## Expressions and Equations

In this 26-lesson module, students examine how the properties of addition, subtraction, multiplication, and division are applied algebraically to write expressions in equivalent forms. Then they use this and other algebraic reasoning to find solutions to equations. They also use linear equations to solve unknown angle problems. Students use the number line to understand the properties of inequality and interpret solutions within the context of problems. As students work to determine the area of circles in this module, they understand the meaning of pi and what the symbol, π, represents in terms of the value of the ratio. Students will work with expressions and equations to solve problems involving composite area in the plane, as well as volume and surface area of right prisms.

Volume and Surface Area of Right Prisms

15 in

10 in

25 in
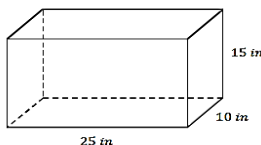
Volume of right rectangular prism:

$$3,750 \ in^3$$

The surface area of the right rectangular prism:

$$1,550 in^2$$

Key Words

An Expression in Expanded Form *(description)*: An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form.

An Expression in Standard Form *(description)*: An expression that is in expanded form where all like-terms have been collected is said to be in *standard form*.

An Expression in Factored Form *(middle school description)*: An expression that is a product of two or more expressions is said to be in *factored form*.
***Coefficient of the Term*:** The number found by multiplying just the numbers in a term together is called the coefficient of the term.

***Circle*:** Given a point C in the plane and a number $r > 0$, the *circle with center C and radius r* is the set of all points in the plane that are distance r from the point C.

***Diameter of a Circle*:** The *diameter of a circle* is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If r is the radius of a circle, then the diameter is $2r$.

***Circumference*:** The length around a circle.

***Pi*:** The number *pi*, denoted π, is the value of the ratio given by the circumference to the diameter, that is, π = (circumference)/(diameter).

***Circular Region or Disk*:** Given a point C in the plane and a number $r > 0$, the *circular region (or disk) with center C and radius r* is the set of all points in the plane whose distance from the point C is less than or equal to r. The *interior of a circle with center C and radius r* is the set of all points in the plane whose distance from the point C is less than r.

Combining Like Terms

Note: *Combining Like Terms* is actually a direct application of the distributive property:
$2x + 5x = (2 + 5)x$.

Write an expression equivalent to

$2x + 3 + 5x + 6$ by combining like terms.

Solution: $2x + 3 + 5x + 6$

$2x + 5x + 3 + 6$

$7x + 9$

This problem can be solved by using either the commutative and associative addition, or the "any order, any grouping" property.

What Came Before this Module:

Students built on their understanding of rational numbers to add, subtract, multiply, and divide signed numbers.

What Comes After this Module:

Students will deepen their understanding of ratios and proportional relationships by solving a variety of percent problems. They will also convert between fractions, decimals, and percents to further develop a conceptual understanding of percent and use algebraic expressions and equations to solve multi-step percent problems.

How can you help at home?

✓ Every day, ask your child what they learned in school and ask them to show you an example.

✓ Ask your child to explain the units used when determining volume and surface area. Why does this make sense?

✓ In the example above, Volume and Surface Area of Right Prisms, ask your child to prove the two solutions shown.

✓ Discuss with your child the meaning of π. Why is this ratio so important?

Key Common Core Standards:

Use properties of operations to generate equivalent expressions.

- Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

- Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

- Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure.
- Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Our model in the spotlight is a tape diagram, also known as a strip diagram, bar model, fraction strip, or length model. Tape diagrams are drawings that look like a segment of tape and are used to illustrate number relationships. They help students see relationships, enabling them to make sense of complex problems, strengthening their critical thinking skills, and promoting the development of problem solving abilities that will be critical in their middle school and high school years.

Problem: A sum of money was shared between George and Brian in a ratio of $3:4$. If the sum of money was $56, how much did George get?

Solution (shown below):

George

Benjamin

$56
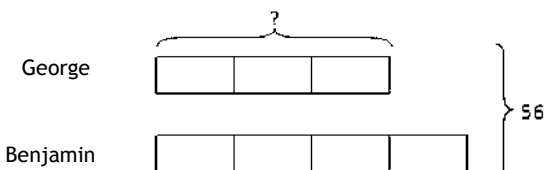
7 units = 56

1 unit = 8

3 units = 24

George received $24.

Explanation:

We know that, by the given ratio, George had three equal size parts of the money and Benjamin had 4 equal size parts of the money. This is represented by equal size units labeled with each of the boys' names. The total amount of money is $56, and there are a total of 7 equal units. Thus, the total amount of money must be shared equally among the 7 units. $56 divided by 7 units tells us that 1 unit is equal to $8 so 3 units is equal to $24.

Model in the Spotlight!

Tape Diagram

Below is another example of how to use the tape diagram (our model in the spotlight) and an algebraic approach to solve a problem in this module.

Problem: The total number of participants who went on the 6th grade field trip to the Natural Science Museum consisted of all of the 6th grade students and 7 adult chaperones. $\frac{2}{3}$ of the total participants rode a large bus and the rest rode a smaller bus. If 54 of them rode the large bus, how many students went on the field trip?

Arithmetic Approach
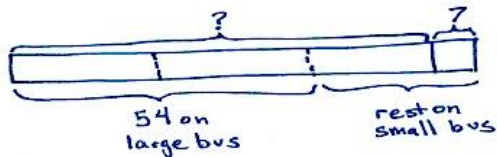
Total on both buses: $(54 \div 2) \times 3 = 81$

Total number of students: $81 - 7 = 74$;

74 students went on the field trip.

Algebraic Approach

Number of students: s

Total number of participants: $s + 7$

$$\frac{2}{3}(s + 7) = 54$$

$$\frac{3}{2}\left(\frac{2}{3}(s+7)\right) = \frac{3}{2}(54)$$

$$\left(\frac{3}{2} \cdot \frac{2}{3}\right)(s+7) = 81$$

$$1(s+7) = 81$$

$$s + 7 = 81$$

$$(s+7) - 7 = 81 - 7$$

$$s + 0 = 74$$

$$s = 74$$

If-then move: Multiply both sides by $\frac{3}{2}$ (to make a 1).

If-then move: Subtract 7 from both sides (to make a 0).

74 students went on the field trip.

In this module, students discover the most famous ratio of all, π, and begin to appreciate why it has been chosen as the symbol to represent the Grades 6-8 mathematics curriculum, *A Story of Ratios*. Below is an example of how to determine the area and circumference of a circle using π, as well as the commonly used approximation, 3.14.
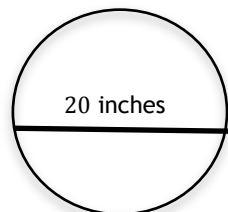
20 inches

*Taken from Lesson 17

A circle has a diameter of 20 inches.

Find the exact area and find an approximate area using $\pi \approx 3.14$.

If the diameter is 20 in., then the radius is 10 in. If $A = \pi r^2$, then $A = \pi \cdot (10 \text{ in.})^2$ or $100\pi \text{ in}^2$.
$A \approx (100 \cdot 3.14) \text{ in}^2 \approx 314 \text{ in}^2$.

What is the circumference of the circle using $\pi \approx 3.14$?

If the diameter is 20 in., then the circumference is $C = \pi d$ or $C \approx 3.14 \cdot 20 \text{ in.} \approx 62.8 \text{ in.}$