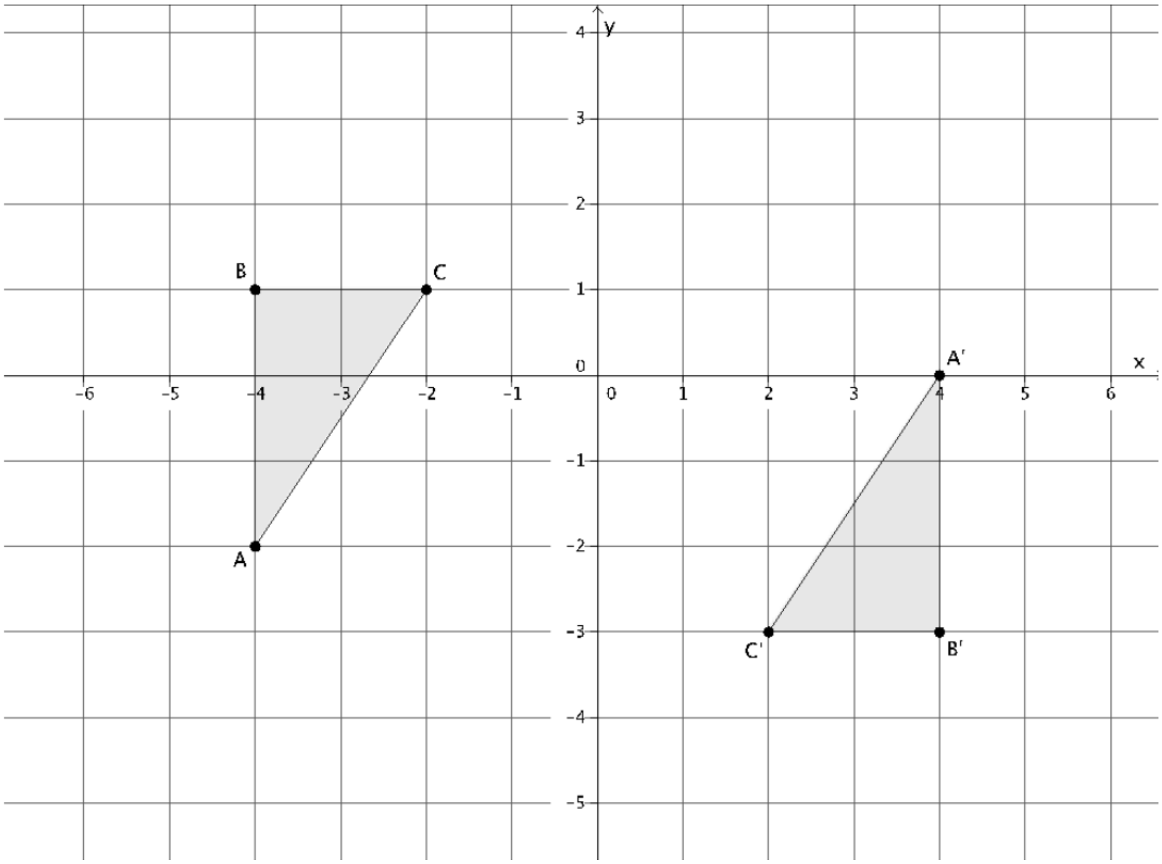


Proof:

Lesson 6 summary

Lesson 6 Independent Practice

Use the following diagram for problems 1-5. Use your transparency as needed.



1. Looking only at segment BC, is it possible that a 180° rotation would map BC to B'C'?

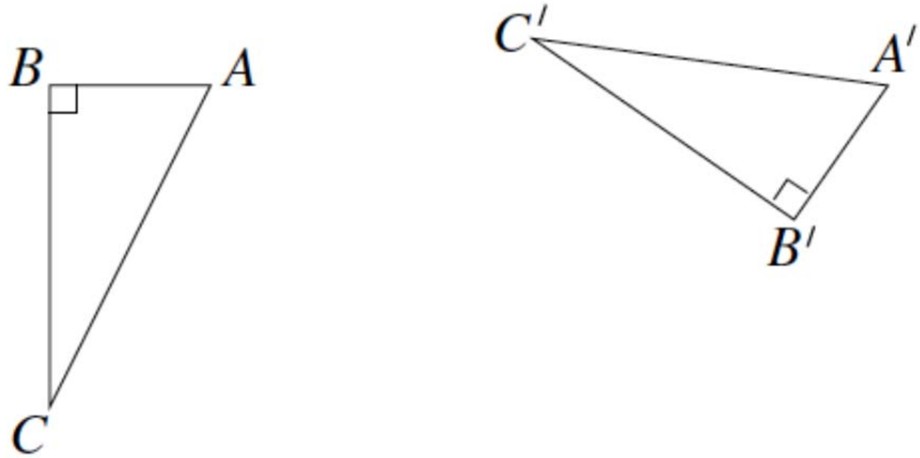
2. Looking only at segment AB, is it possible that a 180° rotation would map AB to A'B'?

3. Looking only at segment AC, is it possible that a 180° rotation would map AC to A'C'?

4. Connect point B to point B', point C to point C', and point A to point A'. What do you notice? What do you think that point is?

5. Would a rotation map triangle ABC to triangle A'B'C'? If so, define the rotation (i.e. degree and center). If not, explain why not.

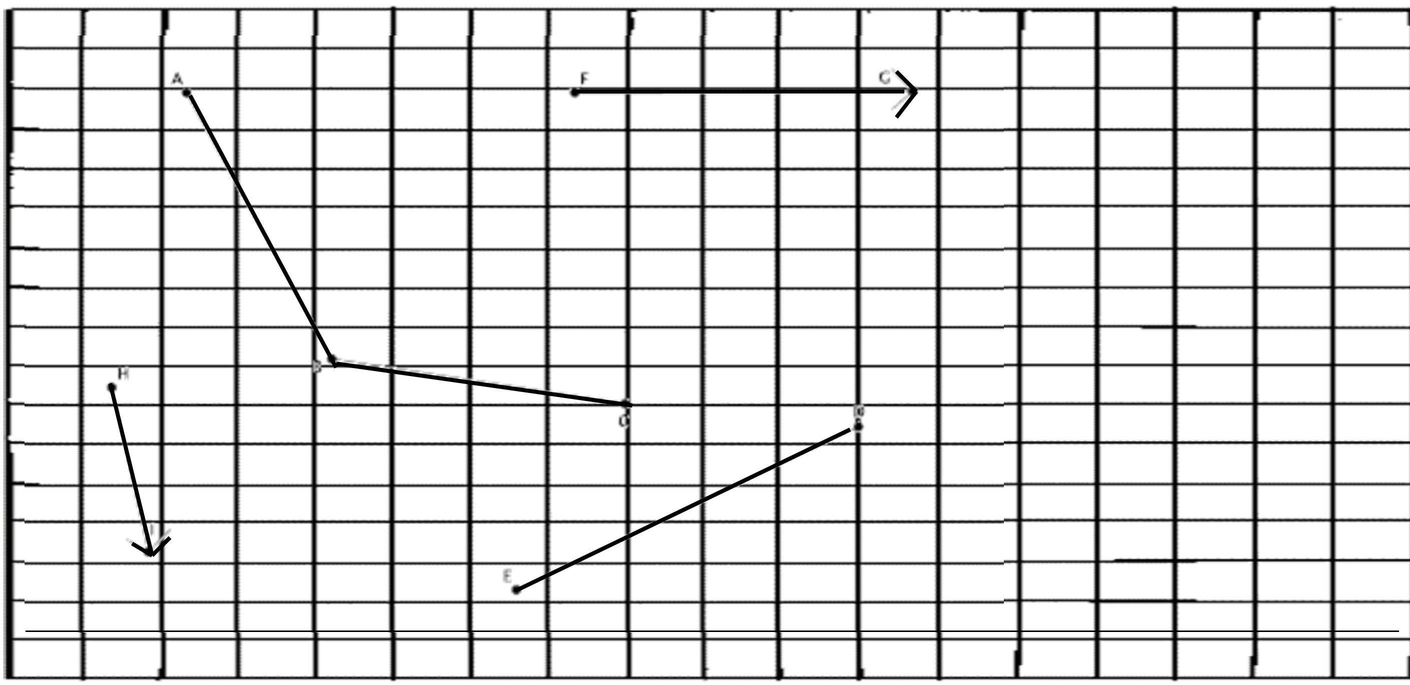
6. The picture below shows right triangles ABC and A'B'C', where the right angles are at B and B'. Given that AB = A'B' = 1, and BC = B'C' = 2, and that AB is not parallel to A'B', is there a 180° rotation that would map triangle ABC to triangle A'B'C'? Explain.



Lesson 7: Sequencing Translations

Essential Questions:

Exploratory Challenge



a. Translate $\angle ABC$ and segment ED along vector $\overrightarrow{FG}$. Label the translated images appropriately, i.e., $A'B'C'$ and $E'D'$.

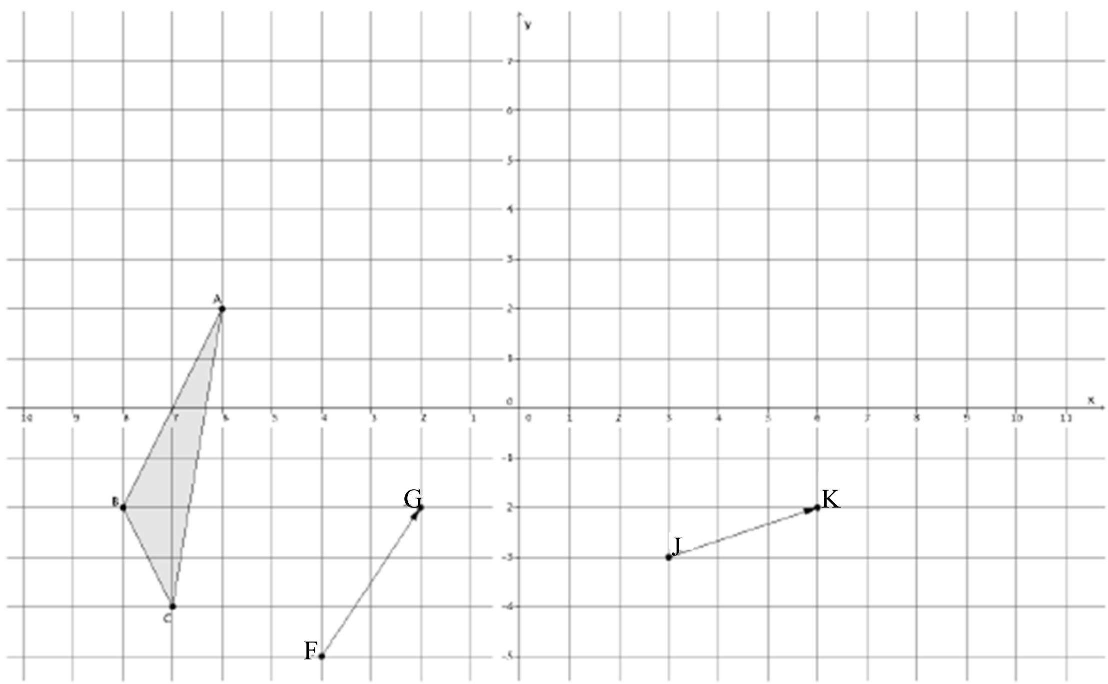
b. Translate $\angle A'B'C'$ and segment $E'D'$ along vector $\overrightarrow{HI}$. Label the translated images appropriately, i.e., $A''B''C''$ and $E''D''$.

c. How does the size of $\angle ABC$ compare to the size of $\angle A''B''C''$?

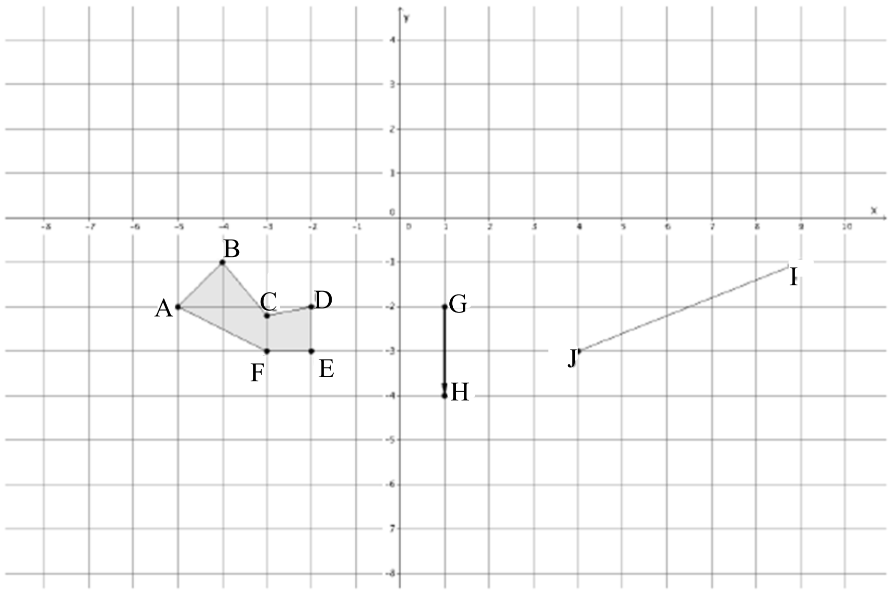
d. How does the length of segment ED compare to the length of the segment $E''D''$?

e. Why do you think what you observed in parts (d) and (e) were true?

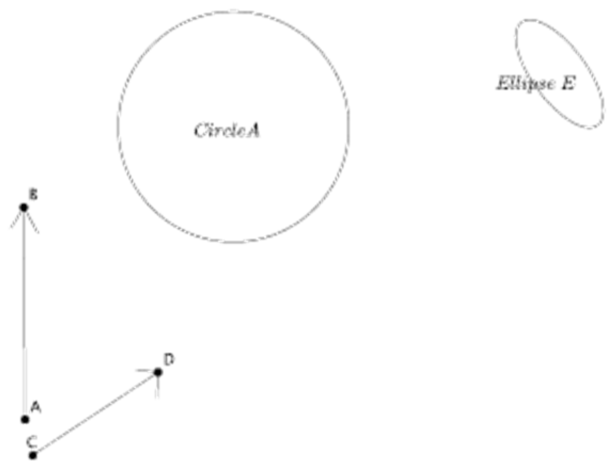
2. Translate $\triangle ABC$ along vector $\overrightarrow{FG}$ and then translate its image along vector $\overrightarrow{JK}$. Be sure to label the images appropriately.



3. Translate figure ABCDEF along vector $\overrightarrow{GH}$. Then translate its image along vector $\overrightarrow{JI}$ Label each image appropriately.



4.



a. Translate Circle A and Ellipse E along vector $\overrightarrow{AB}$. Label the images appropriately.

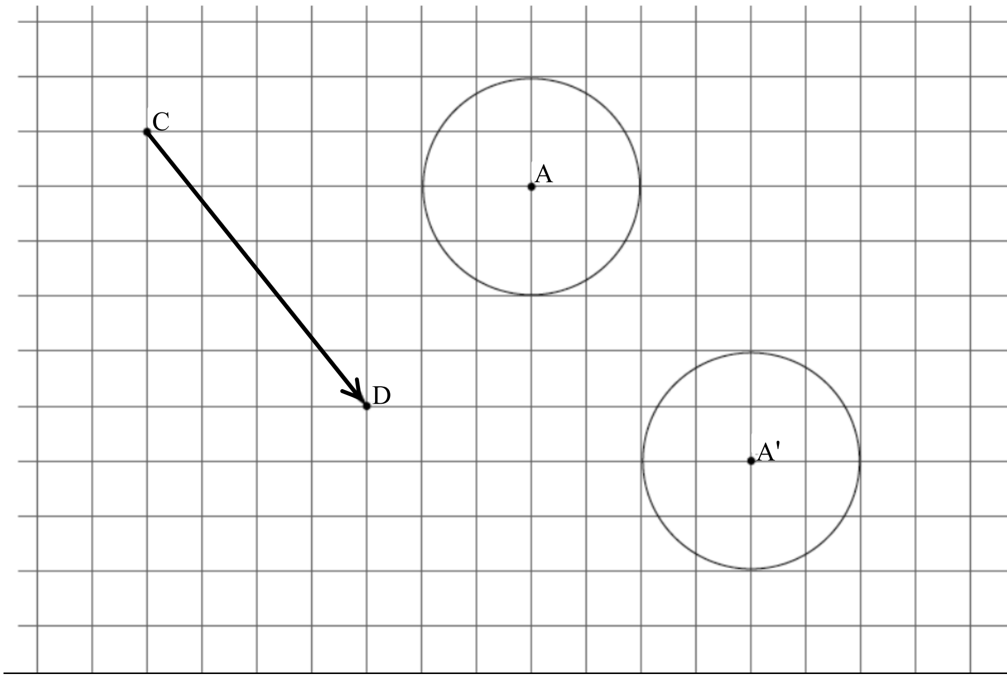
b. Translate Circle A' and Ellipse E' along vector$\overrightarrow{CD}$. Label each image appropriately.

c. Did the size or shape of either figure change after performing the sequence of translations? Explain.

Complete exercises 5 and 6 independently

Exercise 5

The picture below shows the translation of Circle A along vector $\overrightarrow{CD}$. Name the vector that will map the image of Circle A back to its original position.



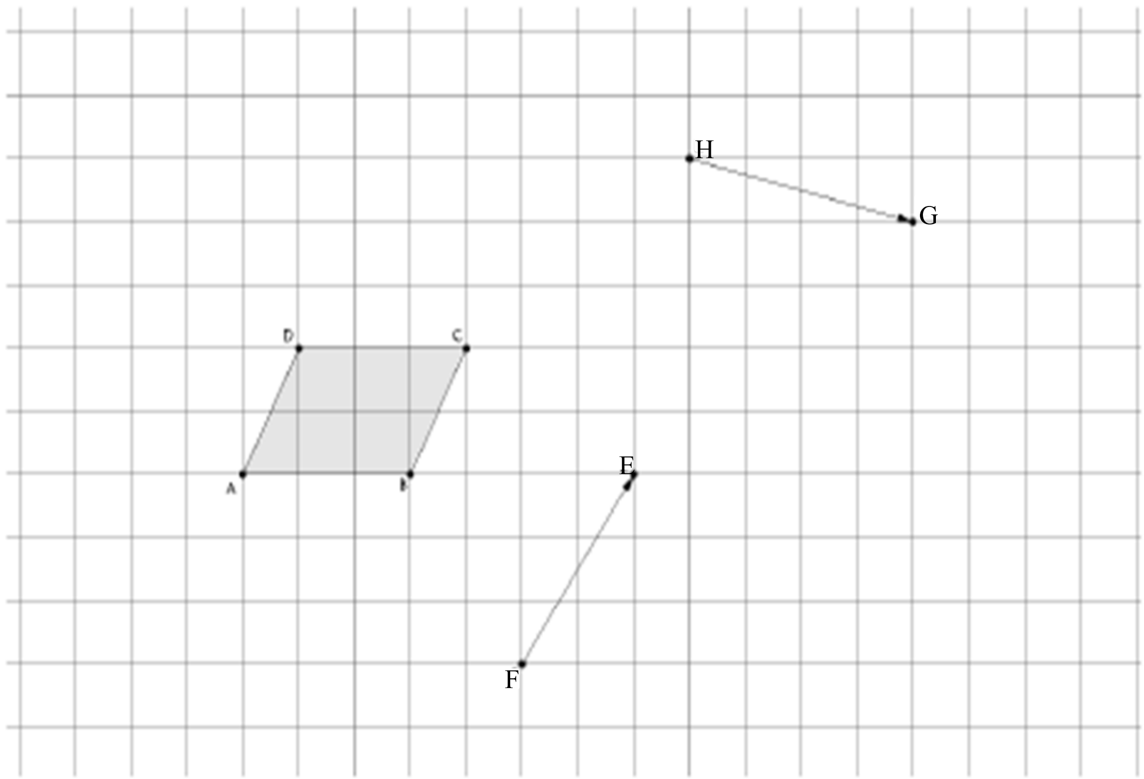
Exercise 6

If a figure is translated along vector $\overrightarrow{QR}$, what translation takes the figure back to its original location?

Lesson 7 Summary

Lesson 7 Independent Practice

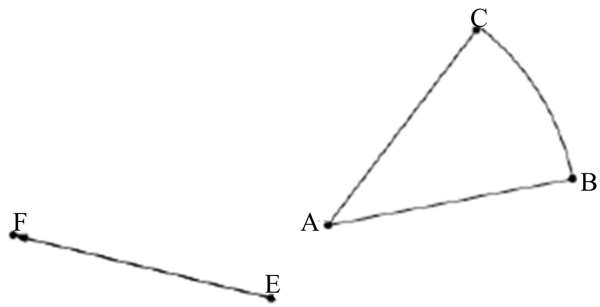
1. Sequence translations of parallelogram ABCD (a quadrilateral in which both pairs of opposite sides are parallel) along vectors $\overrightarrow{HG}$ and $\overrightarrow{FE}$. Label the translated images.



2. What do you know about AD and BC compared with A'B' and B'C'? Explain.

3. Are A'B' and A''B'' equal in length? How do you know?

4. Translate the curved shape ABC along the given vector $\overrightarrow{EF}$. Label the image

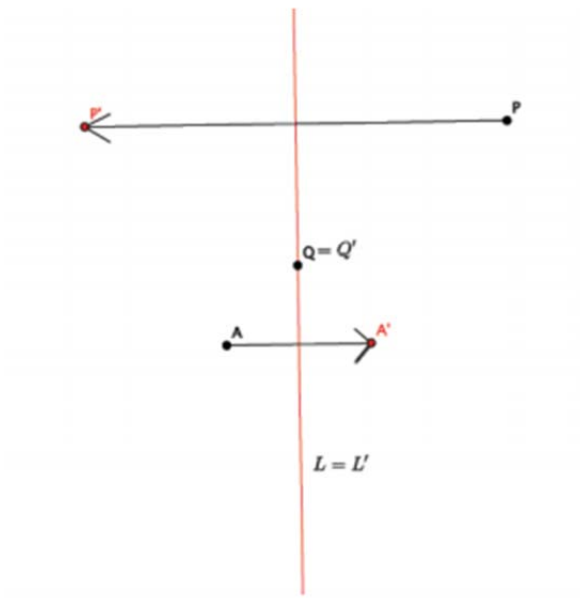


5. What vector would map the shape A'B'C' back onto ABC?

Lesson 8: Sequencing Reflections and Translations

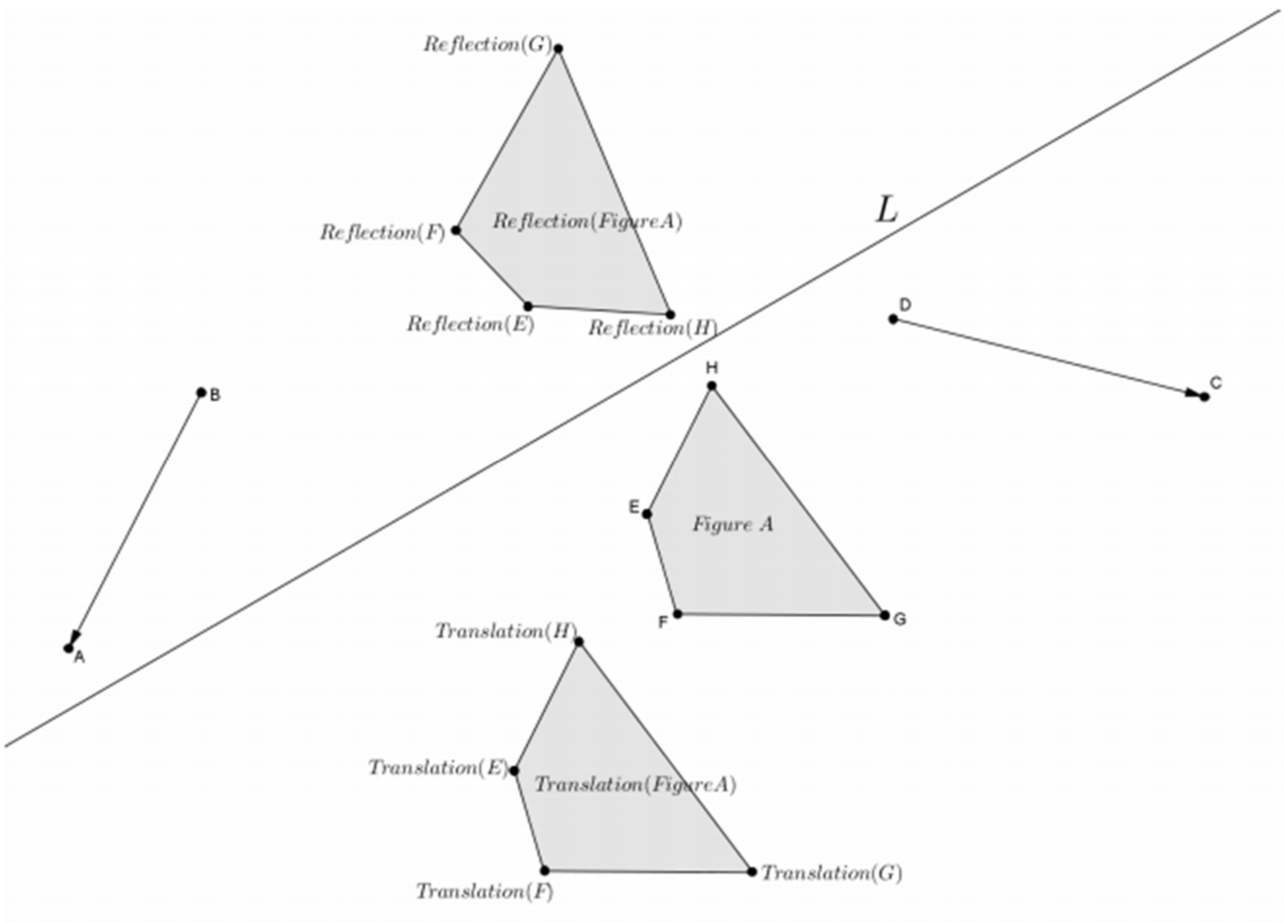
Essential Questions:

Consider the picture below of a reflection across a vertical line L. The arrows show the direction of the reflected image.	



On Your Own

Use the figure for 1-3



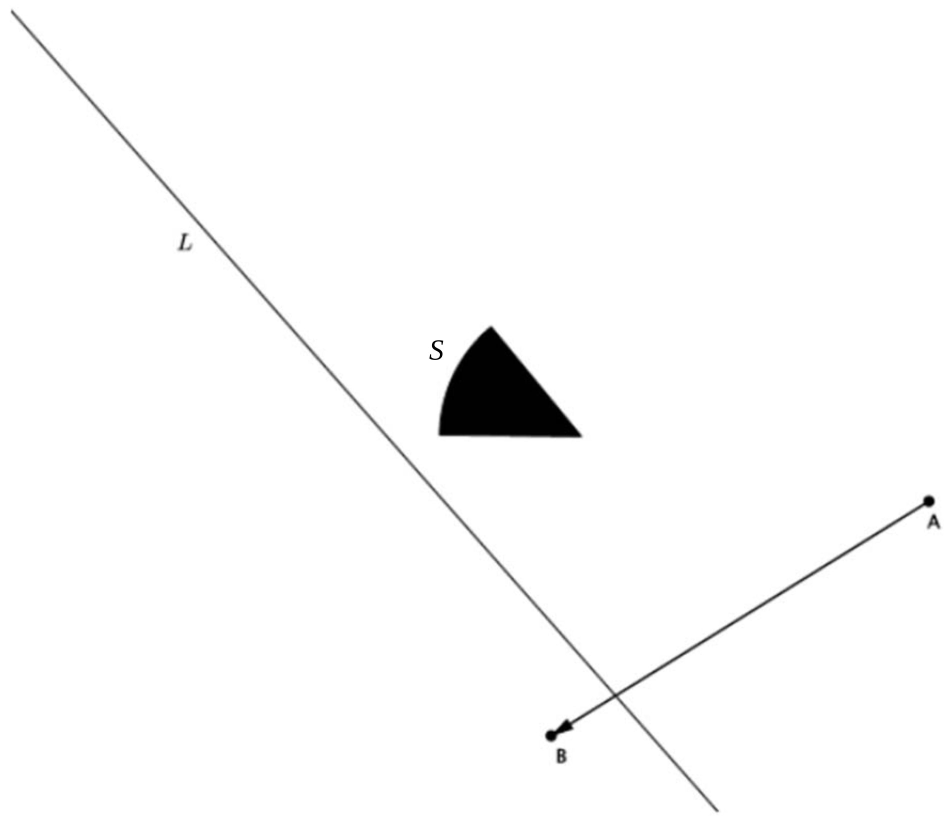
1. Figure A was translated along vector $\overrightarrow{BA}$ resulting in *Translation*$(Figure\ A)$. Describe a sequence of translations that would map Figure A back onto its original position.

2. Figure A was reflected across line L resulting in *Reflection*$(Figure\ A)$. Describe a sequence of reflections that would map Figure A back onto its original position.

3. Can *Translation* $\overrightarrow{BA}$ of Figure A undo the transformation of *Translation* $\overrightarrow{DC}$ of Figure A? Why or why not?

Exercise 4-7

Let S be the black figure.



4. Let there be the translation along vector $\overrightarrow{AB}$ and a reflection across line L. Use a transparency to perform the following sequence: Translate figure S; then, reflect figure S. Label the image S'.

5. Let there be the translation along vector $\overrightarrow{AB}$ and a reflection across line L. Use a transparency to perform the following sequence: Reflect figure S; then, translate figure S. Label the image S''.

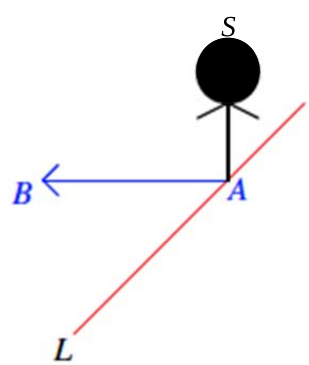
6. Using your transparency, show that under a sequence of any two translations, *Translation* and *Translation*$_0$ (along different vectors), that the sequence of the *Translation* followed by the *Translation*$_0$ is equal to the sequence of the *Translation*$_0$ followed by the *Translation*.. That is, draw a figure, A, and two vectors. Show that the translation along the first vector, followed by a translation along the second vector, places the figure in the same location as when you perform the translations in the reverse order. (This fact will be proven in high school). Label the transformed image A'. Now, draw two new vectors and translate along them just as before. This time, label the transformed image A''. Compare your work with a partner. Was the statement "the sequence of the *Translation* followed by the *Translation*$_0$ is equal to the sequence of the *Translation*$_0$ followed by the *Translation*" true in all cases? Do you think it will always be true?

7. Does the same relationship you noticed in Exercise 6 hold true when you replace one of the translations with a reflection. That is, is the following statement true: A translation followed by a reflection is equal to a reflection followed by a translationn?

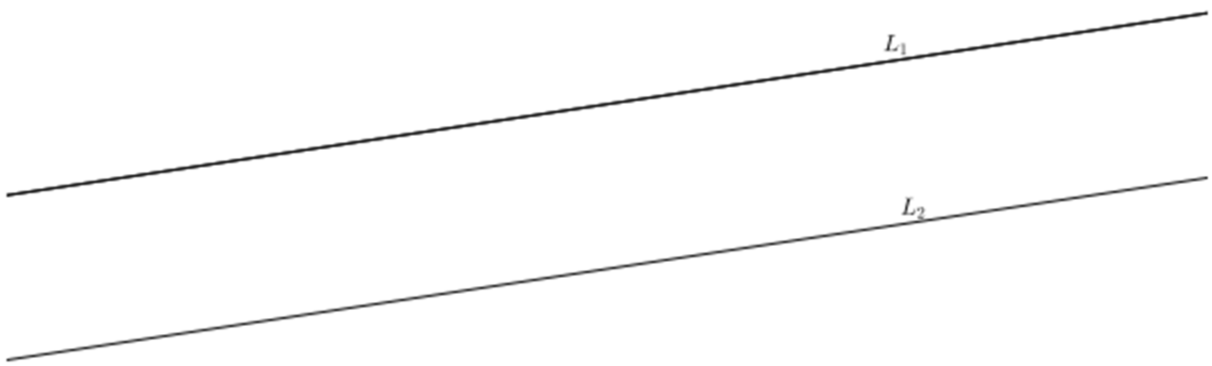
Lesson 8 Summary:

Lesson 8 Independent Practice

1. Let there be a reflection across line L, and let there be a translation along vector $\overrightarrow{AB}$, as shown. If S denotes the black figure, compare the translation of S followed by the reflection of S with the reflection of S followed by the translation of S.



2. Let $L1$ and $L2$ be parallel lines, and let *Reflection*$_1$ and *Reflection*$_2$ be the reflections across $L1$ and $L2$, respectively (in that order). Show that a *Reflection*$_2$ followed by *Reflection*$_1$ is not equal to a *Reflection*$_1$ followed by *Reflection*$_2$. (Hint: Take a point on $L1$ and see what each of the sequences does to it.)



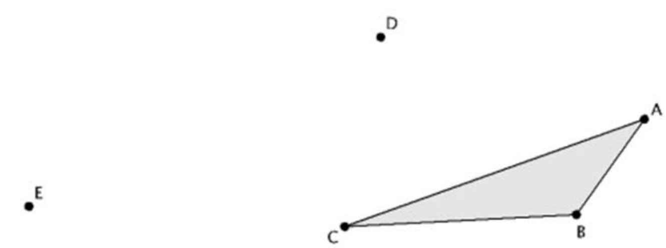
3. Let $L1$ and $L2$ be parallel lines, and let *Reflection*$_1$ and *Reflection*$_2$ be the reflections across $L1$ and $L2$, respectively (in that order). Can you guess what *Reflection*$_1$ followed by *Reflection*$_2$ is? Give as persuasive an argument as you can. (Hint: Examine the work you just finished for the last problem.)

Lesson 9: Sequencing Rotations

Essential Questions:

Classwork (Explore)

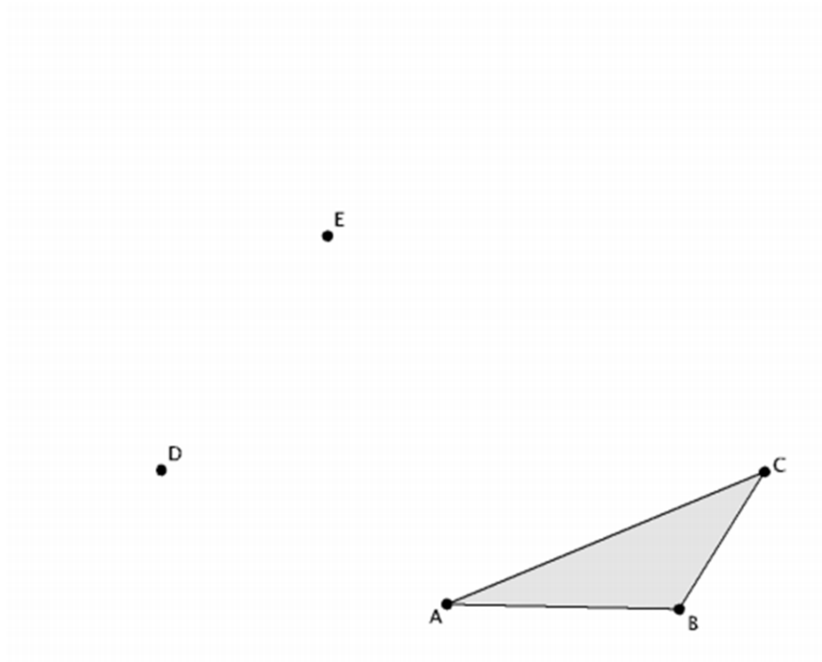
1.



a. Rotate $\triangle ABC$ d degrees around center D. Label the rotated image as $\triangle A'B'C'$	
b. Rotate $\triangle A'B'C'$ d degrees around center E. Label the rotated image as $\triangle A''B''C''$	
c. Measure and label the angles and side lengths of $\triangle ABC$. How do they compare with the images $\triangle A'B'C'$ and $\triangle A''B''C''$?	
d. How can you explain what you observed in part (c)? What statement can you make about properties of sequences of rotations as they relate to a single rotation?	

Explore

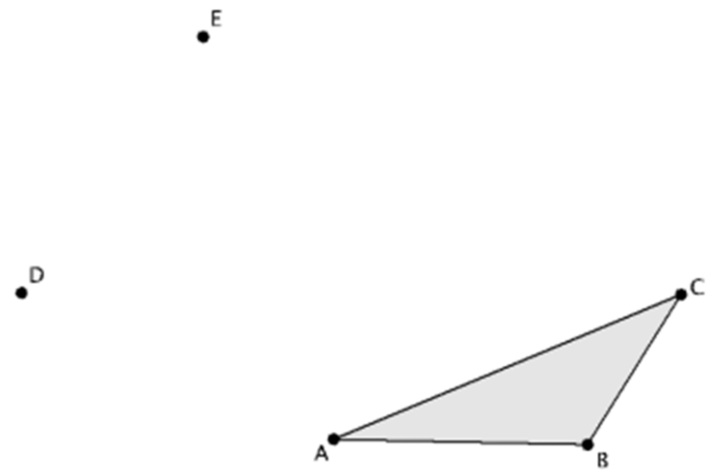
2.



a. Rotate $\triangle ABC$ d degrees around center D, and then rotate again d degrees around center E. Label the image as $\triangle A'B'C'$ after you have completed both rotations.	
b. Can a single rotation around center D map $\triangle A'B'C'$ onto $\triangle ABC$?	
c. Can a single rotation around center E map $\triangle A'B'C'$ onto $\triangle ABC$?	
d. Can you find a center that would map $\triangle A'B'C'$ onto $\triangle ABC$ in one rotation? If so, label the center F.	

Explore

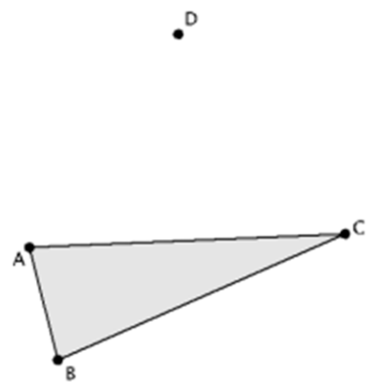
3.



a. Rotate $\triangle ABC$ 90° (counterclockwise) around center D, and then rotate the image another 90° (counterclockwise) around center E. Label the image $\triangle A'B'C'$	
b. Rotate $\triangle ABC$ 90° (counterclockwise) around center E and then rotate the image another 90° (counterclockwise) around center D. Label the image $\triangle A''B''C''$	
c. What do you notice about the locations of $\triangle A'B'C'$ and $\triangle A''B''C''$? Does the order in which you rotate a figure around different centers have an impact on the final location of the figure's image?	

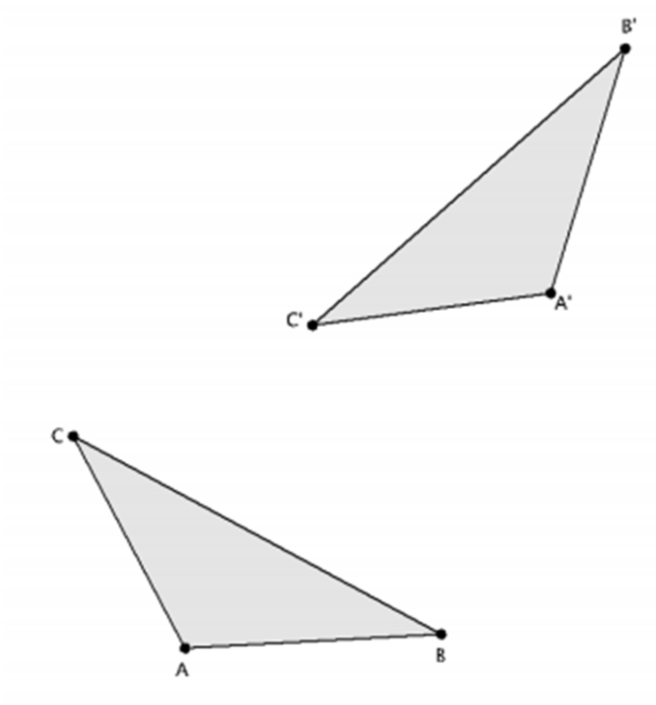
Explore

4.



a. Rotate △ ABC 90° (counterclockwise) around center D, and then rotate the image another 45° (counterclockwise) around center D. Label the△ $A'B'C'$.	
b. Rotate △ ABC 45° (counterclockwise) around center D, and then rotate the image another 90° (counterclockwise) around center D. Label the△ $A''B''C''$.	
c. What do you notice about the locations of △ $A'B'C'$ and △ $A''B''C''$? Does the order in which you rotate a figure around the same center have an impact on the final location of the figure's image?	

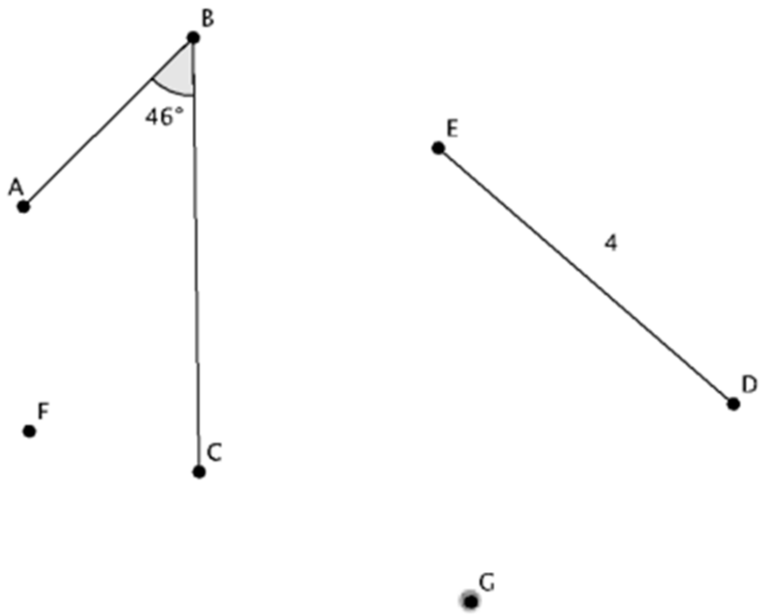
5. $\triangle ABC$ has been rotated around two different centers, and its image is $\triangle A'B'C'$. Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle A'B'C'$.



Lesson 9: Summary

Lesson 9: Independent Practice

1. Refer to the figure below.

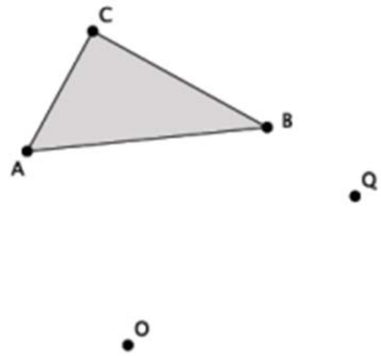


a. Rotate $\angle A$BC and segment DE d degrees around center F, then d degrees around center G. Label the final location of the images as $\angle A'B'C'$ and $D'E'$.

b. What is the size of $\angle A$BC, and how does it compare to the size of $\angle A'B'C'$? Explain.

c. What is the length of segment DE, and how does it compare to the length of segment $D'E'$? Explain.

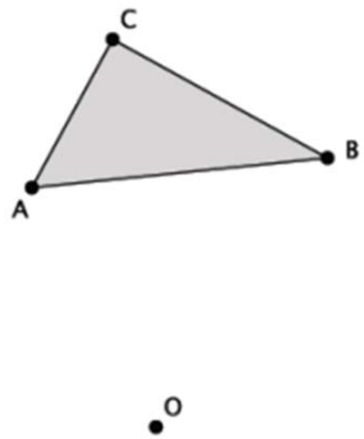
2. Refer to the figure given below.



a. Let $Rotation_1$ be a counterclockwise rotation of 90° around the center O. Let $Rotation_2$ be a clockwise rotation of (−45)° around the center Q. Determine the approximate location of $Rotation_1$ ($\triangle ABC$) followed by $Rotation_2$. Label the image of triangle ABC as $A'B'C'$.

b. Describe the sequence of rigid motions that would map $\triangle ABC$ onto $\triangle A'B'C'$.

3. Refer to the figure given below.



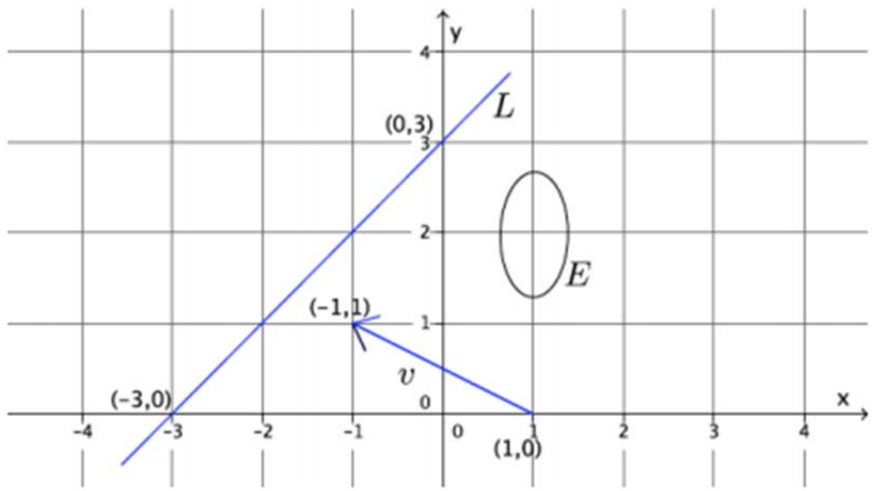
Let R be a rotation of $(-90)°$ around the center O. Let $Rotation_2$ be a rotation of $(-45)°$ around the same center O. Determine the approximate location of $Rotation_1(\triangle ABC)$ followed by $Rotation_2(\triangle ABC)$. Label the image of triangle ABC as $A'B'C'$.

Lesson 10: Sequencing of Rigid Motions

ESSENTIAL QUESTIONS:

Example 1

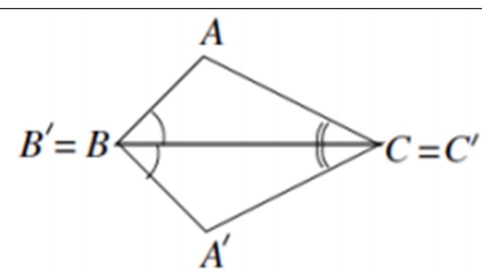
Let E denote the ellipse in the coordinate plane as shown.



Let *Translation₁* be the translation along the vector $\vec{v}$, from (1,0) to (-1,1), let *Rotation₂* be the 90 degree rotation around (-1,1), and let *Reflection₃* be the reflection across line L joining (-3,0) and (0,3). What is the *Translation₁* (E) followed by the *Rotation₂* (E) followed by the *Reflection₃* (E)?

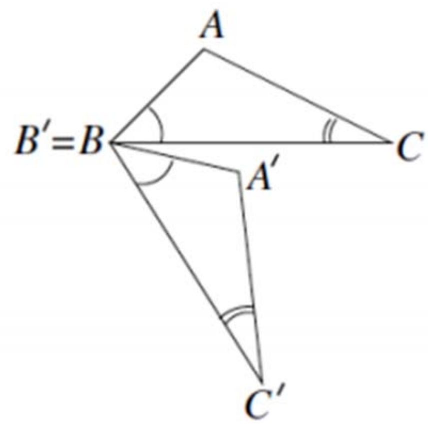
On Your Own

1. In the following picture, triangle ABC can be traced onto a transparency and mapped onto triangle $A'B'C'$. Which basic rigid motion, or sequence of, would map one triangle onto the other?

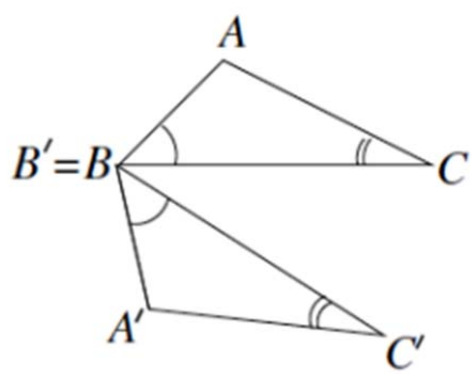


On Your Own

2. In the following picture, triangle ABC can be traced onto a transparency and mapped onto triangle $A'B'C'$. Which basic rigid motion, or sequence of, would map one triangle onto the other?

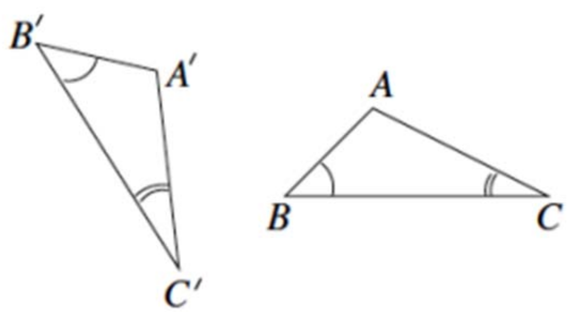


3. In the following picture, triangle ABC can be traced onto a transparency and mapped onto triangle $A'B'C'$. Which basic rigid motion, or sequence of, would map one triangle onto the other?

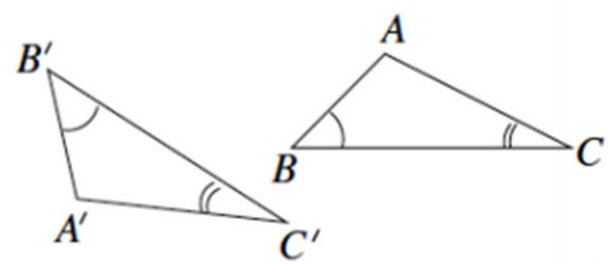


4. In the following picture, we have two pairs of triangles. In each pair, triangle ABC can be traced onto a transparency and mapped onto triangle $A'B'C'$. Which basic rigid motion, or sequence of, would map one triangle onto the other?

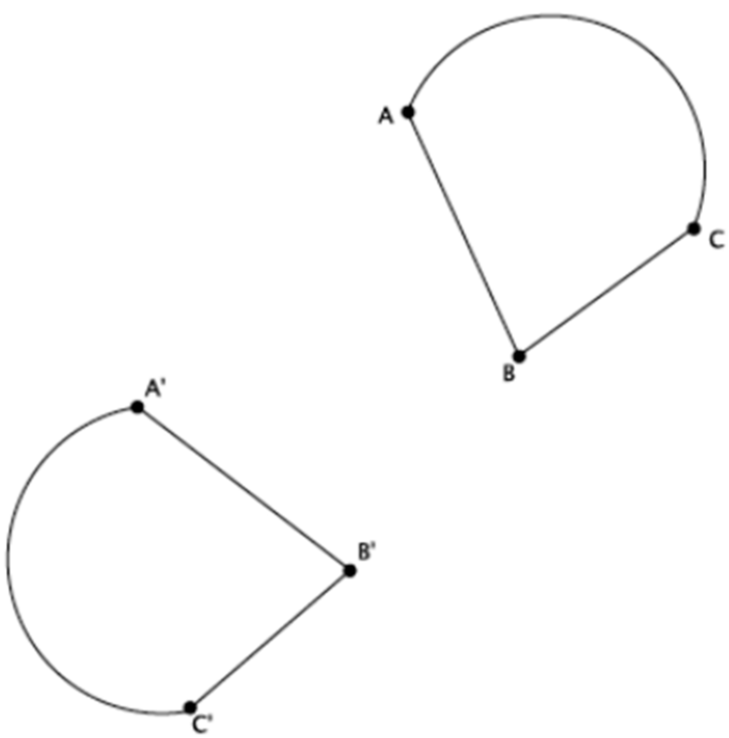
Scenario 1:



Scenario 2:



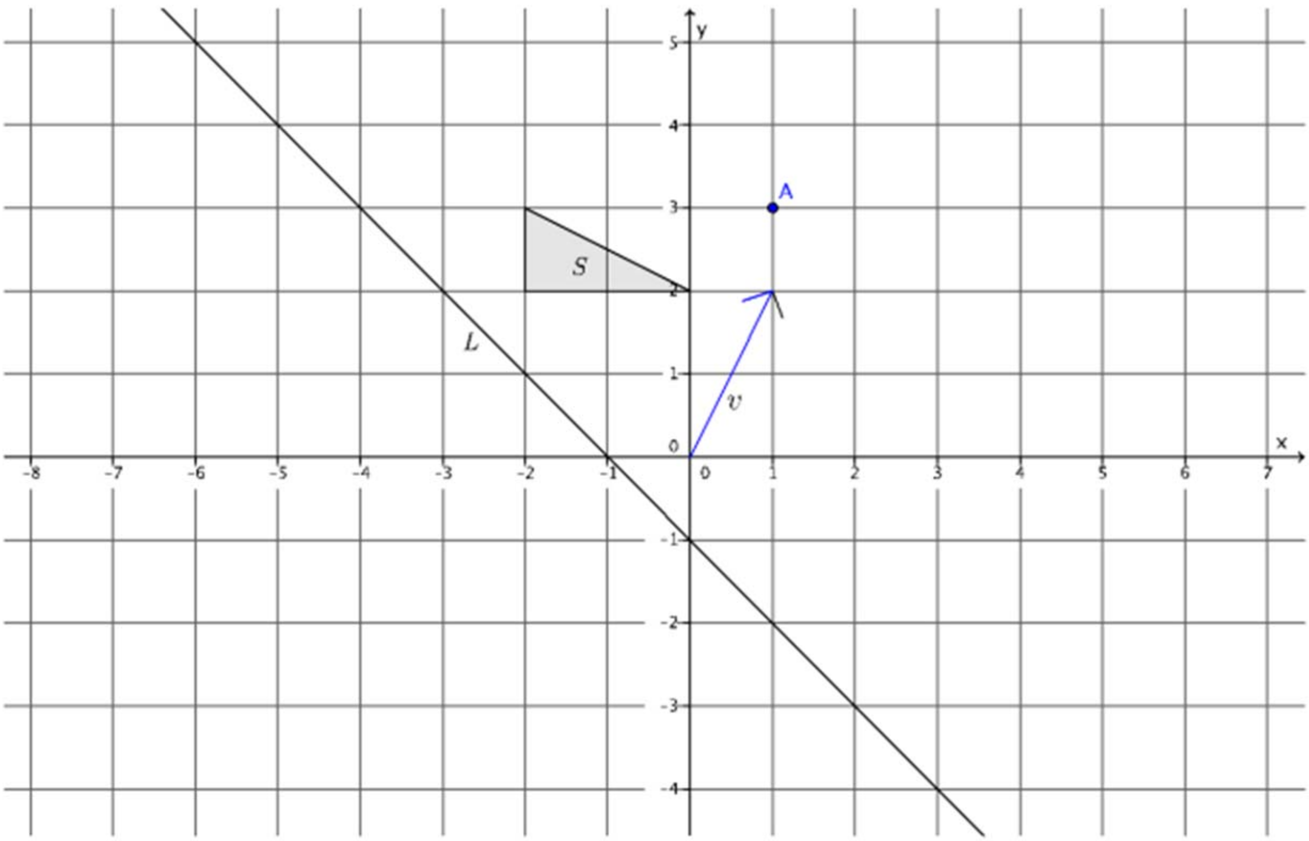
5. Let two figures ABC and $A'B'C'$ be given so that the length of curved segment AC equals the length of curved segment $A'C'$, $\|\angle B\| = \|\angle B'\| = 80°$, and $\|AB\| = \|A'B'\| = 5$. With clarity and precision, describe a sequence of rigid motions that would map figure ABC onto figure $A'B'C'$.	



Lesson 10 Summary:

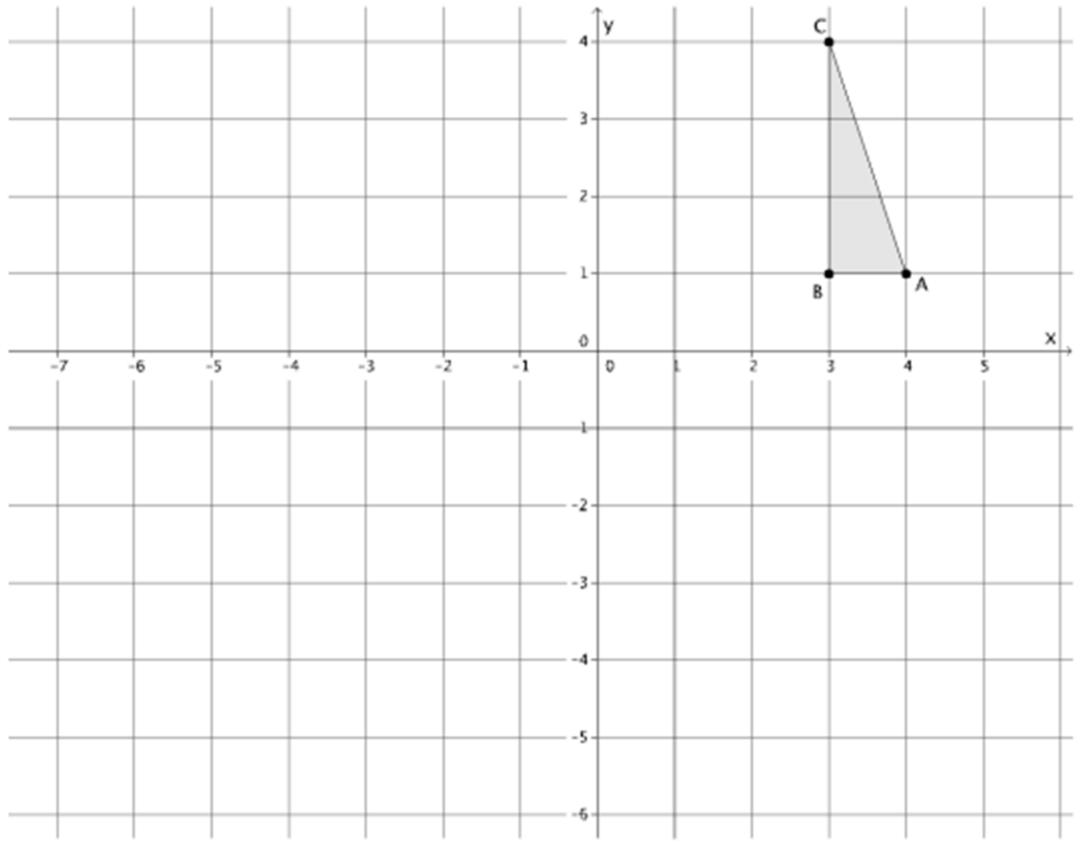
Lesson 10 Independent Practice

1. Let there be the translation along vector $\vec{v}$, let there be the rotation around point A, –90 degrees (clockwise), and let there be the reflection across line L. Let S be the figure as shown below. Show the location of S after performing the following sequence: a translation followed by a rotation followed by a reflection.



2. Would the location of the image of S in the previous problem be the same if the translation was performed last instead of first, i.e., does the sequence: translation followed by a rotation followed by a reflection equal a rotation followed by a reflection followed by a translation? Explain.

3. Use the same coordinate grid to complete parts (a)–(c).



a. Reflect triangle ABC across the vertical line, parallel to the y-axis, going through point (1, 0). Label the transformed points A,B,C as A',B',C', respectively.

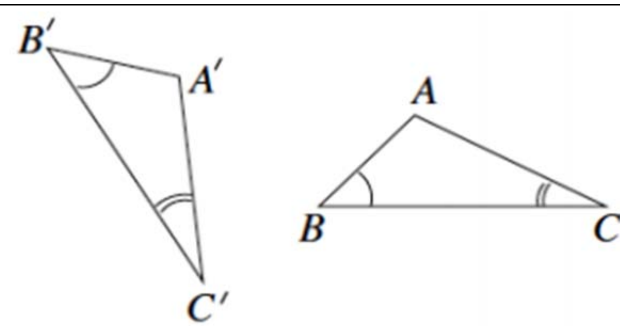
b. Reflect triangle $A'B'C'$ across the horizontal line, parallel to the x-axis going through point (0, –1). Label the transformed points of A',B',C' as A'',B'',C'', respectively.

c. Is there a single rigid motion that would map triangle ABC to triangle $A''B''C''$?

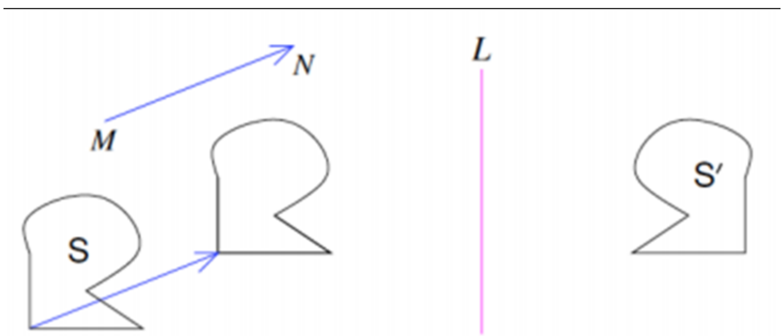
Lesson 11: Definition of Congruence and Some Basic Properties

ESSENTIAL QUESTIONS:

Classwork

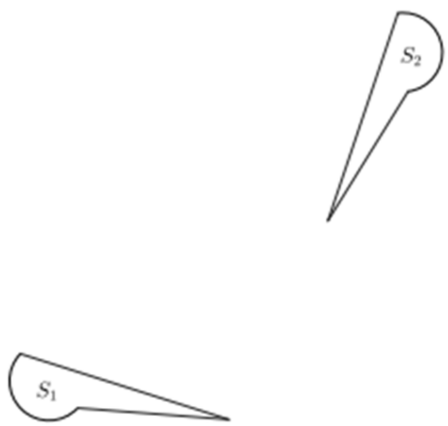
Example 1:	
Describe the sequence of rigid motions that demonstrates the two triangles shown are congruent, i.e., $\triangle ABC \cong \triangle A'B'C'$	

Example 2:

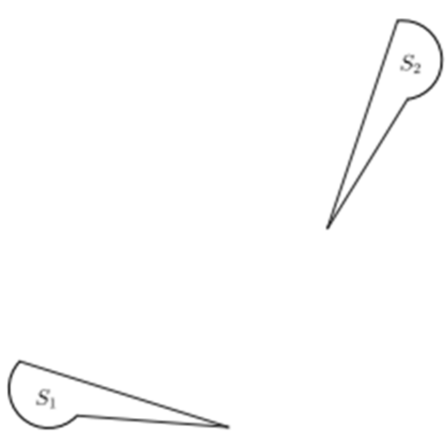


On Your Own

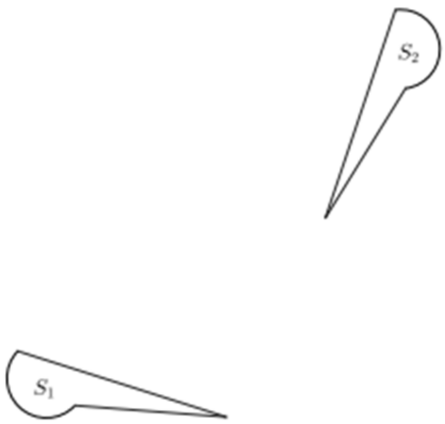
Exercise 1: a. Describe the sequence of basic rigid motions that shows $S_1 \cong S_2$.	



b. Describe the sequence of basic rigid motions that shows $S_2 \cong S_3$.	



c. Describe a sequence of basic rigid motions that shows $S_1 \cong S_3$.	



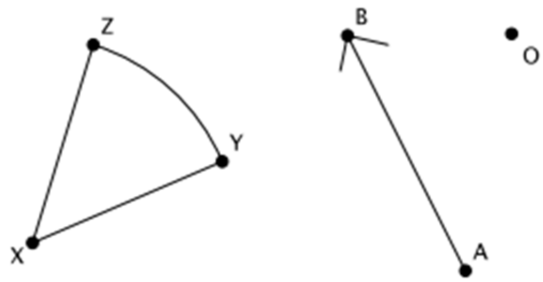
Discussion

PROPERTIES OF CONGRUENCE

Congruence 1	
Congruence 2	
Congruence 3	

Exercise 2

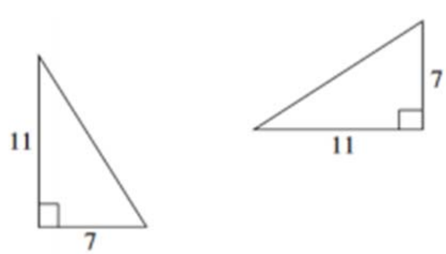
Perform the sequence of a translation followed by a rotation of Figure XYZ, where T is a translation along a vector $\overrightarrow{AB}$, and R is a rotation of d degrees (you choose d) around a center O. Label the transformed figure $X'Y'Z'$. Will XYZ $\cong X'Y'Z'$?



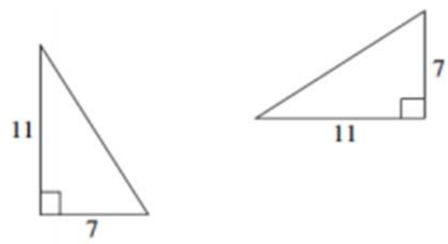
Lesson 11 Summary:

Lesson 11 Independent Practice

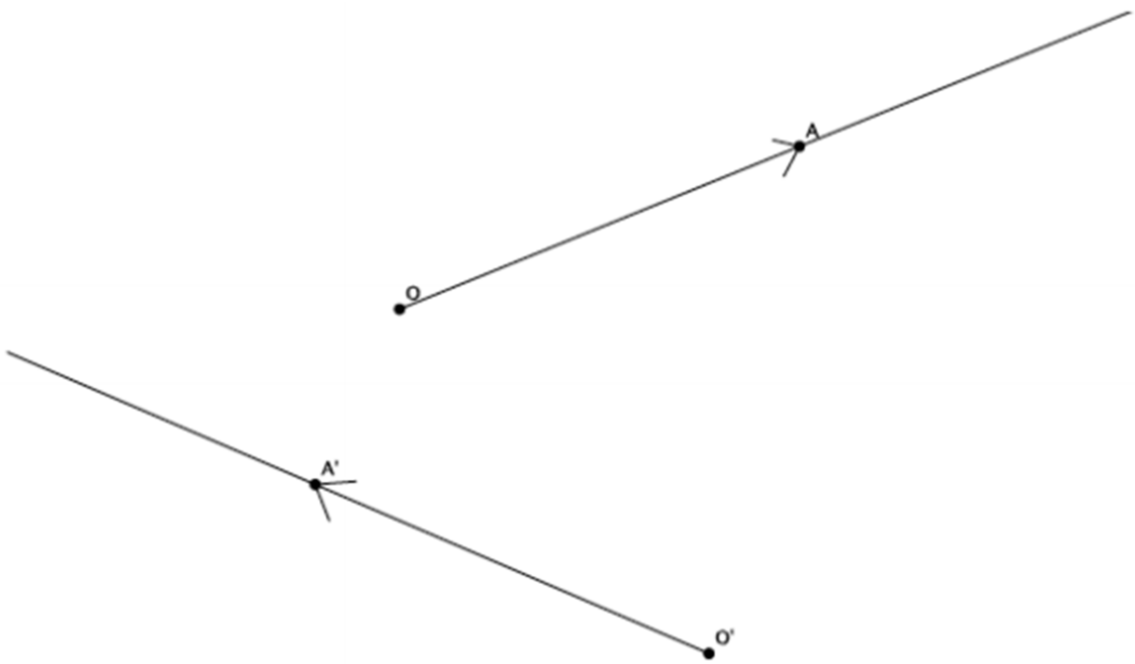
1. Given two right triangles with lengths shown below, is there one basic rigid motion that maps one to the other? Explain.



2. Are the two right triangles shown below congruent? If so, describe a congruence that would map one triangle onto the other.



3. Given two rays, $\overrightarrow{OA}$ and $O'A'$:



a. Describe a congruence that maps $\overrightarrow{OA}$ to $\overrightarrow{O'A'}$.

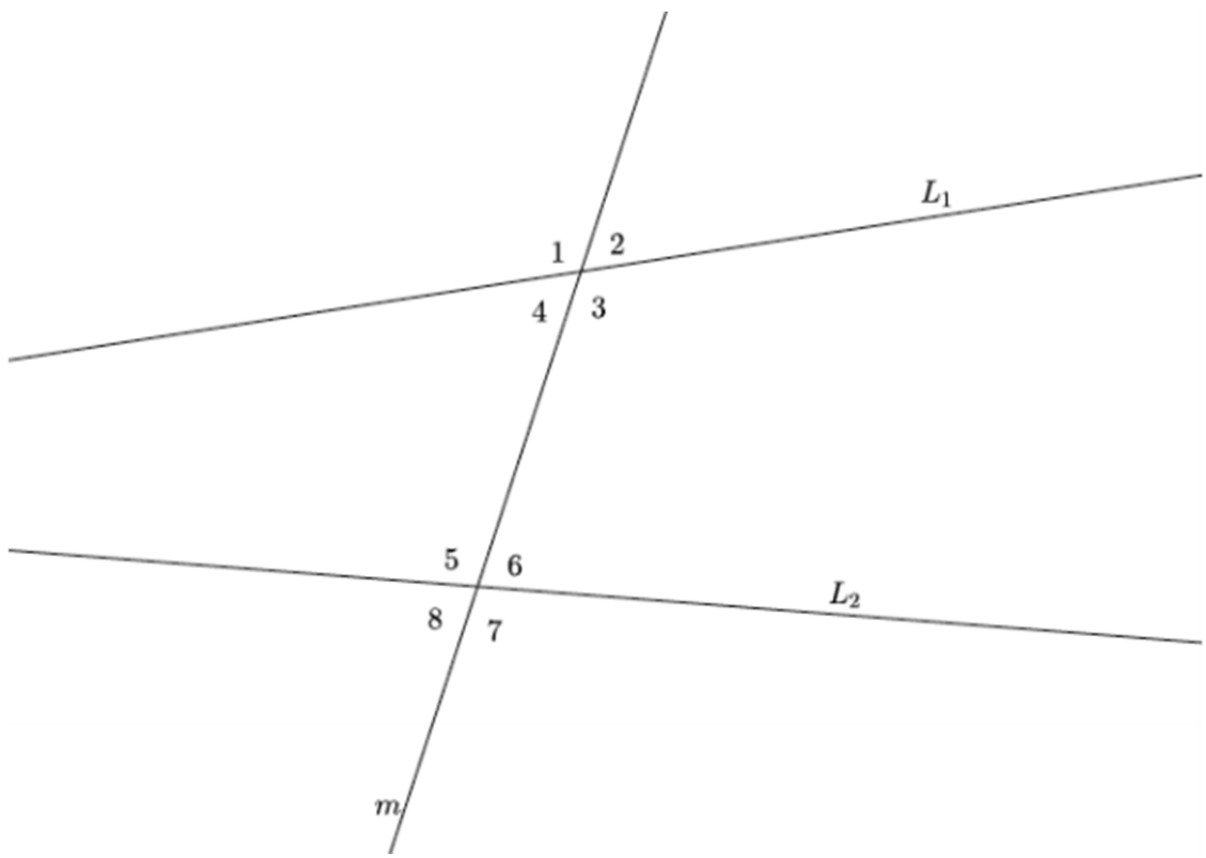
b. Describe a congruence that maps $\overrightarrow{O'A'}$ to $\overrightarrow{OA}$.

Lesson 12: Angles Associated with Parallel Lines

ESSENTIAL QUESTIONS:

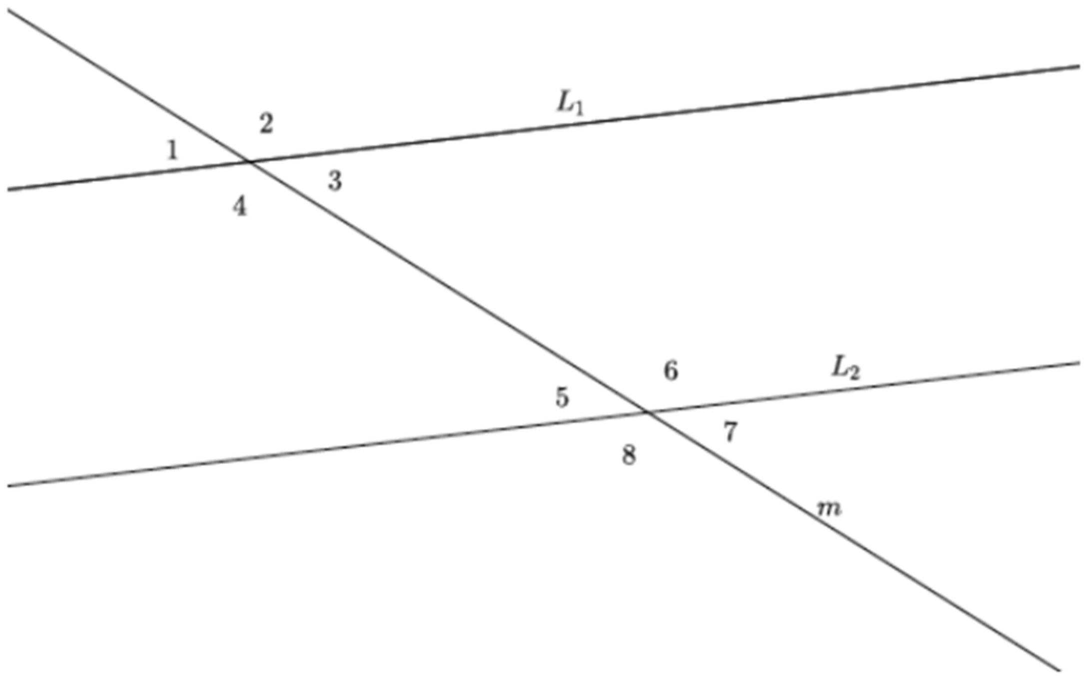
Exploratory Challenge 1

In the figure below, $L1$ is not parallel to $L2$, and m is a transversal. Use a protractor to measure angles 1–8. Which, if any, are equal? Explain why. (Use your transparency if needed.)



Exploratory Challenge 2

In the figure below, $L1 \parallel L2$, and m is a transversal. Use a protractor to measure angles 1–8. List the angles that are equal in measure.



a. What did you notice about the measures of $\angle 1$ and $\angle 5$? Why do you think this is so? (Use your transparency if needed.)	
b. What did you notice about the measures of $\angle 3$ and $\angle 7$? Why do you think this is so? (Use your transparency if needed.) Are there any other pairs of angles with this same relationship? If so, list them.	
c. What did you notice about the measures of $\angle 4$ and $\angle 6$? Why do you think this is so? (Use your transparency if needed.) Is there another pair of angles with this same relationship?	

Discussion

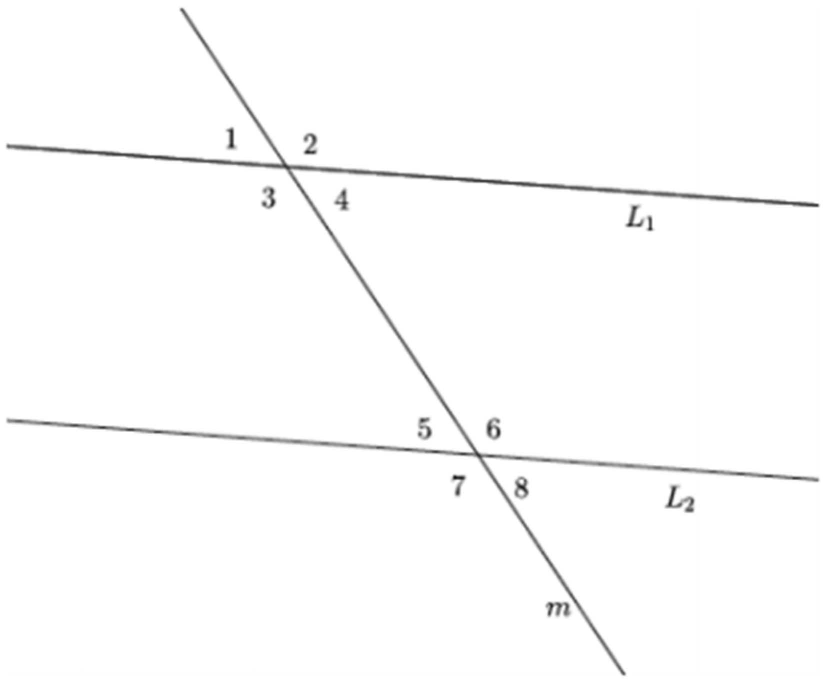
If you know that pairs of corresponding angles, alternate interior angles, and alternate exterior angles are congruent, what do you think is true about the lines?	

Theorem:	
<u>Converse</u> <u>of Theorem:</u>	

Lesson 12 Summary:

Lesson 12 Independent Practice

Use the diagram below to do Problems 1–6.



1. Identify all pairs of corresponding angles. Are the pairs of corresponding angles equal in measure? How do you know?

2. Identify all pairs of alternate interior angles. Are the pairs of alternate interior angles equal in measure? How do you know?

3. Use an informal argument to describe why ∠1 and ∠8 are equal in measure if $L1 \parallel L2$.

4. Assuming $L1 \parallel L2$ if the measure of ∠4 is 73°, what is the measure of ∠8? How do you know?

5. Assuming $L1 \parallel L2$, if the measure of $\angle 3$ is $107°$ degrees, what is the measure of $\angle 6$? How do you know?

6. Assuming $L1 \parallel L2$, if the measure of $\angle 2$ is $107°$, what is the measure of $\angle 7$? How do you know?

7. Would your answers to Problems 4–6 be the same if you had not been informed that $L1 \parallel L2$? Why, or why not?

8. Use an informal argument to describe why $\angle 1$ and $\angle 5$ are equal in measure if $L1 \parallel L2$.

9. Use an informal argument to describe why $\angle 4$ and $\angle 5$ are equal in measure if $L1 \parallel L2$.

10. Assume that $L1$ is not parallel to $L2$. Explain why $\angle 3 \neq \angle 7$.

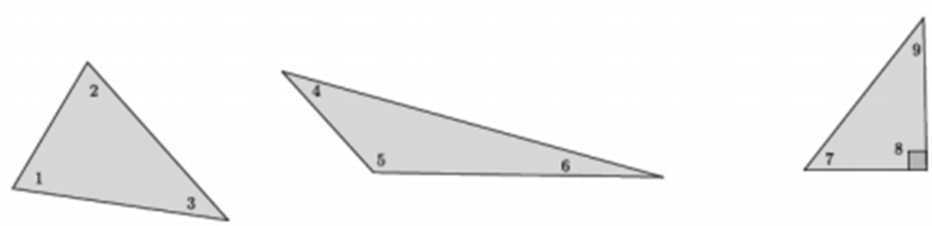
Lesson 13: Angle Sum of a Triangle

ESSENTIAL QUESTIONS:

Concept Development

The angle sum theorem for triangles states	

Concept Development

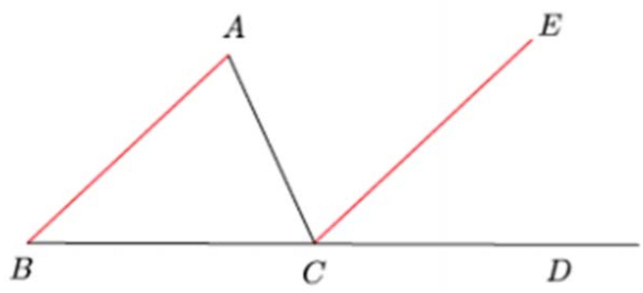


$$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 6 = \angle 7 + \angle 8 + \angle 9 = 180$$

Note that the sum of angles 7 and 9 must equal 90° because of the known right angle in the right triangle.

Exploratory Challenge 1

Let triangle ABC be given. On the ray from B to C, take a point D so that CC is between B and$D$. Through point C, draw a line parallel to AB, as shown. Extend the parallel lines AB and CE. Line AC is the transversal that intersects the parallel lines.



a. Name the three interior angles of triangle A

b. Name the straight angle.

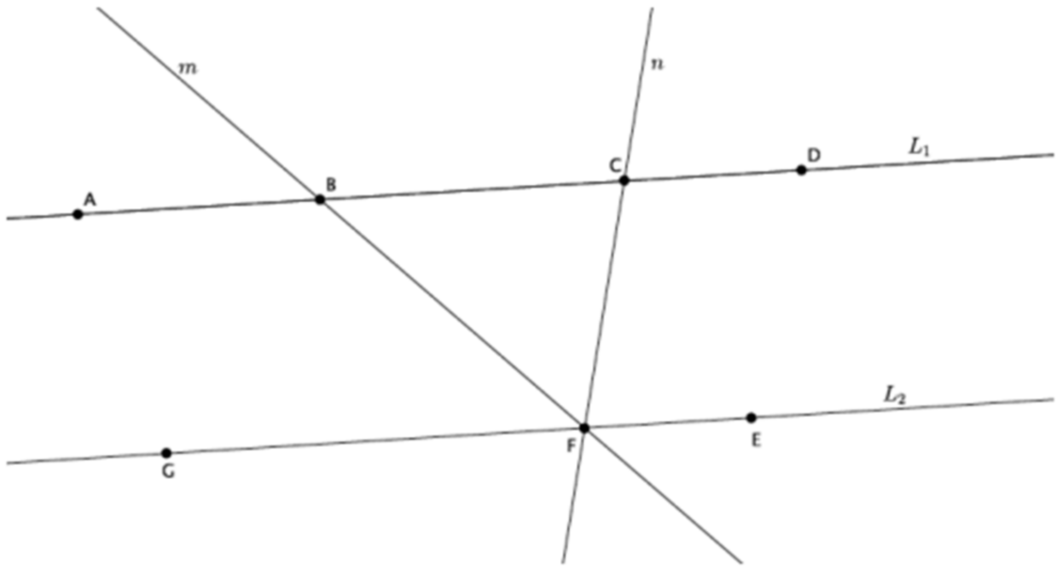
c. What kinds of angles are $\angle A$BC and $\angle E$CD? What does that mean about their measures?

d. What kinds of angles are $\angle B$AC and $\angle E$CA? What does that mean about their measures?

e. We know that $\angle B$CD = $\angle B$CA + $\angle E$CA + $\angle E$CD = 180°. Use substitution to show that the three interior angles of the triangle have a sum of 180°.

Exploratory Challenge 2

The figure below shows parallel lines $L1$ and $L2$. Let m and n be transversals that intersect $L1$ at points B and C, respectively, and $L2$ at point F, as shown. Let A be a point on $L1$ to the left of B, D be a point on $L1$ to the right of C, G be a point on $L2$ to the left of F, and E be a point on $L2$ to the right of F.



a. Name the triangle in the figure.

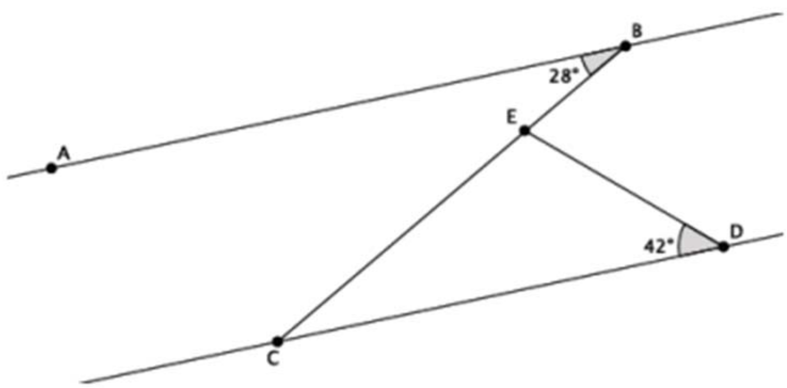
b. Name a straight angle that will be useful in proving that the sum of the interior angles of the triangle is 180°.

c. Write your proof below.

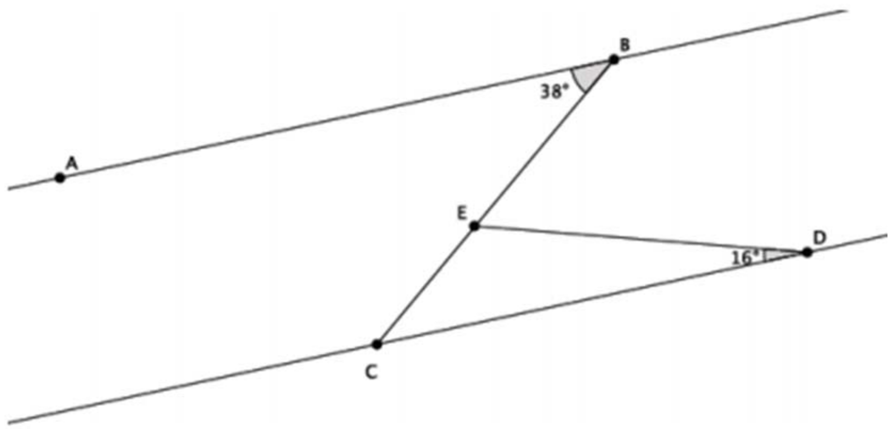
Lesson 13 Summary:

Lesson 13 Independent Practice

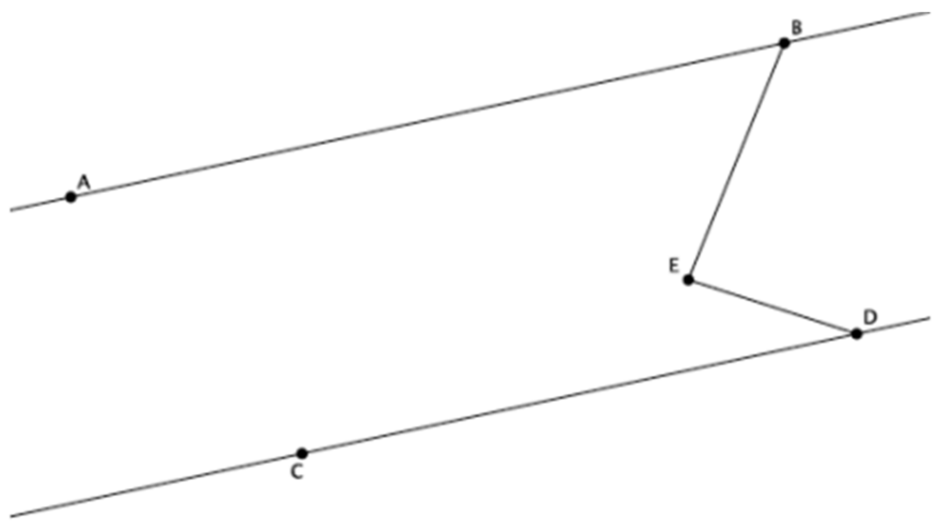
1. In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle A$BC = 28°, and the measure of angle $\angle E$DC = 42°. Find the measure of angle $\angle C$ED. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle.



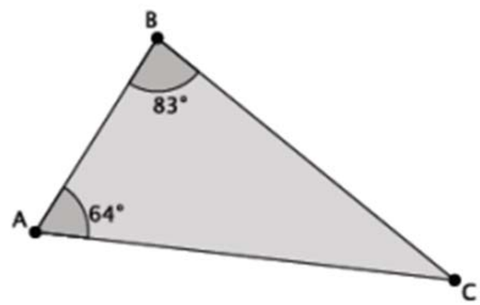
2. In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle A$BE = 38°, and the measure of angle $\angle E$DC = 16°. Find the measure of angle $\angle B$ED. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Find the measure of angle $\angle C$ED first, and then use that measure to find the measure of angle $\angle B$ED.)



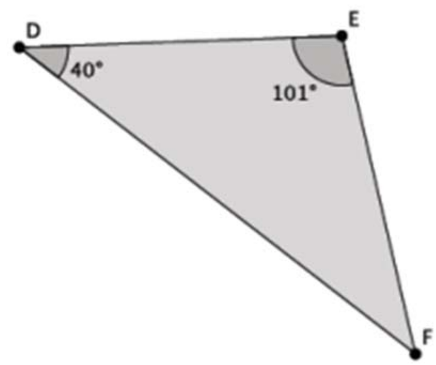
3. In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle A$BE = 56°, and the measure of angle $\angle E$DC = 22°. Find the measure of angle $\angle B$ED. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Extend the segment BE so that it intersects line CD.)



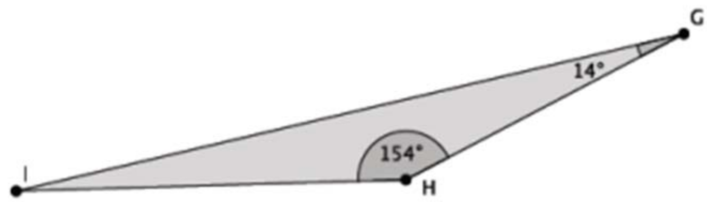
4. What is the measure of $\angle ACB$?



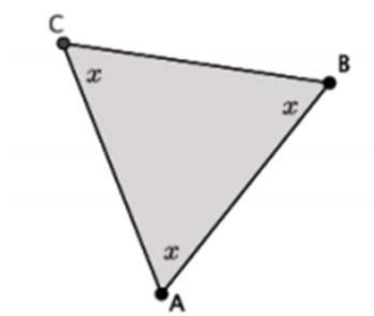
5. What is the measure of $\angle EFD$?



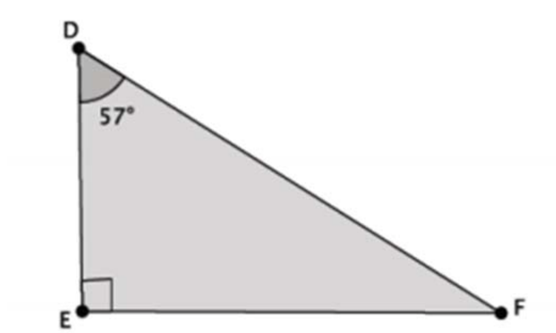
6. What is the measure of $\angle HIG$?



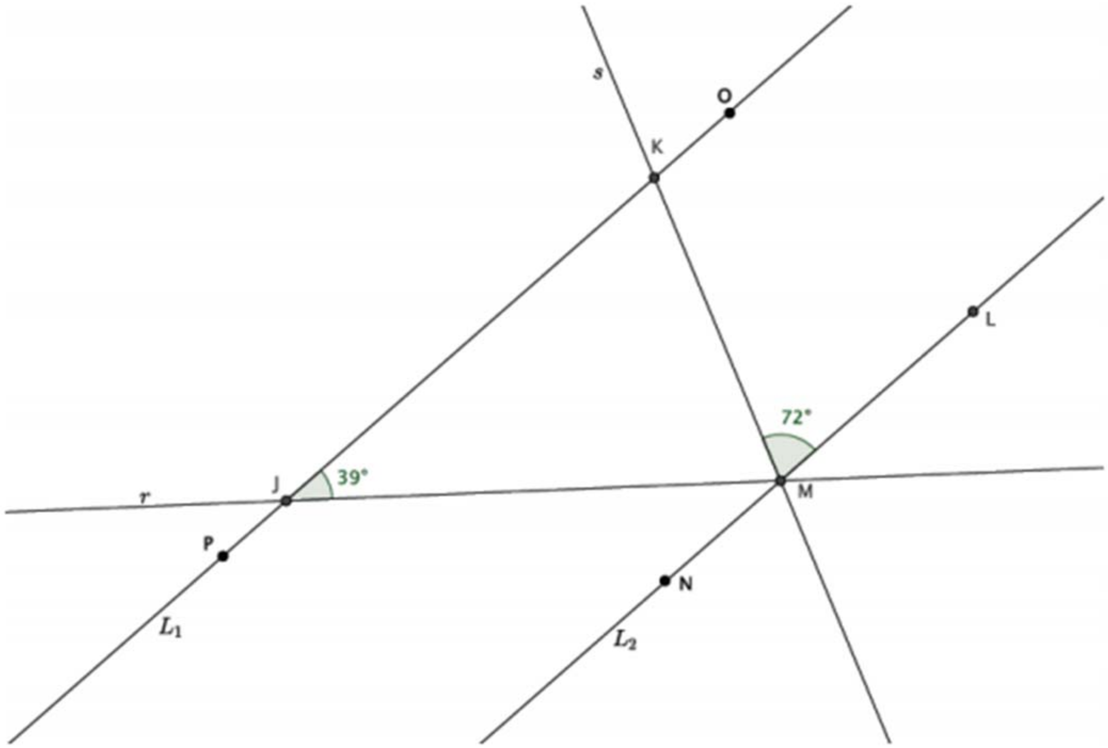
7. What is the measure of $\angle ABC$?



8. Triangle DEF is a right triangle. What is the measure of $\angle EFD$?



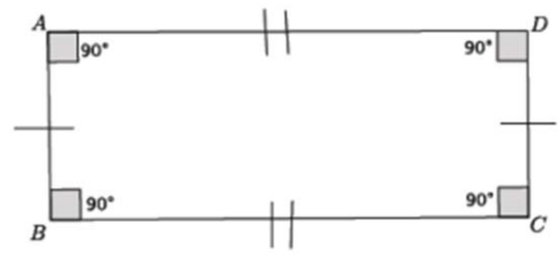
9. In the diagram below, lines $L1$ and $L2$ are parallel. Transversals r and s intersect both lines at the points shown below. Determine the measure of $\angle J$MK. Explain how you know you are correct.



Lesson 14: More on the Angles of a Triangle

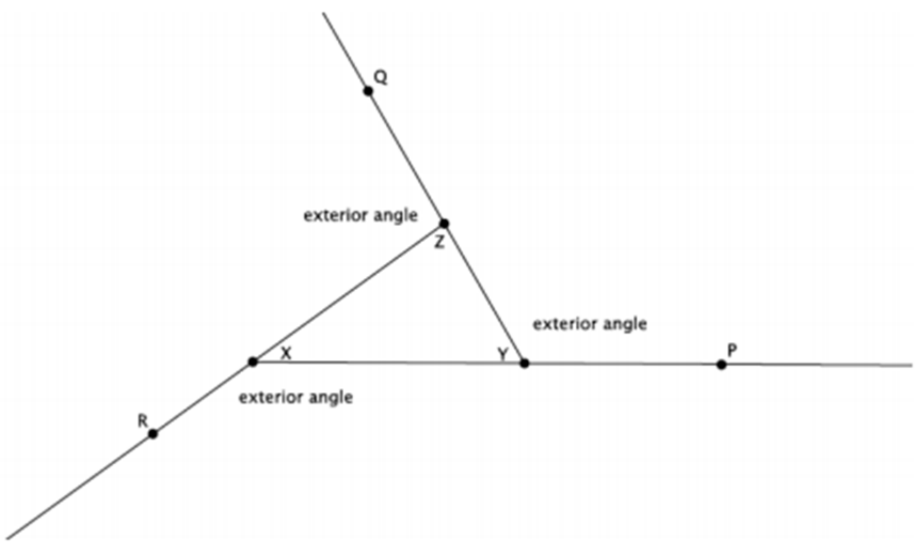
ESSENTIAL QUESTIONS:

Discussion

Start with a rectangle. What properties do rectangles have?	

On Your Own

Use the diagram below to complete Exercises 1–4.



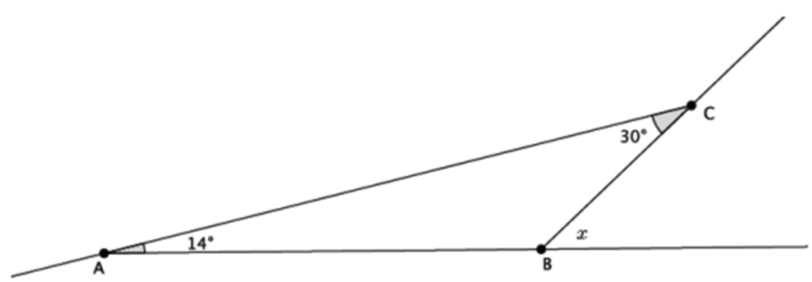
1. Name an exterior angle and the related remote interior angles.

2. Name a second exterior angle and the related remote interior angles.

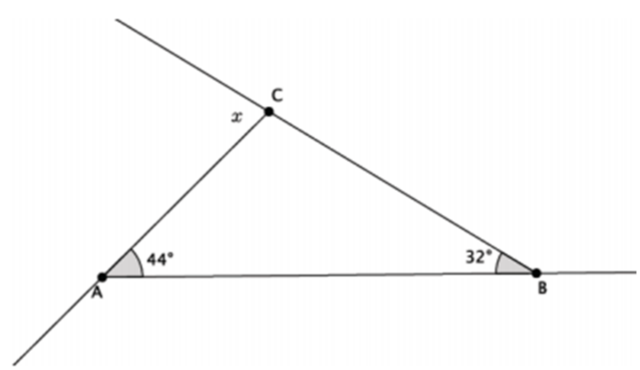
3. Name a third exterior angle and the related remote interior angles.

4. Show that the measure of an exterior angle is equal to the sum of the related remote interior angles.

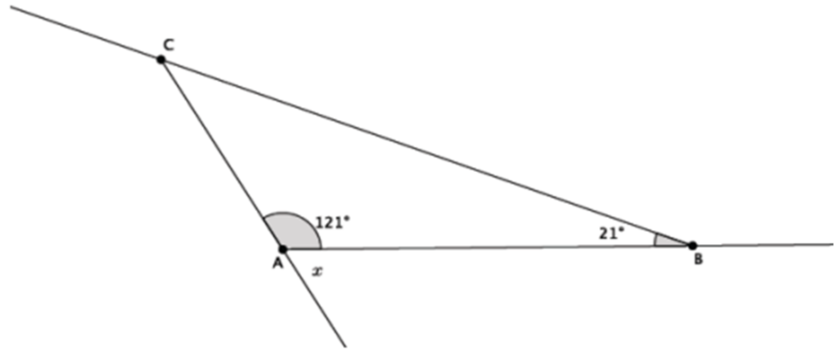
Example 1:
Find the measure of angle *x*



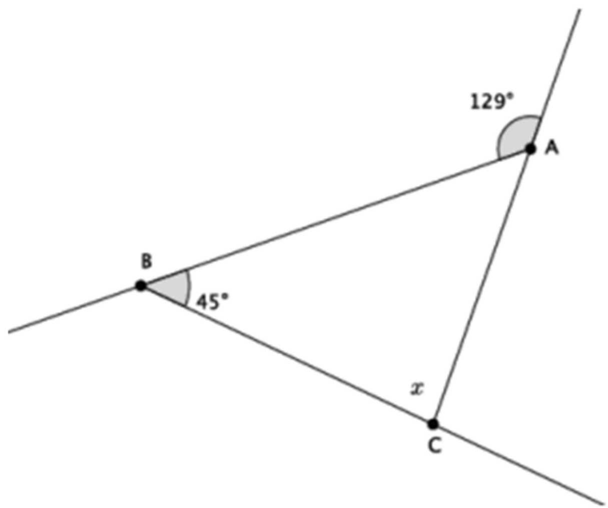
Example 2:
Find the measure of angle *x*



Example 3:
Find the measure of angle x

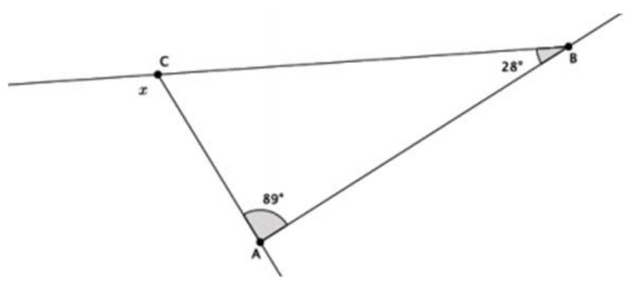


Example 4:
Find the measure of angle x

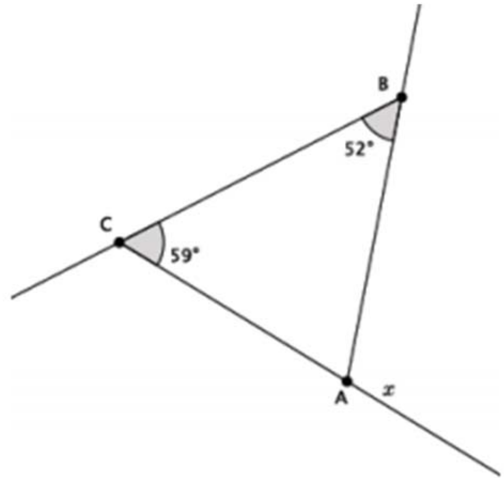


On Your Own

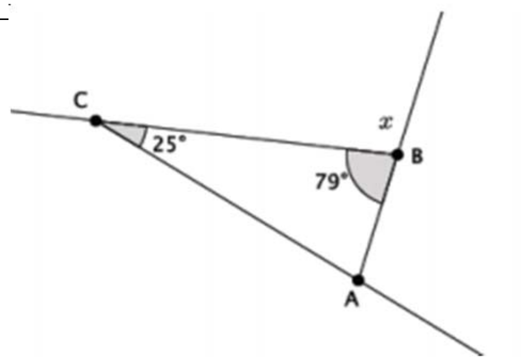
5. Find the measure of angle x.
Present an informal argument showing that your answer is correct.



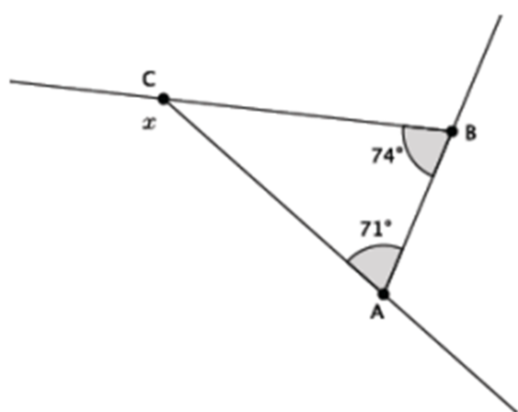
6. Find the measure of angle x. Present an informal argument showing that your answer is correct.



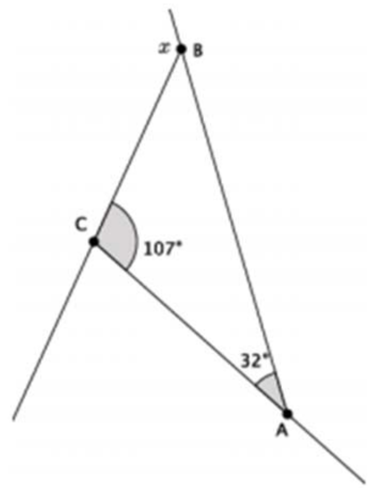
7. Find the measure of angle x. Present an informal argument showing that your answer is correct.



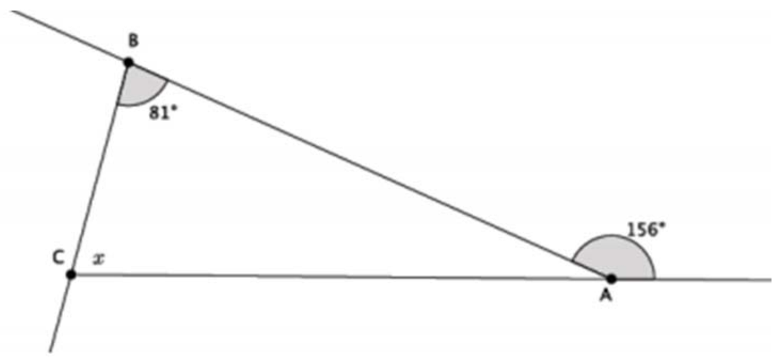
8. Find the measure of angle x. Present an informal argument showing that your answer is correct.



9. Find the measure of angle x. Present an informal argument showing that your answer is correct.



10. Find the measure of angle x. Present an informal argument showing that your answer is correct.

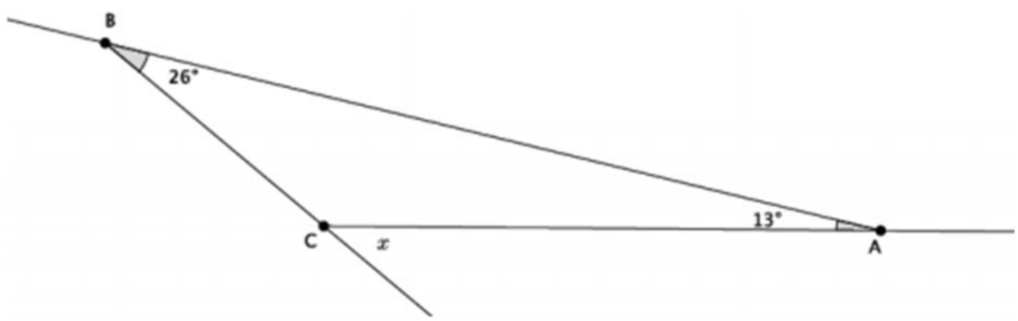


Lesson 14 Summary:

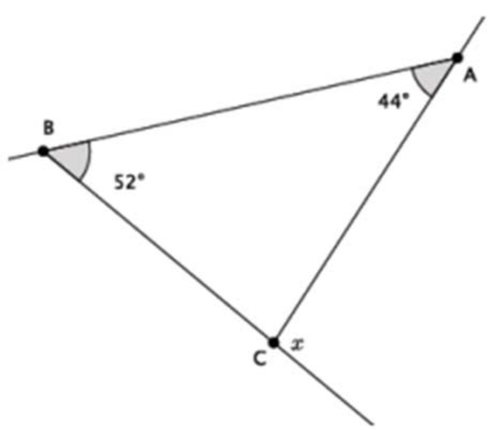
Lesson 14 Independent Practice

For each of the problems below, use the diagram to find the missing angle measure. Show your work.

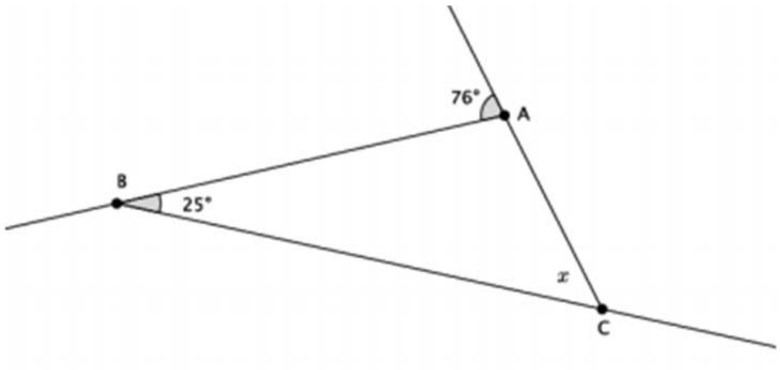
1. Find the measure of angle x. Present an informal argument showing that your answer is correct.



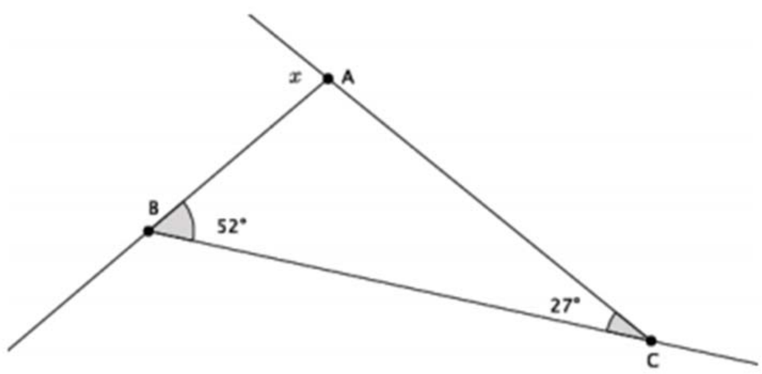
2. Find the measure of angle x.



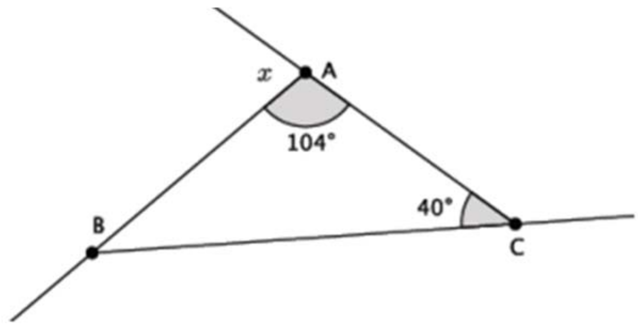
3. Find the measure of angle x. Present an informal argument showing that your answer is correct.



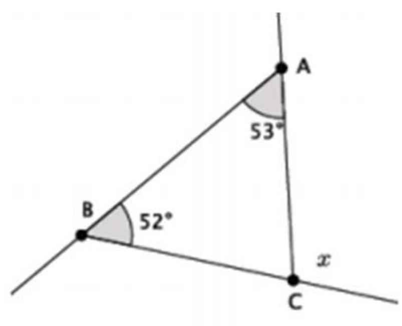
4. Find the measure of angle x.



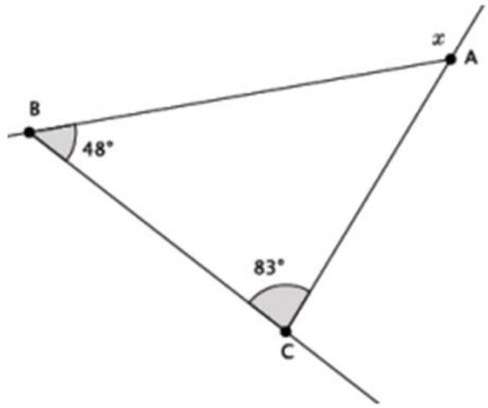
5. Find the measure of angle x.



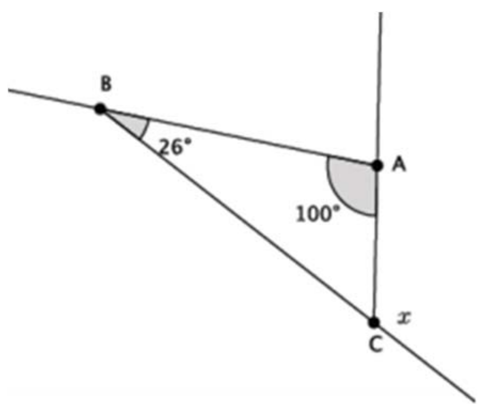
6. Find the measure of angle x.



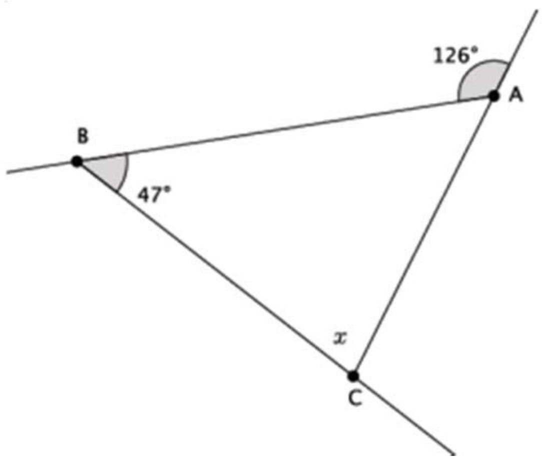
7. Find the measure of angle x.



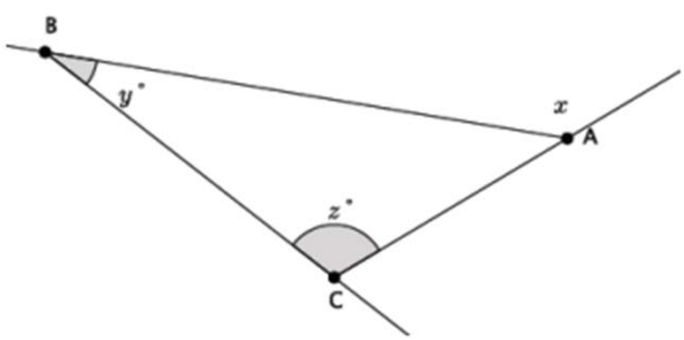
8. Find the measure of angle x.



9. Find the measure of angle x.



10. Write an equation that would allow you to find the measure of angle x. Present an informal argument showing that your answer is correct.

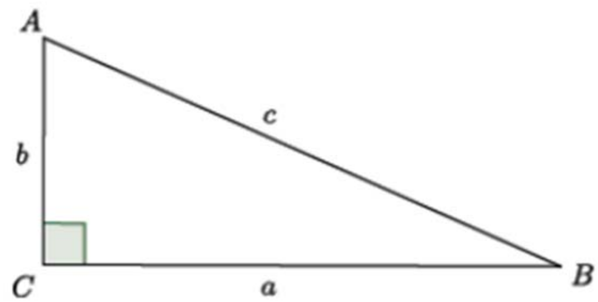


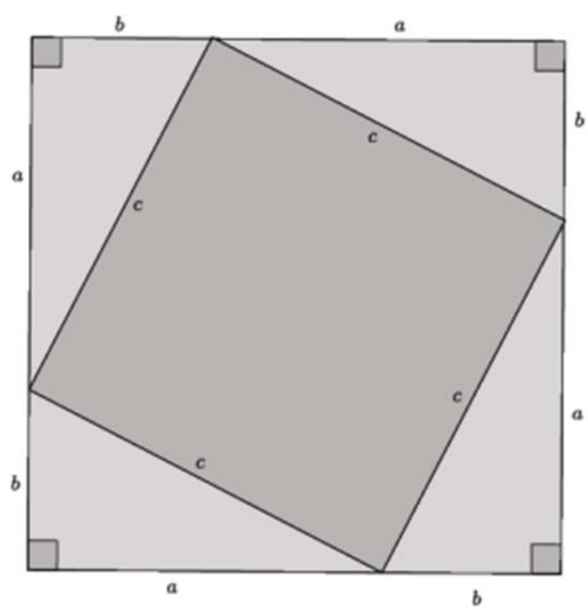
Lesson 15: Informal Proof of the Pythagorean Theorem

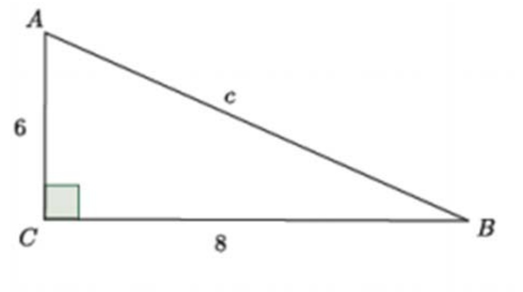
ESSENTIAL QUESTIONS:

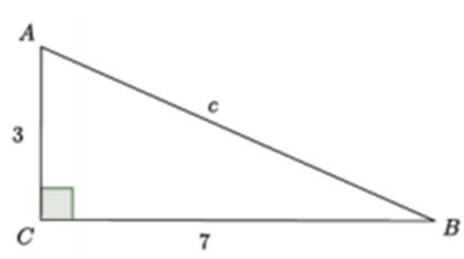
Concept Development

Pythagorean Theorem:





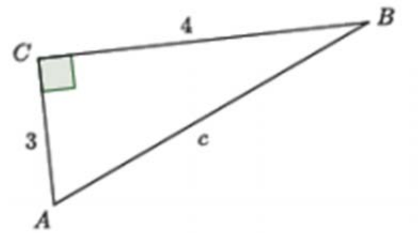
Example 1: Determine the length of the hypotenuse of the right triangle.	
The Pythagorean theorem states that for right triangles $a^2 + b^2 = c^2$, where a and b are the legs and c is the hypotenuse. Then,	

Example 2: Determine the length of the hypotenuse of the right triangle.	

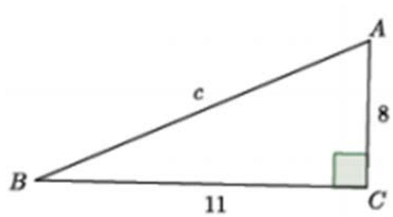
On Your Own

For each of the exercises, determine the length of the hypotenuse of the right triangle shown. Note: Figures not drawn to scale.

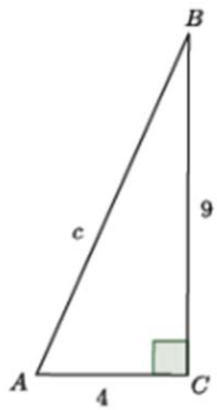
1.



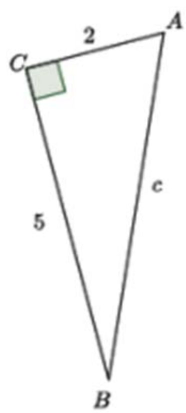
2.



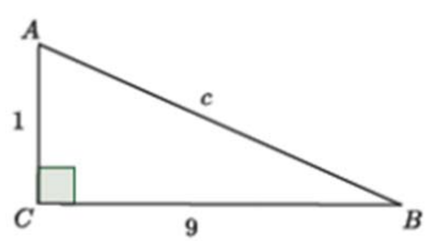
3.



4.



5.

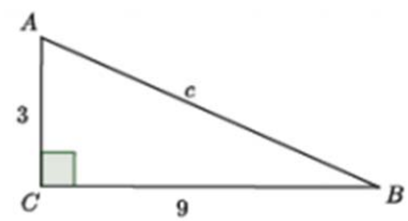


Lesson 15 Summary:

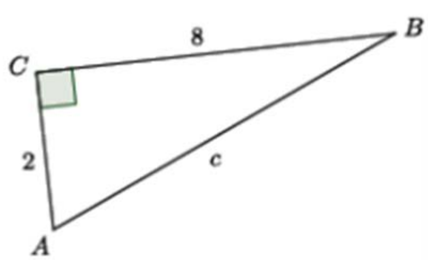
Lesson 15 Independent Practice

For each of the problems below, determine the length of the hypotenuse of the right triangle shown. Note: Figures not drawn to scale.

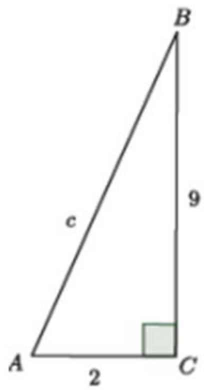
1.



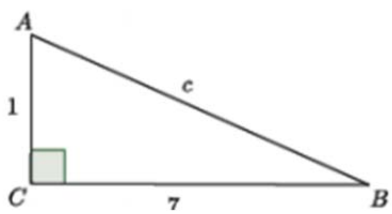
2.



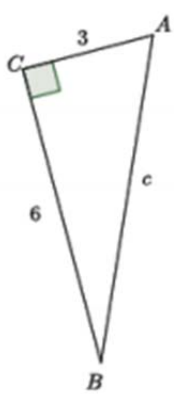
3.



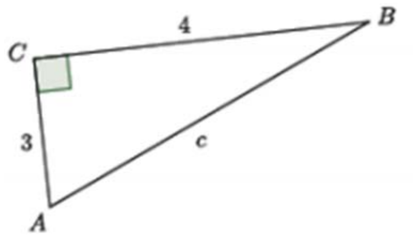
4.



5.



6.



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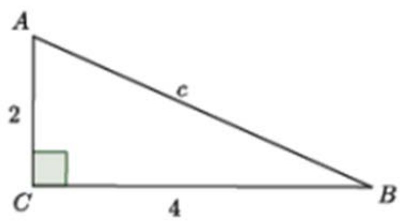

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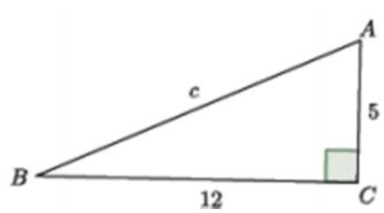

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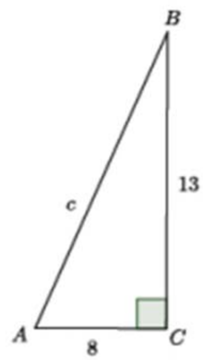

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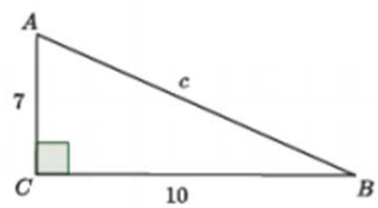

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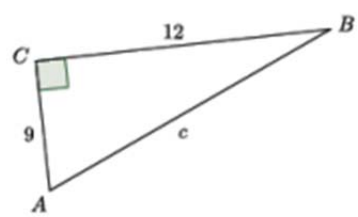

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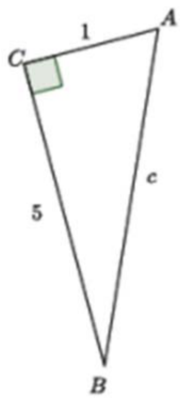
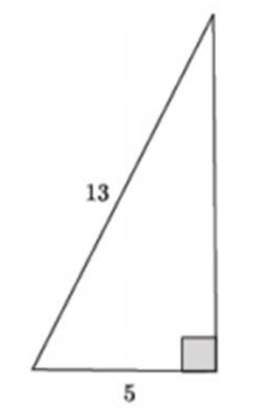


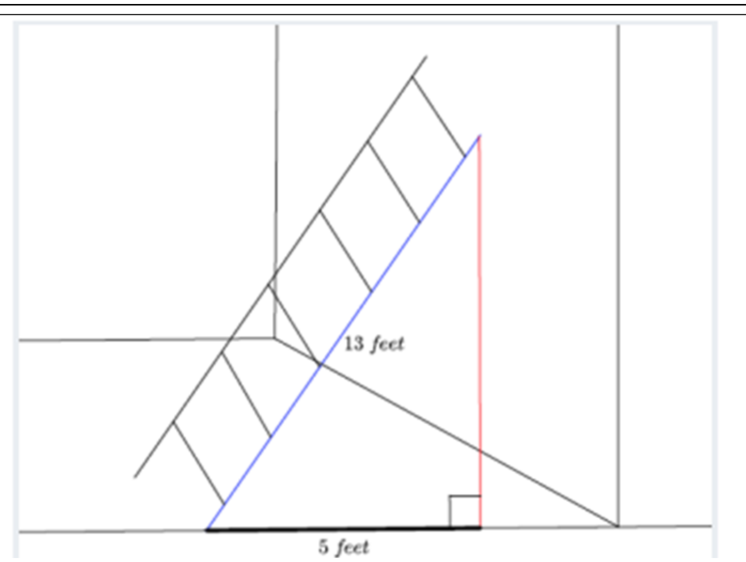
Lesson 16: Applications of the Pythagorean Theorem

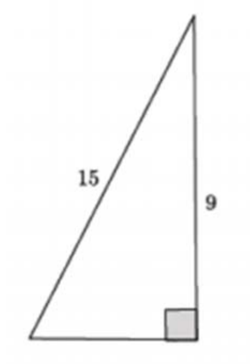
ESSENTIAL QUESTIONS:

Example 1:
Given a right triangle with a hypotenuse with length 13 units and a leg with length 5 units, as shown, determine the length of the other leg.



Example 2:
Suppose you have a ladder of length 13 feet. Suppose that to make it sturdy enough to climb, you must place the ladder exactly 5 feet from the wall of a building. You need to post a banner on the building 10 feet above the ground. Is the ladder long enough for you to reach the location you need to post the banner?

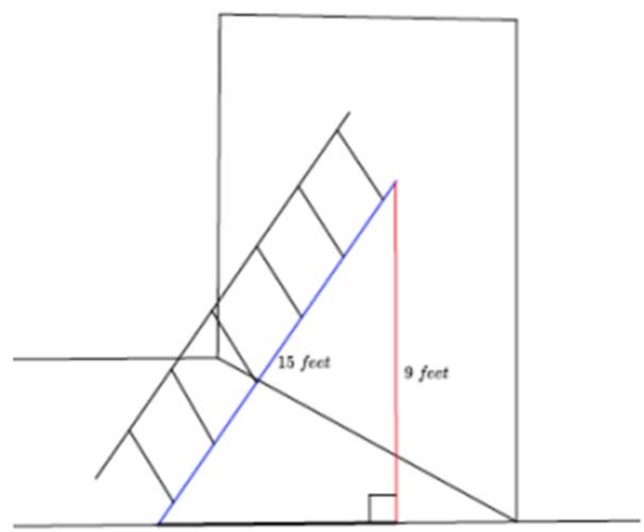


Example 3: Given a right triangle with a hypotenuse of length 15 units and a leg of length 9, what is the length of the other leg?	

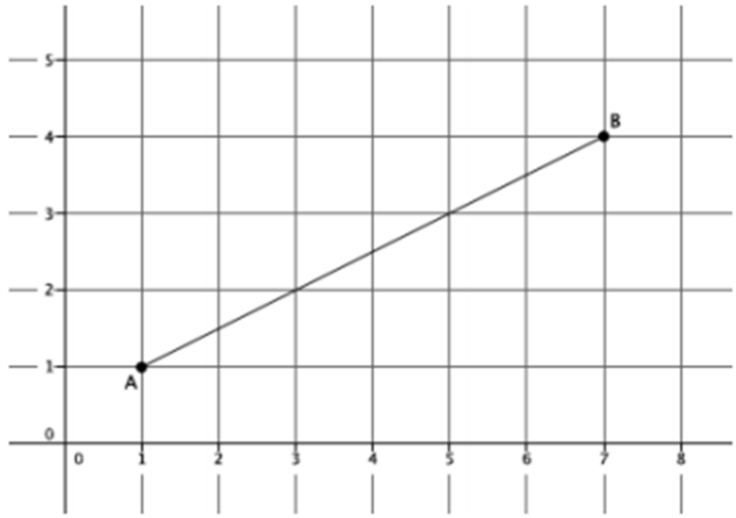
On Your Own

1. Use the Pythagorean Theorem to find the missing length of the leg in the right triangle.

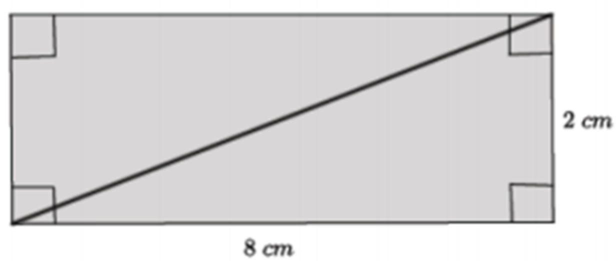
2. You have a 15-foot ladder and need to reach exactly 9 feet up the wall. How far away from the wall should you place the ladder so that you can reach your desired location?



Example 4:
We want to find the length of the segment AB on the coordinate plane, as shown.

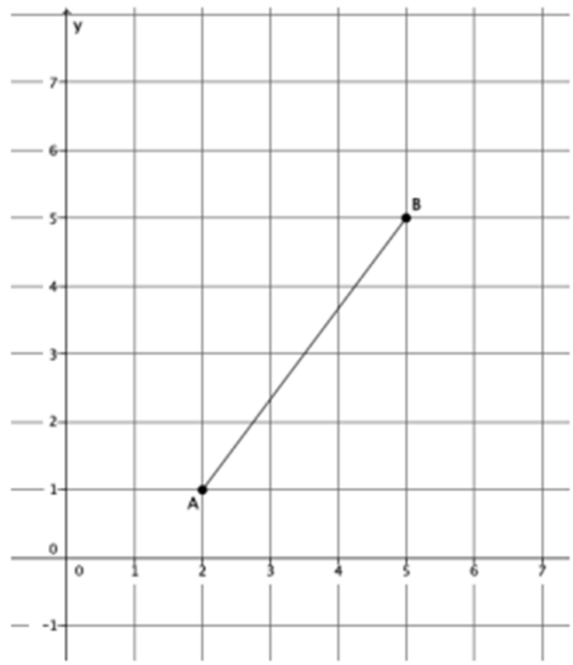


Example 5:
Given a rectangle with side lengths of 8 cm and 2 cm, as shown, what is the length of the diagonal?

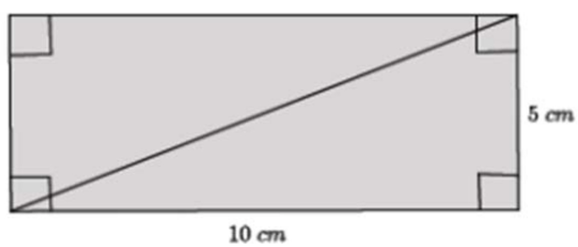


On Your Own

3. Find the length of the segment AB, if possible.

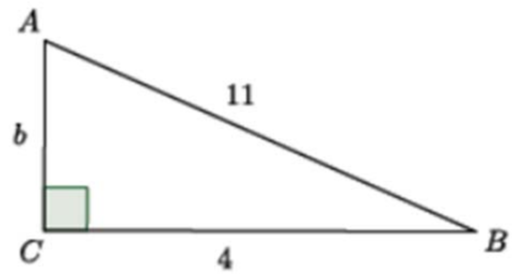


4. Given a rectangle with dimensions 5 cm and 10 cm, as shown, find the length of the diagonal, if possible.



5. A right triangle has a hypotenuse of length 13 in. and a leg with length 4 in. What is the length of the other leg?

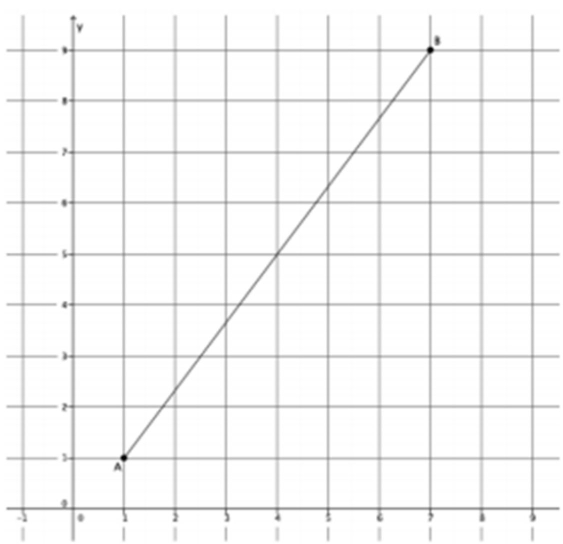
6. Find the length of b in the right triangle below, if possible.



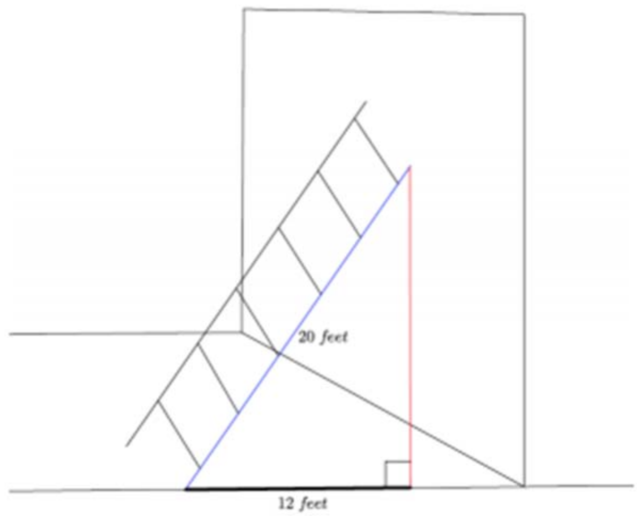
Lesson 15 Summary:

Lesson 16 Independent Practice

1. Find the length of the segment AB shown below, if possible.



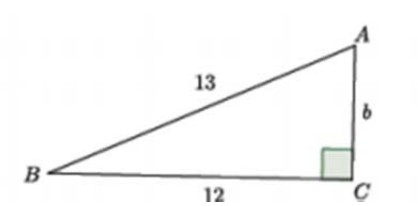
2. A 20-foot ladder is placed 12 feet from the wall, as shown. How high up the wall will the ladder reach?



3. A rectangle has dimensions 6 in. by 12 in. What is the length of the diagonal of the rectangle?

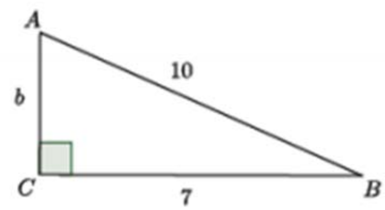
Use the Pythagorean Theorem to find the missing side lengths for the triangles shown in Problems 4–8.

4. Determine the length of the missing side, if possible.

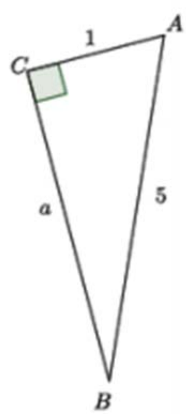


5. Determine the length of the missing side, if possible.

6. Determine the length of the missing side, if possible.



7. Determine the length of the missing side, if possible.



8. Determine the length of the missing side, if possible.