Integer Exponents and Scientific Notation

In this 13- lesson module, students expand their knowledge of operations on numbers to include integer exponents and use this knowledge to transform expressions. Students will also make conjectures about how zero and negative exponents of a number should be defined and prove the properties of integer exponents. Students will also make sense out of very large and very small numbers and will use the number line to guide them in determining the relationship between numbers.

The Laws of Exponents

For $x, y > 0$, and all integers a, b, the following holds:

$$x^a \cdot x^b = x^{a+b}$$

$$(x^b)^a = x^{ab}$$

$$(xy)^a = x^a y^a$$

In general, if x is any number and m, n are positive integers, then

$$x^m \cdot x^n = x^{m+n}$$

In general, if x is nonzero and m, n are positive inte

$$\frac{x^m}{x^n} = x^{m-n}, \text{ if } m > n.$$

For any positive number x and for any positive integer n, we define

$$x^{-n} = \frac{1}{x^n}.$$

x^{-1} is just the reciprocal, $\frac{1}{x}$, of x.

We use the definition above to prove that the following statement is true for *all integer* exponents b.

$$x^{-b} = \frac{1}{x^b}.$$

Key Words

Scientific Notation:

The scientific notation is the representation of a number as the product of a finite decimal, *d*, and a power of 10. The decimal *d* must be greater than or equal to 1 and less than 10. The exponent of the power of 10 must be an integer. For example, the scientific notation for 192.7 is 1.927×10^2. An example of a number that is not written in scientific notation is 0.234567×10^3 because 0.234567 is not greater than or equal to 1 and less than 10.

Order of Magnitude:

The order of magnitude of a *finite decimal* is the exponent in the power of 10 when that decimal is expressed in scientific notation. For example, the order of magnitude of 192.7 is 2 because when 192.7 is expressed in scientific notation as 1.927×10^2, 2 is the exponent of 10^2.

How can you help at home?

- ✓ Ask your child what they learned in school today and ask them to show you an example.
- ✓ Complete the *Sprint* activity on the next page with your child.
- ✓ Ask your child to show you why the equation below is true.

$$x^{-5} \cdot x^{-7} = x^{-12}.$$

- ✓ Ask your child to determine the value of *n* and explain to you why they think their solution is correct.

$$2^3 \cdot 4^3 = 2^3 \cdot 2^n = 2^9$$

What Came Before this Module: Students used whole number exponents to denote powers of ten, expanded the use of exponents to include bases other than ten and evaluated expressions limited to whole-number exponents. Students also learned how to apply exponents in various formulas.

What Comes After this Module: Students will learn about translations, reflections, and rotations in the plane and more importantly, how to use them to precisely define the concept of *congruence*.

Key Common Core Standards:

Work with radicals and integer exponents.

- Know and apply the properties of integer exponents to general equivalent numerical expressions.
- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
- Use, interpret and perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.

Sprinting Towards Fluency!

Sprints help develop fluency, build excitement towards mathematics, and encourage students to do their personal best! They are not necessarily a competition among classmates, but a quest to improve upon a student's previous time, ultimately helping them achieve the desired fluency when they are working with numbers as well as provide a feeling of achievement when their second sprint shows improvement.

During the Sprint activity below, your role as the parent will be the same as the role of the teacher when the class is completing this activity. You will keep track of the time as well as be an exciting and encouraging coach for your child. You will give your child the following: a copy of Sprint A and Sprint B. You can make a copy of this newsletter or use the original and fold the newsletter in half so your child only sees one Sprint at a time. You can use a stopwatch to record the time. For these modified sprints, please give your child 15 seconds to complete the 11 problems. The answers for both Sprints are provided at the bottom of the newsletter.

Have fun!

Timed Challenge:

The SPRINT!

Can you beat your personal best?

The Sprints!

Directions: Rewrite each item as an equivalent expression in exponential notation. All letters denote numbers.

Sprint A

1.	$2^2 \cdot 2^3$	
2.	$2^2 \cdot 2^4$	
3.	$2^2 \cdot 2^5$	
4.	$99^5 \cdot 99^2$	
5.	$99^6 \cdot 99^3$	
6.	$99^7 \cdot 99^4$	
7.	$r^8 \cdot r^2$	
8.	$s^8 \cdot s^2$	
9.	$x^3 \cdot x^2$	
10.	$5^4 \cdot 125$	
11.	$8 \cdot 2^9$	

Sprint B

1.	$5^2 \cdot 5^3$	
2.	$5^2 \cdot 5^4$	
3.	$5^2 \cdot 5^5$	
4.	$11^{12} \cdot 11^2$	
5.	$11^{12} \cdot 11^4$	
6.	$11^{12} \cdot 11^6$	
7.	$x^7 \cdot x^3$	
8.	$y^7 \cdot y^3$	
9.	$z^9 \cdot z^8$	
10.	$2^{11} \cdot 4$	
11.	$2^{11} \cdot 16$	

Sample Problem from the Module

Compare 2.01×10^{15} and 2.8×10^{13}. Which number is larger?

Sample Solution:

$2.01 \times 10^{15} = 2.01 \times 10^2 \times 10^{13} = 201 \times 10^{13}$

Since $201 > 2.8$, *we have* $201 \times 10^{13} > 2.8 \times 10^{13}$, *and since* $201 \times 10^{13} = 2.01 \times 10^{15}$, *we conclude* $2.01 \times 10^{15} > 2.8 \times 10^{13}$.

Answers to the Sprints.

Sprint A

1.	2^5
2.	2^6
3.	2^7
4.	99^7
5.	99^9
6.	99^{11}
7.	r^{10}
8.	s^{10}
9.	x^5
10.	5^7
11.	2^{12}

Sprint B

1.	5^5
2.	5^6
3.	5^7
4.	11^{14}
5.	11^{16}
6.	11^{18}
7.	x^{10}
8.	y^{10}
9.	z^{17}
10.	2^{13}
11.	2^{15}