

A Story of Ratios[®]

Eureka Math[™]

Grade 6, Module 3

Student File_A

*Contains copy-ready classwork and homework
as well as templates (including cut outs)*

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Printed in the U.S.A.

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10 9 8 7 6 5 4 3 2 1

Lesson 1: Positive and Negative Numbers on the Number Line— Opposite Direction and Value

Classwork

Exploratory Challenge: Constructing the Number Line

Exercises

Complete the diagrams. Count by ones to label the number lines.



1. Plot your point on both number lines.
2. Show and explain how to find the opposite of your number on both number lines.
3. Mark the opposite on both number lines.
4. Choose a group representative to place the opposite number on the class number lines.
5. Which group had the opposite of the number on your index card?

Problem Set

1. Draw a number line, and create a scale for the number line in order to plot the points -2 , 4 , and 6 .
 - a. Graph each point and its opposite on the number line.
 - b. Explain how you found the opposite of each point.
2. Carlos uses a vertical number line to graph the points -4 , -2 , 3 , and 4 . He notices that -4 is closer to zero than -2 . He is not sure about his diagram. Use what you know about a vertical number line to determine if Carlos made a mistake or not. Support your explanation with a number line diagram.
3. Create a scale in order to graph the numbers -12 through 12 on a number line. What does each tick mark represent?
4. Choose an integer between -5 and -10 . Label it R on the number line created in Problem 3, and complete the following tasks.
 - a. What is the opposite of R ? Label it Q .
 - b. State a positive integer greater than Q . Label it T .
 - c. State a negative integer greater than R . Label it S .
 - d. State a negative integer less than R . Label it U .
 - e. State an integer between R and Q . Label it V .
5. Will the opposite of a positive number always, sometimes, or never be a positive number? Explain your reasoning.
6. Will the opposite of zero always, sometimes, or never be zero? Explain your reasoning.
7. Will the opposite of a number always, sometimes, or never be greater than the number itself? Explain your reasoning. Provide an example to support your reasoning.

Lesson 2: Real-World Positive and Negative Numbers and Zero

Classwork

Example 1: Take It to the Bank

Read Example 1 silently. In the first column, write down any words and definitions you know. In the second column, write down any words you do not know.

For Tim's 13th birthday, he received \$150 in cash from his mom. His dad took him to the bank to open a savings account. Tim gave the cash to the banker to deposit into the account. The banker credited Tim's new account \$150 and gave Tim a receipt. One week later, Tim deposited another \$25 that he had earned as allowance. The next month, Tim's dad gave him permission to withdraw \$35 to buy a new video game. Tim's dad explained that the bank would charge a \$5 fee for each withdrawal from the savings account and that each withdrawal and charge results in a debit to the account.

Words I <u>Already Know</u> :	Words I <u>Want to Know</u> :	Words I <u>Learned</u> :

In the third column, write down any new words and definitions that you learn during the discussion.

Exercises 1–2

1. Read Example 1 again. With your partner, number the events in the story problem. Write the number above each sentence to show the order of the events.

For Tim's 13th birthday, he received \$150 in cash from his mom. His dad took him to the bank to open a savings account.

Tim gave the cash to the banker to deposit into the account. The banker credited Tim's new account \$150 and gave Tim

a receipt. One week later, Tim deposited another \$25 that he had earned as allowance. The next month, Tim's dad gave

him permission to withdraw \$35 to buy a new video game. Tim's dad explained that the bank would charge a \$5 fee for

each withdrawal from the savings account and that each withdrawal and charge results in a debit to the account.

2. Write each individual description below as an integer. Model the integer on the number line using an appropriate scale.

EVENT	INTEGER	NUMBER LINE MODEL
Open a bank account with \$0.		
Make a \$150 deposit.		
Credit an account for \$150.		
Make a deposit of \$25.		
A bank makes a charge of \$5.		
Tim withdraws \$35.		

Example 2: How Hot, How Cold?

Temperature is commonly measured using one of two scales, Celsius or Fahrenheit. In the United States, the Fahrenheit system continues to be the accepted standard for nonscientific use. All other countries have adopted Celsius as the primary scale in use. The thermometer shows how both scales are related.

- a. The boiling point of water is 100°C . Where is 100 degrees Celsius located on the thermometer to the right?

- b. On a vertical number line, describe the position of the integer that represents 100°C .

- c. Write each temperature as an integer.

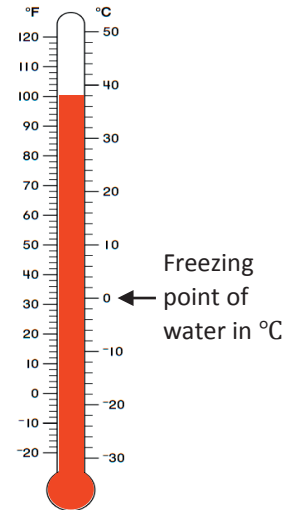
- i. The temperature shown on the thermometer in degrees Fahrenheit:

- ii. The temperature shown on the thermometer in degrees Celsius:

- iii. The freezing point of water in degrees Celsius:

- d. If someone tells you your body temperature is 98.6° , what scale is being used? How do you know?

- e. Does the temperature 0 degrees mean the same thing on both scales?



Exercises 3–5

3. Write each word under the appropriate column, “Positive Number” or “Negative Number.”

Gain Loss Deposit Credit Debit Charge Below Zero Withdraw Owe Receive

Positive Number	Negative Number

4. Write an integer to represent each of the following situations:

a. A company loses \$345,000 in 2011. _____

b. You earned \$25 for dog sitting. _____

c. Jacob owes his dad \$5. _____

d. The temperature at the sun’s surface is about 5,500°C. _____

e. The temperature outside is 4 degrees below zero. _____

f. A football player lost 10 yards when he was tackled. _____

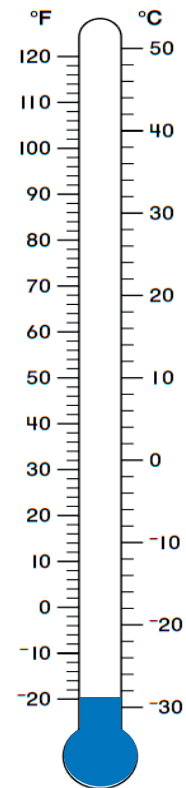
5. Describe a situation that can be modeled by the integer -15 . Explain what zero represents in the situation.

Problem Set

- Express each situation as an integer in the space provided.
 - A gain of 56 points in a game _____
 - A fee charged of \$2 _____
 - A temperature of 32 degrees below zero _____
 - A 56-yard loss in a football game _____
 - The freezing point of water in degrees Celsius _____
 - A \$12,500 deposit _____

For Problems 2–5, use the thermometer to the right.

- Each sentence is stated *incorrectly*. Rewrite the sentence to correctly describe each situation.
 - The temperature is -10 degrees Fahrenheit below zero.
 - The temperature is -22 degrees Celsius below zero.
- Mark the integer on the thermometer that corresponds to the temperature given.
 - 70°F
 - 12°C
 - 110°F
 - -4°C
- The boiling point of water is 212°F . Can this thermometer be used to record the temperature of a boiling pot of water? Explain.
- Kaylon shaded the thermometer to represent a temperature of 20 degrees below zero Celsius as shown in the diagram. Is she correct? Why or why not? If necessary, describe how you would fix Kaylon's shading.

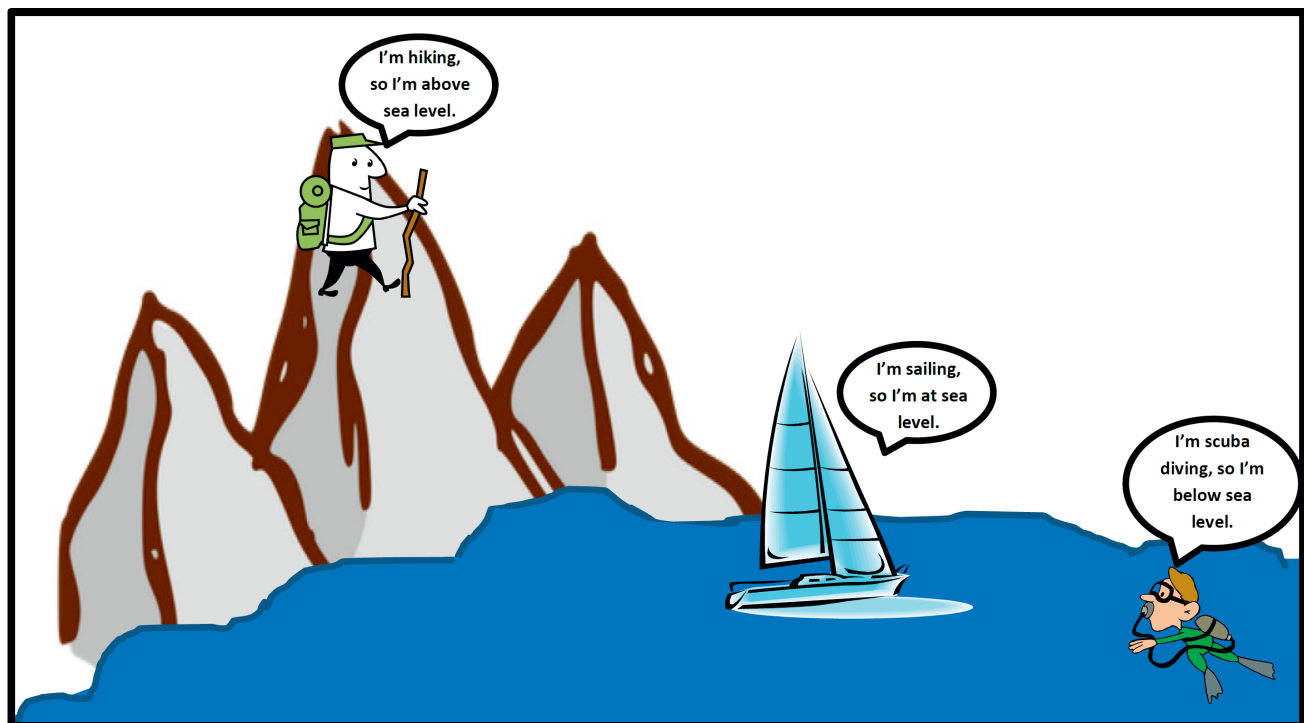


Lesson 3: Real-World Positive and Negative Numbers and Zero

Classwork

Example 1: A Look at Sea Level

The picture below shows three different people participating in activities at three different elevations. With a partner, discuss what you see. What do you think the word *elevation* means in this situation?



Exercises

Refer back to Example 1. Use the following information to answer the questions.

- The scuba diver is 30 feet below sea level.
- The sailor is at sea level.
- The hiker is 2 miles (10,560 feet) above sea level.

1. Write an integer to represent each situation.

2. Use an appropriate scale to graph each of the following situations on the number line to the right. Also, write an integer to represent both situations.

a. A hiker is 15 feet above sea level.

b. A diver is 20 feet below sea level.

3. For each statement, there are two related statements: (i) and (ii). Determine which related statement ((i) or (ii)) is expressed correctly, and circle it. Then, correct the other related statement so that both parts, (i) and (ii), are stated correctly.

a. A submarine is submerged 800 feet below sea level.

i. The depth of the submarine is -800 feet below sea level.

ii. 800 feet below sea level can be represented by the integer -800 .

b. The elevation of a coral reef with respect to sea level is given as -150 feet.

i. The coral reef is 150 feet below sea level.

ii. The depth of the coral reef is -150 feet below sea level.



Problem Set

1. Write an integer to match the following descriptions.
 - a. A debit of \$40 _____
 - b. A deposit of \$225 _____
 - c. 14,000 feet above sea level _____
 - d. A temperature increase of 40°F _____
 - e. A withdrawal of \$225 _____
 - f. 14,000 feet below sea level _____

For Problems 2–4, read each statement about a real-world situation and the two related statements in parts (a) and (b) carefully. Circle the correct way to describe each real-world situation; *possible answers include either (a), (b), or both (a) and (b)*.

2. A whale is 600 feet below the surface of the ocean.
 - a. The depth of the whale is 600 feet from the ocean’s surface.
 - b. The whale is -600 feet below the surface of the ocean.
3. The elevation of the bottom of an iceberg with respect to sea level is given as -125 feet.
 - a. The iceberg is 125 feet above sea level.
 - b. The iceberg is 125 feet below sea level.
4. Alex’s body temperature decreased by 2°F .
 - a. Alex’s body temperature dropped 2°F .
 - b. The integer -2 represents the change in Alex’s body temperature in degrees Fahrenheit.
5. A credit of \$35 and a debit of \$40 are applied to your bank account.
 - a. What is an appropriate scale to graph a credit of \$35 and a debit of \$40? Explain your reasoning.
 - b. What integer represents “a credit of \$35” if zero represents the original balance? Explain.
 - c. What integer describes “a debit of \$40” if zero represents the original balance? Explain.
 - d. Based on your scale, describe the location of both integers on the number line.
 - e. What does zero represent in this situation?

Name _____

Date _____

Exploratory Challenge Station Record Sheet

	#1	#2	#3	#4	#5
Poster # _____ Integers: _____ Number Line Scale: _____	↑	↑	↑	↑	↑
Poster # _____ Integers: _____ Number Line Scale: _____	↑	↑	↑	↑	↑
Poster # _____ Integers: _____ Number Line Scale: _____	↑	↑	↑	↑	↑
Poster # _____ Integers: _____ Number Line Scale: _____	↑	↑	↑	↑	↑
Poster # _____ Integers: _____ Number Line Scale: _____	↑	↑	↑	↑	↑
Poster # _____ Integers: _____ Number Line Scale: _____	↓	↓	↓	↓	↓

Lesson 4: The Opposite of a Number

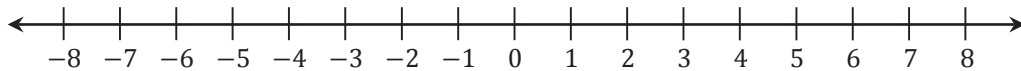
Classwork

Exercise 1: Walk the Number Line

- Each nonzero integer has an opposite, denoted $-a$; $-a$ and a are opposites if they are on opposite sides of zero and the same distance from zero on the number line.

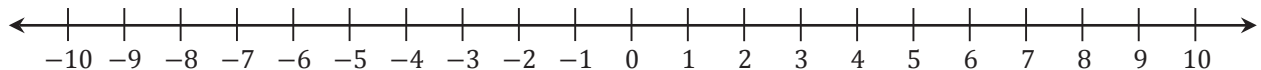
Example 1: Every Number Has an Opposite

Locate the number 8 and its opposite on the number line. Explain how they are related to zero.



Exercises 2–3

- Locate and label the opposites of the numbers on the number line.
 - 9
 - 2
 - 4
 - 7



3. Write the integer that represents the opposite of each situation. Explain what zero means in each situation.
- 100 feet above sea level
 - 32°C below zero
 - A withdrawal of \$25

Example 2: A Real-World Example

Maria decides to take a walk along Central Avenue to purchase a book at the bookstore. On her way, she passes the Furry Friends Pet Shop and goes in to look for a new leash for her dog. Furry Friends Pet Shop is seven blocks west of the bookstore. She leaves Furry Friends Pet Shop and walks toward the bookstore to look at some books. After she leaves the bookstore, she heads east for seven blocks and stops at Ray's Pet Shop to see if she can find a new leash at a better price. Which location, if any, is the farthest from Maria while she is at the bookstore?

Determine an appropriate scale, and model the situation on the number line below.



Explain your answer. What does zero represent in the situation?

Exercises 4–6

Read each situation carefully, and answer the questions.

4. On a number line, locate and label a credit of \$15 and a debit for the same amount from a bank account. What does zero represent in this situation?



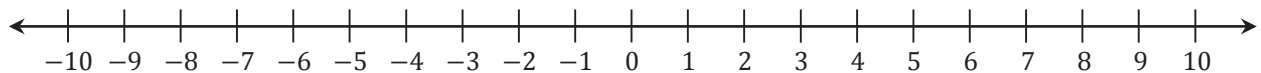
5. On a number line, locate and label 20°C below zero and 20°C above zero. What does zero represent in this situation?



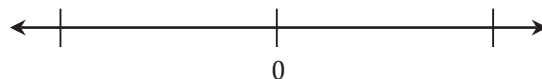
6. A proton represents a positive charge. Write an integer to represent 5 protons. An electron represents a negative charge. Write an integer to represent 3 electrons.

Problem Set

- Find the opposite of each number, and describe its location on the number line.
 - 5
 - 10
 - 3
 - 15
- Write the opposite of each number, and label the points on the number line.
 - Point *A*: the opposite of 9
 - Point *B*: the opposite of -4
 - Point *C*: the opposite of -7
 - Point *D*: the opposite of 0
 - Point *E*: the opposite of 2



- Study the first example. Write the integer that represents the opposite of each real-world situation. In words, write the meaning of the opposite.
 - An atom's positive charge of 7
 - A deposit of \$25
 - 3,500 feet below sea level
 - A rise of 45°C
 - A loss of 13 pounds
- On a number line, locate and label a credit of \$38 and a debit for the same amount from a bank account. What does zero represent in this situation?



- On a number line, locate and label 40°C below zero and 40°C above zero. What does zero represent in this situation?

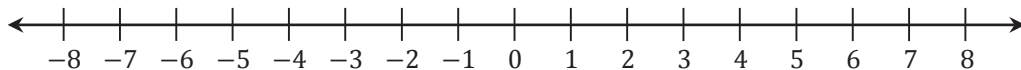


Lesson 5: The Opposite of a Number's Opposite

Classwork

Opening Exercise

- a. Locate the number -2 and its opposite on the number line below.



- b. Write an integer that represents each of the following.
- 90 feet below sea level
 - \$100 of debt
 - 2°C above zero
- c. Joe is at the ice cream shop, and his house is 10 blocks north of the shop. The park is 10 blocks south of the ice cream shop. When he is at the ice cream shop, is Joe closer to the park or his house? How could the number zero be used in this situation? Explain.

Example 1: The Opposite of an Opposite of a Number

What is the opposite of the opposite of 8? How can we illustrate this number on a number line?

- What number is 8 units to the right of 0? _____
- How can you illustrate locating the opposite of 8 on this number line?
- What is the opposite of 8? _____
- Use the same process to locate the opposite of -8 . What is the opposite of -8 ? _____



- The opposite of an opposite of a number is _____.

Exercises

Complete the table using the cards in your group.

Person	Card (a)	Opposite of Card ($-a$)	Opposite of Opposite of Card $-(-a)$

- Write the opposite of the opposite of -10 as an equation.
- In general, the opposite of the opposite of a number is the _____.
- Provide a real-world example of this rule. Show your work.

Problem Set

- Read each description carefully, and write an equation that represents the description.
 - The opposite of negative seven
 - The opposite of the opposite of twenty-five
 - The opposite of fifteen
 - The opposite of negative thirty-six
- Jose graphed the opposite of the opposite of 3 on the number line. First, he graphed point P on the number line 3 units to the right of zero. Next, he graphed the opposite of P on the number line 3 units to the left of zero and labeled it K . Finally, he graphed the opposite of K and labeled it Q .



- Is his diagram correct? Explain. If the diagram is not correct, explain his error, and correctly locate and label point Q .
 - Write the relationship between the points:

P and K _____

K and Q _____

P and Q _____
- Read each real-world description. Write the integer that represents the opposite of the opposite. Show your work to support your answer.
 - A temperature rise of 15 degrees Fahrenheit
 - A gain of 55 yards
 - A loss of 10 pounds
 - A withdrawal of \$2,000
 - Write the integer that represents the statement. Locate and label each point on the number line below.
 - The opposite of a gain of 6
 - The opposite of a deposit of \$10
 - The opposite of the opposite of 0
 - The opposite of the opposite of 4
 - The opposite of the opposite of a loss of 5



Lesson 6: Rational Numbers on the Number Line

Classwork

Opening Exercise

a. Write the decimal equivalent of each fraction.

i. $\frac{1}{2}$

ii. $\frac{4}{5}$

iii. $6\frac{7}{10}$

b. Write the fraction equivalent of each decimal.

i. 0.42

ii. 3.75

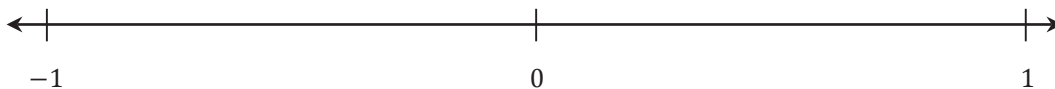
iii. 36.90

Example 1: Graphing Rational Numbers

If b is a nonzero whole number, then the unit fraction $\frac{1}{b}$ is located on the number line by dividing the segment between 0 and 1 into b segments of equal length. One of the b segments has 0 as its left end point; the right end point of this segment corresponds to the unit fraction $\frac{1}{b}$.

The fraction $\frac{a}{b}$ is located on the number line by joining a segments of length $\frac{1}{b}$ so that (1) the left end point of the first segment is 0, and (2) the right end point of each segment is the left end point of the next segment. The right end point of the last segment corresponds to the fraction $\frac{a}{b}$.

Locate and graph the number $\frac{3}{10}$ and its opposite on a number line.

**Exercise 1**

Use what you know about the point $-\frac{7}{4}$ and its opposite to graph both points on the number line below. The fraction $-\frac{7}{4}$ is located between which two consecutive integers? Explain your reasoning.



On the number line, each segment will have an equal length of _____. The fraction is located between _____ and _____.

Explanation:

Example 2: Rational Numbers and the Real World

The water level of a lake rose 1.25 feet after it rained. Answer the following questions using the number line below.

- a. Write a rational number to represent the situation.

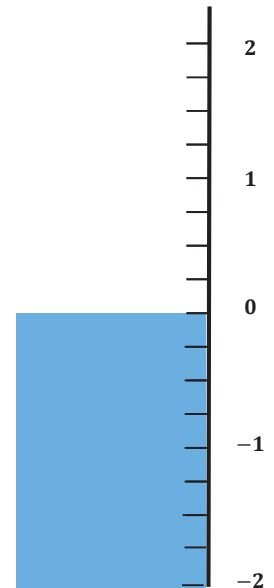
- b. What two integers is 1.25 between on a number line?

- c. Write the length of each segment on the number line as a decimal and a fraction.

- d. What will be the water level after it rained? Graph the point on the number line.

- e. After two weeks have passed, the water level of the lake is now the opposite of the water level when it rained. What will be the new water level? Graph the point on the number line. Explain how you determined your answer.

- f. State a rational number that is not an integer whose value is less than 1.25, and describe its location between two consecutive integers on the number line.



Exercise 2

Our Story Problem

Problem Set

1. In the space provided, write the opposite of each number.

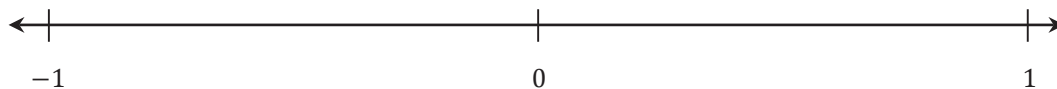
a. $\frac{10}{7}$

b. $-\frac{5}{3}$

c. 3.82

d. $-6\frac{1}{2}$

2. Choose a non-integer between 0 and 1. Label it point A and its opposite point B on the number line. Write values below the points.



- a. To draw a scale that would include both points, what could be the length of each segment?
- b. In words, create a real-world situation that could represent the number line diagram.
3. Choose a value for point P that is between -6 and -7 .
- a. What is the opposite of point P ?
- b. Use the value from part (a), and describe its location on the number line in relation to zero.
- c. Find the opposite of the opposite of point P . Show your work, and explain your reasoning.
4. Locate and label each point on the number line. Use the diagram to answer the questions.
- Jill lives one block north of the pizza shop.*
- Janette's house is $\frac{1}{3}$ block past Jill's house.*
- Jeffrey and Olivia are in the park $\frac{4}{3}$ blocks south of the pizza shop.*
- Jenny's Jazzy Jewelry Shop is located halfway between the pizza shop and the park.*
- a. Describe an appropriate scale to show all the points in this situation.
- b. What number represents the location of Jenny's Jazzy Jewelry Shop? Explain your reasoning.

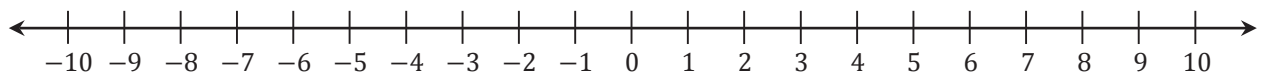


Lesson 7: Ordering Integers and Other Rational Numbers

Classwork

Exercise 1

- a. Graph the number 7 and its opposite on the number line. Graph the number 5 and its opposite on the number line.



- b. Where does 7 lie in relation to 5 on the number line?
- c. Where does the opposite of 7 lie on the number line in relation to the opposite of 5?
- d. I am thinking of two numbers. The first number lies to the right of the second number on a number line. What can you say about the location of their opposites? (If needed, refer to your number line diagram.)

Example 1

The record low temperatures for a town in Maine are -20°F for January and -19°F for February. Order the numbers from least to greatest. Explain how you arrived at the order.

Exercises 2–4

For each problem, order the rational numbers from least to greatest by first reading the problem, then drawing a number line diagram, and finally, explaining your answer.

- Jon's time for running the mile in gym class is 9.2 minutes. Jacky's time is 9.18 minutes. Who ran the mile in less time?
- Mrs. Rodriguez is a teacher at Westbury Middle School. She gives bonus points on tests for outstanding written answers and deducts points for answers that are not written correctly. She uses rational numbers to represent the points. She wrote the following on students' papers: Student A: -2 points, Student B: -2.5 points. Did Student A or Student B perform worse on the test?
- A carp is swimming approximately $8\frac{1}{4}$ feet beneath the water's surface, and a sunfish is swimming approximately $3\frac{1}{2}$ feet beneath the water's surface. Which fish is swimming farther beneath the water's surface?

Example 2

Henry, Janon, and Clark are playing a card game. The object of the game is to finish with the most points. The scores at the end of the game are Henry: -7 , Janon: 0 , and Clark: -5 . Who won the game? Who came in last place? Use a number line model, and explain how you arrived at your answer.

Exercises 5–6

For each problem, order the rational numbers from least to greatest by first reading the problem, then drawing a number line diagram, and finally, explaining your answer.

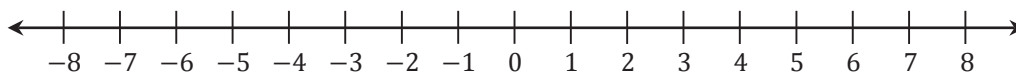
5. Henry, Janon, and Clark are playing another round of the card game. Their scores this time are as follows: Clark: -1 , Janon: -2 , and Henry: -4 . Who won? Who came in last place?

6. Represent each of the following elevations using a rational number. Then, order the numbers from least to greatest.

Cayuga Lake	122 meters above sea level
Mount Marcy	1,629 meters above sea level
New York Stock Exchange Vault	15.24 meters below sea level

Closing: What Is the Value of Each Number, and Which Is Larger?

Use your teacher's verbal clues and this number line to determine which number is larger.



Problem Set

1. In the table below, list each set of rational numbers in order from least to greatest. Then, list their opposites. Finally, list the opposites in order from least to greatest. The first example has been completed for you.

Rational Numbers	Ordered from Least to Greatest	Opposites	Opposites Ordered from Least to Greatest
-7.1, -7.25	-7.25, -7.1	7.25, 7.1	7.1, 7.25
$\frac{1}{4}, -\frac{1}{2}$			
2, -10			
0, $3\frac{1}{2}$			
-5, -5.6			
$24\frac{1}{2}, 24$			
-99.9, -100			
-0.05, -0.5			
-0.7, 0			
100.02, 100.04			

2. For each row, what pattern do you notice between the numbers in the second and fourth columns? Why is this so?

Lesson 8: Ordering Integers and Other Rational Numbers

Classwork

Example 1: Ordering Rational Numbers from Least to Greatest

Sam has \$10.00 in the bank. He owes his friend Hank \$2.25. He owes his sister \$1.75. Consider the three rational numbers related to this story of Sam's money. Write and order them from least to greatest.

Exercises 2–4

For each problem, list the rational numbers that relate to each situation. Then, order them from least to greatest, and explain how you made your determination.

- During their most recent visit to the optometrist (eye doctor), Kadijsha and her sister, Beth, had their vision tested. Kadijsha's vision in her left eye was -1.50 , and her vision in her right eye was the opposite number. Beth's vision was -1.00 in her left eye and $+0.25$ in her right eye.
- There are three pieces of mail in Ms. Thomas's mailbox: a bill from the phone company for \$38.12, a bill from the electric company for \$67.55, and a tax refund check for \$25.89. (A bill is money that you owe, and a tax refund check is money that you receive.)

4. Monica, Jack, and Destiny measured their arm lengths for an experiment in science class. They compared their arm lengths to a standard length of 22 inches. The listing below shows, in inches, how each student's arm length compares to 22 inches.

Monica: $-\frac{1}{8}$

Jack: $1\frac{3}{4}$

Destiny: $-\frac{1}{2}$

Example 2: Ordering Rational Numbers from Greatest to Least

Jason is entering college and has opened a checking account, which he will use for college expenses. His parents gave him \$200.00 to deposit into the account. Jason wrote a check for \$85.00 to pay for his calculus book and a check for \$25.34 to pay for miscellaneous school supplies. Write the three rational numbers related to the balance in Jason's checking account in order from greatest to least.

Exercises 5–6

For each problem, list the rational numbers that relate to each situation in order from greatest to least. Explain how you arrived at the order.

5. The following are the current monthly bills that Mr. McGraw must pay:

\$122.00 Cable and Internet

\$73.45 Gas and Electric

\$45.00 Cell Phone

6. $-\frac{1}{3}$, 0 , $-\frac{1}{5}$, $\frac{1}{8}$

Lesson Summary

When we order rational numbers, their opposites are in the opposite order. For example, if 7 is greater than 5, -7 is less than -5 .

Problem Set

- 1.
- a. In the table below, list each set of rational numbers from greatest to least. Then, in the appropriate column, state which number was farthest right and which number was farthest left on the number line.

Column 1	Column 2	Column 3	Column 4
Rational Numbers	Ordered from Greatest to Least	Farthest Right on the Number Line	Farthest Left on the Number Line
$-1.75, -3.25$			
$-9.7, -9$			
$\frac{4}{5}, 0$			
$-70, -70\frac{4}{5}$			
$-15, -5$			
$\frac{1}{2}, -2$			
$-99, -100, -99.3$			
$0.05, 0.5$			
$0, -\frac{3}{4}, -\frac{1}{4}$			
$-0.02, -0.04$			

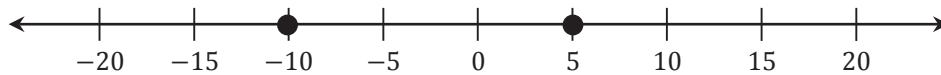
- b. For each row, describe the relationship between the number in Column 3 and its order in Column 2. Why is this?
- c. For each row, describe the relationship between the number in Column 4 and its order in Column 2. Why is this?

2. If two rational numbers, a and b , are ordered such that a is less than b , then what must be true about the order for their opposites: $-a$ and $-b$?
3. Read each statement, and then write a statement relating the *opposites* of each of the given numbers:
 - a. 7 is greater than 6.
 - b. 39.2 is greater than 30.
 - c. $-\frac{1}{5}$ is less than $\frac{1}{3}$.
4. Order the following from least to greatest: -8 , -19 , 0 , $\frac{1}{2}$, $\frac{1}{4}$.
5. Order the following from greatest to least: -12 , 12 , -19 , $1\frac{1}{2}$, 5 .

Lesson 9: Comparing Integers and Other Rational Numbers

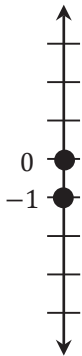
Classwork

Example 1: Interpreting Number Line Models to Compare Numbers



Exercises

1. Create a real-world situation that relates to the points shown in the number line model. Be sure to describe the relationship between the values of the two points and how it relates to their order on the number line.



For each problem, determine if you *agree* or *disagree* with the representation. Then, defend your stance by citing specific details in your writing.

2. Felicia needs to write a story problem that relates to the order in which the numbers $-6\frac{1}{2}$ and -10 are represented on a number line. She writes the following:

“During a recent football game, our team lost yards on two consecutive downs. We lost $6\frac{1}{2}$ yards on the first down. During the second down, our quarterback was sacked for an additional 10-yard loss. On the number line, I represented this situation by first locating $-6\frac{1}{2}$. I located the point by moving $6\frac{1}{2}$ units to the left of zero. Then, I graphed the second point by moving 10 units to the left of 0.”

3. Manuel looks at a number line diagram that has the points $-\frac{3}{4}$ and $-\frac{1}{2}$ graphed. He writes the following related story:

“I borrowed 50 cents from my friend, Lester. I borrowed 75 cents from my friend, Calvin. I owe Lester less than I owe Calvin.”

4. Henry located $2\frac{1}{4}$ and 2.1 on a number line. He wrote the following related story:

“In gym class, both Jerry and I ran for 20 minutes. Jerry ran $2\frac{1}{4}$ miles, and I ran 2.1 miles. I ran a farther distance.”

5. Sam looked at two points that were graphed on a vertical number line. He saw the points -2 and 1.5 . He wrote the following description:

“I am looking at a vertical number line that shows the location of two specific points. The first point is a negative number, so it is below zero. The second point is a positive number, so it is above zero. The negative number is -2 . The positive number is $\frac{1}{2}$ unit more than the negative number.”

6. Claire draws a vertical number line diagram and graphs two points: -10 and 10 . She writes the following related story:

“These two locations represent different elevations. One location is 10 feet above sea level, and one location is 10 feet below sea level. On a number line, 10 feet above sea level is represented by graphing a point at 10, and 10 feet below sea level is represented by graphing a point at -10 .”

7. Mrs. Kimble, the sixth-grade math teacher, asked the class to describe the relationship between two points on the number line, 7.45 and 7.5 , and to create a real-world scenario. Jackson writes the following story:

“Two friends, Jackie and Jennie, each brought money to the fair. Jackie brought more than Jennie. Jackie brought $\$7.45$, and Jennie brought $\$7.50$. Since 7.45 has more digits than 7.5 , it would come after 7.5 on the number line, or to the right, so it is a greater value.”

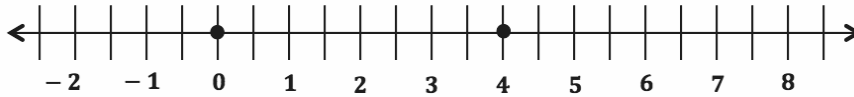
8. Justine graphs the points associated with the following $\frac{1}{2}$ numbers on a vertical number line: $-1\frac{1}{4}$, $-1\frac{1}{2}$, and 1. She then writes the following real-world scenario:

“The nurse measured the height of three sixth-grade students and compared their heights to the height of a typical sixth grader. Two of the students’ heights are below the typical height, and one is above the typical height. The point whose coordinate is 1 represents the student who has a height that is 1 inch above the typical height. Given this information, Justine determined that the student represented by the point associated with $-1\frac{1}{4}$ is the shortest of the three students.”

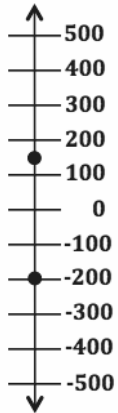
Problem Set

Write a story related to the points shown in each graph. Be sure to include a statement relating the numbers graphed on the number line to their order.

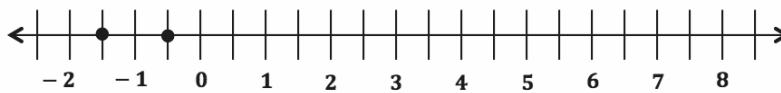
1.



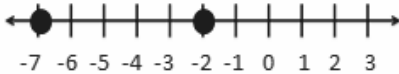
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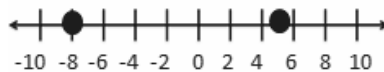
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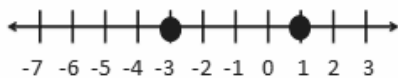
4.



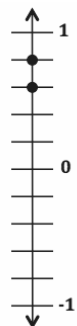
5.



6.



7.



Lesson 10: Writing and Interpreting Inequality Statements Involving Rational Numbers

Classwork

Opening Exercise

“The amount of money I have in my pocket is less than \$5 but greater than \$4.”

- One possible value for the amount of money in my pocket is _____.
- Write an inequality statement comparing the possible value of the money in my pocket to \$4.
- Write an inequality statement comparing the possible value of the money in my pocket to \$5.

Exercises 1–4

- Graph your answer from the Opening Exercise part (a) on the number line below.
- Also, graph the points associated with 4 and 5 on the number line.
- Explain in words how the location of the three numbers on the number line supports the inequality statements you wrote in the Opening Exercise parts (b) and (c).
- Write one inequality statement that shows the relationship among all three numbers.



Exercises 5–8

5. Mark's favorite football team lost yards on two back-to-back plays. They lost 3 yards on the first play. They lost 1 yard on the second play. Write an inequality statement using integers to compare the forward progress made on each play.
6. Sierra had to pay the school for two textbooks that she lost. One textbook cost \$55, and the other cost \$75. Her mother wrote two separate checks for each expense. Write two integers that represent the change to her mother's checking account balance. Then, write an inequality statement that shows the relationship between these two numbers.
7. Jason ordered the numbers -70 , -18 , and -18.5 from least to greatest by writing the following statement:
 $-18 < -18.5 < -70$.
Is this a true statement? Explain.
8. Write a real-world situation that is represented by the following inequality: $-19 < 40$. Explain the position of the numbers on a number line.

Exercise 9: A Closer Look at the Sprint

9. Look at the following two examples from the Sprint.

<input type="text"/>	<	<input type="text"/>	<	<input type="text"/>
$-\frac{1}{4}, -1, 0$				
<input type="text"/>	>	<input type="text"/>	>	<input type="text"/>
$-\frac{1}{4}, -1, 0$				

- Fill in the numbers in the correct order.
 - Explain how the position of the numbers on the number line supports the inequality statements you created.
- c. Create a new pair of greater than and less than inequality statements using three other rational numbers.

Problem Set

For each of the relationships described below, write an inequality that relates the rational numbers.

1. Seven feet below sea level is farther below sea level than $4\frac{1}{2}$ feet below sea level.
2. Sixteen degrees Celsius is warmer than zero degrees Celsius.
3. Three and one-half yards of fabric is less than five and one-half yards of fabric.
4. A loss of \$500 in the stock market is worse than a gain of \$200 in the stock market.
5. A test score of 64 is worse than a test score of 65, and a test score of 65 is worse than a test score of $67\frac{1}{2}$.
6. In December, the total snowfall was 13.2 inches, which is more than the total snowfall in October and November, which was 3.7 inches and 6.15 inches, respectively.

For each of the following, use the information given by the inequality to describe the relative position of the numbers on a horizontal number line.

7. $-0.2 < -0.1$
8. $8\frac{1}{4} > -8\frac{1}{4}$
9. $-2 < 0 < 5$
10. $-99 > -100$
11. $-7.6 < -7\frac{1}{2} < -7$

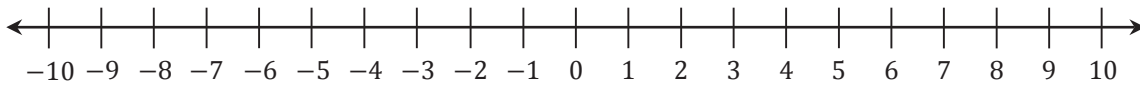
Fill in the blanks with numbers that correctly complete each of the statements.

12. Three integers between -4 and 0 $<$ $<$
13. Three rational numbers between 16 and 15 $<$ $<$
14. Three rational numbers between -1 and -2 $<$ $<$
15. Three integers between 2 and -2 $<$ $<$

Lesson 11: Absolute Value—Magnitude and Distance

Classwork

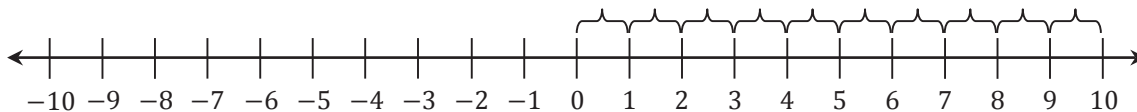
Opening Exercise



Example 1: The Absolute Value of a Number

The absolute value of ten is written as $|10|$. On the number line, count the number of units from 10 to 0. How many units is 10 from 0?

$$|10| =$$

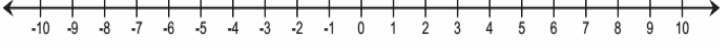
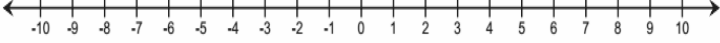
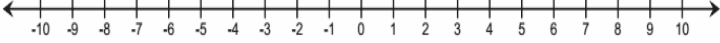


What other number has an absolute value of 10? Why?

The absolute value of a number is the distance between the number and zero on the number line.

Exercises 1–3

Complete the following chart.

	Number	Absolute Value	Number Line Diagram	Different Number with the Same Absolute Value
1.	-6			
2.	8			
3.	-1			

Example 2: Using Absolute Value to Find Magnitude

Mrs. Owens received a call from her bank because she had a checkbook balance of $-\$45$. What was the magnitude of the amount overdrawn?

The magnitude of a measurement is the absolute value of its measure.

Exercises 4–19

For each scenario below, use absolute value to determine the magnitude of each quantity.

4. Maria was sick with the flu, and her weight change as a result of it is represented by -4 pounds. How much weight did Maria lose?

11. Which of the following situations can be represented by the absolute value of 10? Check all that apply.

___ The temperature is 10 degrees below zero. Express this as an integer.

___ Determine the size of Harold's debt if he owes \$10.

___ Determine how far -10 is from zero on a number line.

___ 10 degrees is how many degrees above zero?

12. Julia used absolute value to find the distance between 0 and 6 on a number line. She then wrote a similar statement to represent the distance between 0 and -6 . Below is her work. Is it correct? Explain.

$$|6| = 6 \text{ and } |-6| = -6$$

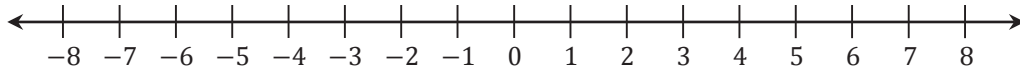
13. Use absolute value to represent the amount, in dollars, of a \$238.25 profit.

14. Judy lost 15 pounds. Use absolute value to represent the number of pounds Judy lost.

15. In math class, Carl and Angela are debating about integers and absolute value. Carl said two integers can have the same absolute value, and Angela said one integer can have two absolute values. Who is right? Defend your answer.

16. Jamie told his math teacher: "Give me any absolute value, and I can tell you two numbers that have that absolute value." Is Jamie correct? For any given absolute value, will there always be two numbers that have that absolute value?

17. Use a number line to show why a number and its opposite have the same absolute value.



18. A bank teller assisted two customers with transactions. One customer made a \$25 withdrawal from a savings account. The other customer made a \$15 deposit. Use absolute value to show the size of each transaction. Which transaction involved more money?

19. Which is farther from zero: $-7\frac{3}{4}$ or $7\frac{1}{2}$? Use absolute value to defend your answer.

Problem Set

For each of the following two quantities in Problems 1–4, which has the greater magnitude? (Use absolute value to defend your answers.)

1. 33 dollars and -52 dollars
2. -14 feet and 23 feet
3. -24.6 pounds and -24.58 pounds
4. $-11\frac{1}{4}$ degrees and 11 degrees

For Problems 5–7, answer true or false. If false, explain why.

5. The absolute value of a negative number will always be a positive number.
6. The absolute value of any number will always be a positive number.
7. Positive numbers will always have a higher absolute value than negative numbers.
8. Write a word problem whose solution is $|20| = 20$.
9. Write a word problem whose solution is $|-70| = 70$.
10. Look at the bank account transactions listed below, and determine which has the greatest impact on the account balance. Explain.
 - a. A withdrawal of \$60
 - b. A deposit of \$55
 - c. A withdrawal of \$58.50

Lesson 12: The Relationship Between Absolute Value and Order

Classwork

Opening Exercise

Record your integer values in order from least to greatest in the space below.

Example 1: Comparing Order of Integers to the Order of Their Absolute Values

Write an inequality statement relating the ordered integers from the Opening Exercise. Below each integer, write its absolute value.

Circle the absolute values that are in increasing numerical order and their corresponding integers. Describe the circled values.

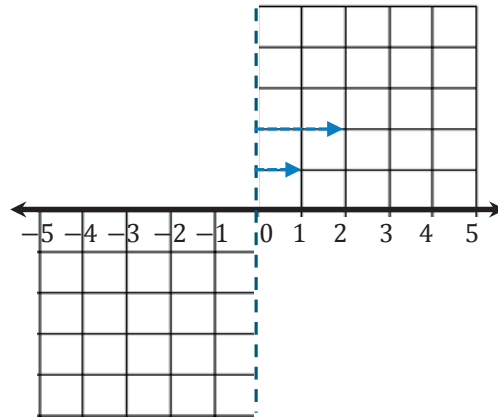
Rewrite the integers that are not circled in the space below. How do these integers differ from the ones you circled?

Rewrite the negative integers in ascending order and their absolute values in ascending order below them.

Describe how the order of the absolute values compares to the order of the negative integers.

Example 2: The Order of Negative Integers and Their Absolute Values

Draw arrows starting at the dashed line (zero) to represent each of the integers shown on the number line below. The arrows that correspond with 1 and 2 have been modeled for you.



As you approach zero from the left on the number line, the integers _____, but the absolute values of those integers _____. This means that the order of negative integers is _____ the order of their absolute values.

Exercise 1

Complete the steps below to order these numbers:

$$\left\{ 2.1, -4\frac{1}{2}, -6, 0.25, -1.5, 0, 3.9, -6.3, -4, 2\frac{3}{4}, 3.99, -9\frac{1}{4} \right\}$$

- a. Separate the set of numbers into positive rational numbers, negative rational numbers, and zero in the top cells below (order does not matter).
- b. Write the absolute values of the rational numbers (order does not matter) in the bottom cells below.

<p>Negative Rational Numbers</p>	<p>Zero</p> <p>0</p>	<p>Positive Rational Numbers</p>
<p>Absolute Values</p>		<p>Absolute Values</p>

- c. Order each subset of absolute values from least to greatest.

	0	
--	---	--

- d. Order each subset of rational numbers from least to greatest.

	0	
--	---	--

- e. Order the whole given set of rational numbers from least to greatest.

--

Exercise 2

- a. Find a set of four integers such that their order and the order of their absolute values are the same.
- b. Find a set of four integers such that their order and the order of their absolute values are opposite.
- c. Find a set of four non-integer rational numbers such that their order and the order of their absolute values are the same.
- d. Find a set of four non-integer rational numbers such that their order and the order of their absolute values are opposite.
- e. Order all of your numbers from parts (a)–(d) in the space below. This means you should be ordering 16 numbers from least to greatest.

Lesson Summary

The absolute values of positive numbers always have the same order as the positive numbers themselves. Negative numbers, however, have exactly the opposite order as their absolute values. The absolute values of numbers on the number line increase as you move away from zero in either direction.

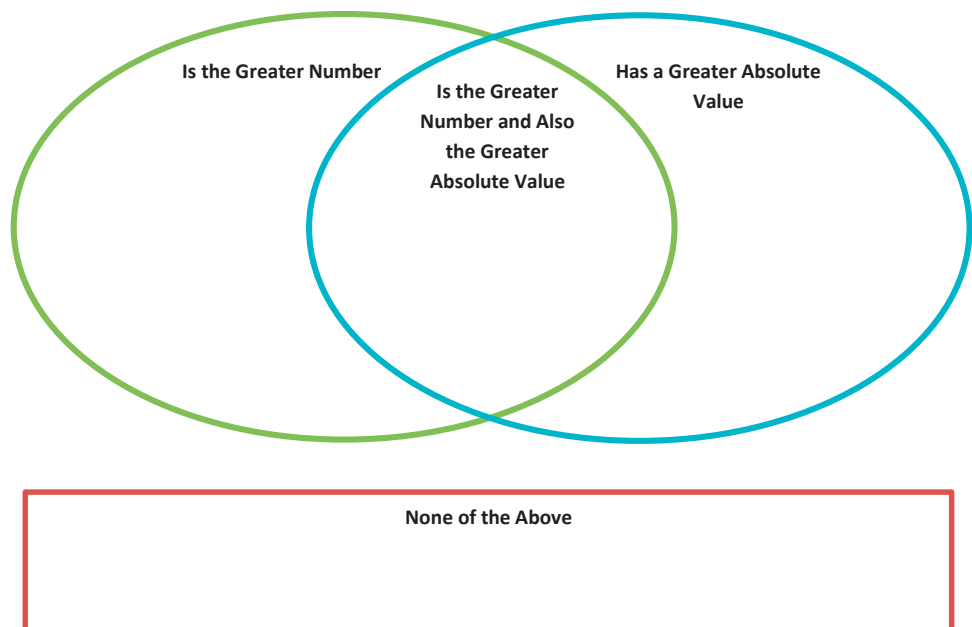
Problem Set

1. Micah and Joel each have a set of five rational numbers. Although their sets are not the same, their sets of numbers have absolute values that are the same. Show an example of what Micah and Joel could have for numbers. Give the sets in order and the absolute values in order.

Enrichment Extension: Show an example where Micah and Joel both have positive and negative numbers.

2. For each pair of rational numbers below, place each number in the Venn diagram based on how it compares to the other.

- $-4, -8$
- $4, 8$
- $7, -3$
- $-9, 2$
- $6, 1$
- $-5, 5$
- $-2, 0$



Lesson 13: Statements of Order in the Real World

Classwork

Opening Exercise

A radio disc jockey reports that the temperature outside his studio has changed 10 degrees since he came on the air this morning. Discuss with your group what listeners can conclude from this report.

Example 1: Ordering Numbers in the Real World

A \$25 credit and a \$25 charge appear similar, yet they are very different.

Describe what is similar about the two transactions.

How do the two transactions differ?

Exercises

- Scientists are studying temperatures and weather patterns in the Northern Hemisphere. They recorded temperatures (in degrees Celsius) in the table below as reported in emails from various participants. Represent each reported temperature using a rational number. Order the rational numbers from least to greatest. Explain why the rational numbers that you chose appropriately represent the given temperatures.

Temperatures as Reported	8 below zero	12	-4	13 below zero	0	2 above zero	6 below zero	-5
Temperature (°C)								

2. Jami's bank account statement shows the transactions below. Represent each transaction as a rational number describing how it changes Jami's account balance. Then, order the rational numbers from greatest to least. Explain why the rational numbers that you chose appropriately reflect the given transactions.

Listed Transactions	Debit \$12.20	Credit \$4.08	Charge \$1.50	Withdrawal \$20.00	Deposit \$5.50	Debit \$3.95	Charge \$3.00
Change to Jami's Account							

3. During the summer, Madison monitors the water level in her parents' swimming pool to make sure it is not too far above or below normal. The table below shows the numbers she recorded in July and August to represent how the water levels compare to normal. Order the rational numbers from least to greatest. Explain why the rational numbers that you chose appropriately reflect the given water levels.

Madison's Readings	$\frac{1}{2}$ inch above normal	$\frac{1}{4}$ inch above normal	$\frac{1}{2}$ inch below normal	$\frac{1}{8}$ inch above normal	$1\frac{1}{4}$ inches below normal	$\frac{3}{8}$ inch below normal	$\frac{3}{4}$ inch below normal
Compared to Normal							

4. Changes in the weather can be predicted by changes in the barometric pressure. Over several weeks, Stephanie recorded changes in barometric pressure seen on her barometer to compare to local weather forecasts. Her observations are recorded in the table below. Use rational numbers to record the indicated changes in the pressure in the second row of the table. Order the rational numbers from least to greatest. Explain why the rational numbers that you chose appropriately represent the given pressure changes.

Barometric Pressure Change (Inches of Mercury)	Rise 0.04	Fall 0.21	Rise 0.2	Fall 0.03	Rise 0.1	Fall 0.09	Fall 0.14
Barometric Pressure Change (Inches of Mercury)							

Example 2: Using Absolute Value to Solve Real-World Problems

The captain of a fishing vessel is standing on the deck at 23 feet above sea level. He holds a rope tied to his fishing net that is below him underwater at a depth of 38 feet.

Draw a diagram using a number line, and then use absolute value to compare the lengths of rope in and out of the water.

Example 3: Making Sense of Absolute Value and Statements of Inequality

A recent television commercial asked viewers, “Do you have over \$10,000 in credit card debt?”

What types of numbers are associated with the word *debt*, and why? Write a number that represents the value from the television commercial.

Give one example of “over \$10,000 in credit card debt.” Then, write a rational number that represents your example.

How do the debts compare, and how do the rational numbers that describe them compare? Explain.

Lesson Summary

When comparing values in real-world situations, descriptive words help you to determine if the number represents a positive or negative number. Making this distinction is critical when solving problems in the real world. Also critical is to understand how an inequality statement about an absolute value compares to an inequality statement about the number itself.

Problem Set

- Negative air pressure created by an air pump makes a vacuum cleaner able to collect air and dirt into a bag or other container. Below are several readings from a pressure gauge. Write rational numbers to represent each of the readings, and then order the rational numbers from least to greatest.

Gauge Readings (pounds per square inch)	25 psi pressure	13 psi vacuum	6.3 psi vacuum	7.8 psi vacuum	1.9 psi vacuum	2 psi pressure	7.8 psi pressure
Pressure Readings (pounds per square inch)							

- The fuel gauge in Nic's car says that he has 26 miles to go until his tank is empty. He passed a fuel station 19 miles ago, and a sign says there is a town only 8 miles ahead. If he takes a chance and drives ahead to the town and there isn't a fuel station there, does he have enough fuel to go back to the last station? Include a diagram along a number line, and use absolute value to find your answer.

Lesson 14: Ordered Pairs

Classwork

Example 1: The *Order* in Ordered Pairs

The first number of an ordered pair is called the _____.

The second number of an ordered pair is called the _____.

Example 2: Using Ordered Pairs to Name Locations

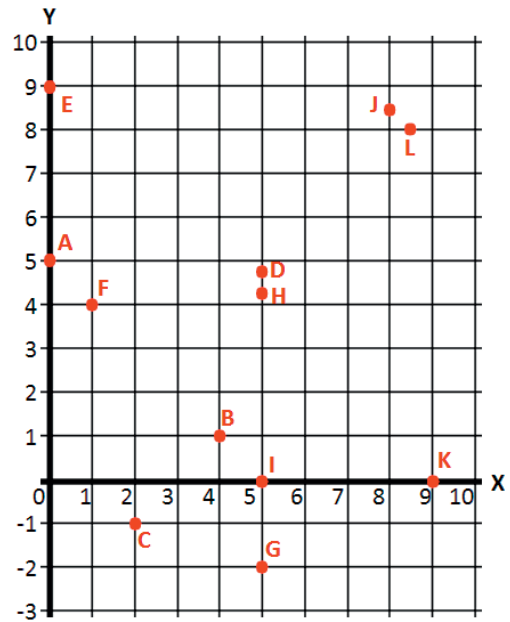
Describe how the ordered pair is being used in your scenario. Indicate what defines the first coordinate and what defines the second coordinate in your scenario.

Exercises

The first coordinates of the ordered pairs represent the numbers on the line labeled x , and the second coordinates represent the numbers on the line labeled y .

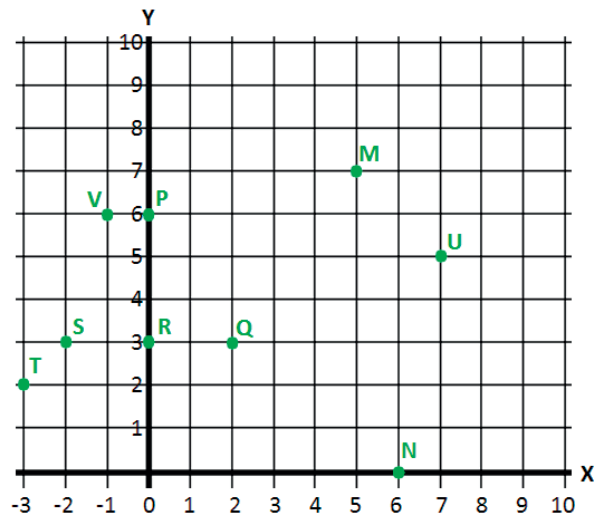
1. Name the letter from the grid below that corresponds with each ordered pair of numbers below.

- a. $(1, 4)$
- b. $(0, 5)$
- c. $(4, 1)$
- d. $(8.5, 8)$
- e. $(5, -2)$
- f. $(5, 4.2)$
- g. $(2, -1)$
- h. $(0, 9)$



2. List the ordered pair of numbers that corresponds with each letter from the grid below.

- a. Point M
- b. Point S
- c. Point N
- d. Point T
- e. Point P
- f. Point U
- g. Point Q
- h. Point V
- i. Point R



Lesson Summary

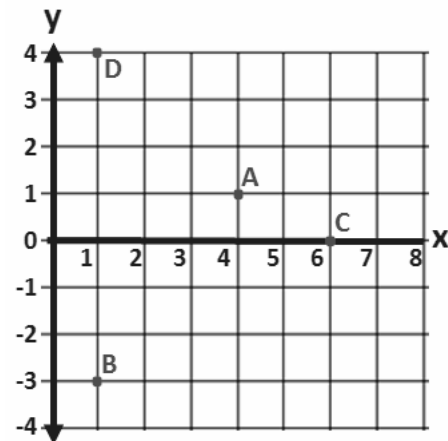
- The order of numbers in an ordered pair is important because the ordered pair should describe one location in the coordinate plane.
- The first number (called the *first coordinate*) describes a location using the horizontal direction.
- The second number (called the *second coordinate*) describes a location using the vertical direction.

Problem Set

1. Use the set of ordered pairs below to answer each question.

$$\{(4, 20), (8, 4), (2, 3), (15, 3), (6, 15), (6, 30), (1, 5), (6, 18), (0, 3)\}$$

- a. Write the ordered pair(s) whose first and second coordinate have a greatest common factor of 3.
 - b. Write the ordered pair(s) whose first coordinate is a factor of its second coordinate.
 - c. Write the ordered pair(s) whose second coordinate is a prime number.
2. Write ordered pairs that represent the location of points A , B , C , and D , where the first coordinate represents the horizontal direction, and the second coordinate represents the vertical direction.



Extension:

3. Write ordered pairs of integers that satisfy the criteria in each part below. Remember that the origin is the point whose coordinates are $(0, 0)$. When possible, give ordered pairs such that (i) both coordinates are positive, (ii) both coordinates are negative, and (iii) the coordinates have opposite signs in either order.
- a. These points' vertical distance from the origin is twice their horizontal distance.
 - b. These points' horizontal distance from the origin is two units more than the vertical distance.
 - c. These points' horizontal and vertical distances from the origin are equal, but only one coordinate is positive.

Lesson 15: Locating Ordered Pairs on the Coordinate Plane

Classwork

Example 1: Extending the Axes Beyond Zero

The point below represents zero on the number line. Draw a number line to the right starting at zero. Then, follow directions as provided by the teacher.



Example 2: Components of the Coordinate Plane

All points on the coordinate plane are described with reference to the origin. What is the origin, and what are its coordinates?

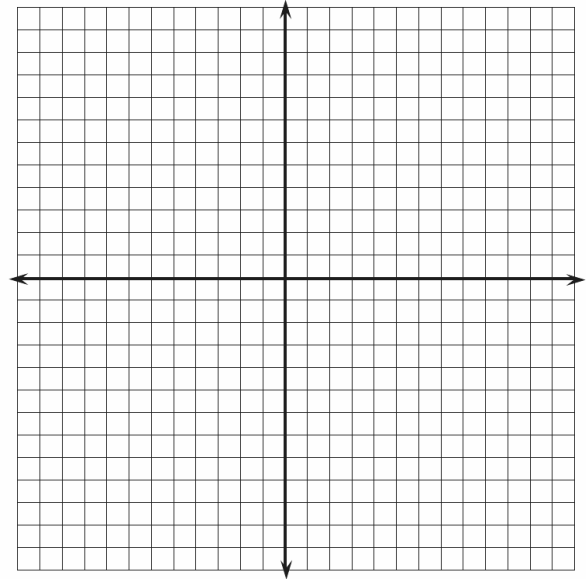
To describe locations of points in the coordinate plane, we use _____ of numbers.

Order is important, so on the coordinate plane, we use the form (_____). The first coordinate represents the point's location from zero on the _____-axis, and the second coordinate represents the point's location from zero on the _____-axis.

Exercises 1–3

1. Use the coordinate plane below to answer parts (a)–(c).

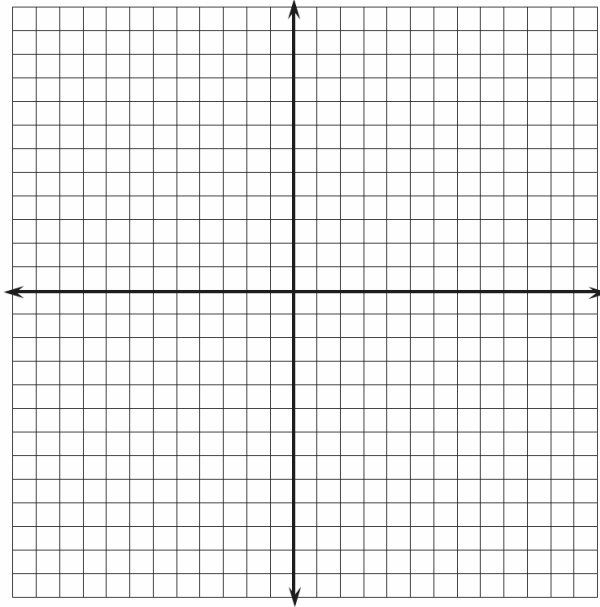
- Graph at least five points on the x -axis, and label their coordinates.
- What do the coordinates of your points have in common?
- What must be true about any point that lies on the x -axis? Explain.



2. Use the coordinate plane to answer parts (a)–(c).

- Graph at least five points on the y -axis, and label their coordinates.
- What do the coordinates of your points have in common?
- What must be true about any point that lies on the y -axis? Explain.

3. If the origin is the only point with 0 for both coordinates, what must be true about the origin?

Example 3: Quadrants of the Coordinate Plane**Exercises 4–6**

4. Locate and label each point described by the ordered pairs below. Indicate which of the quadrants the points lie in.

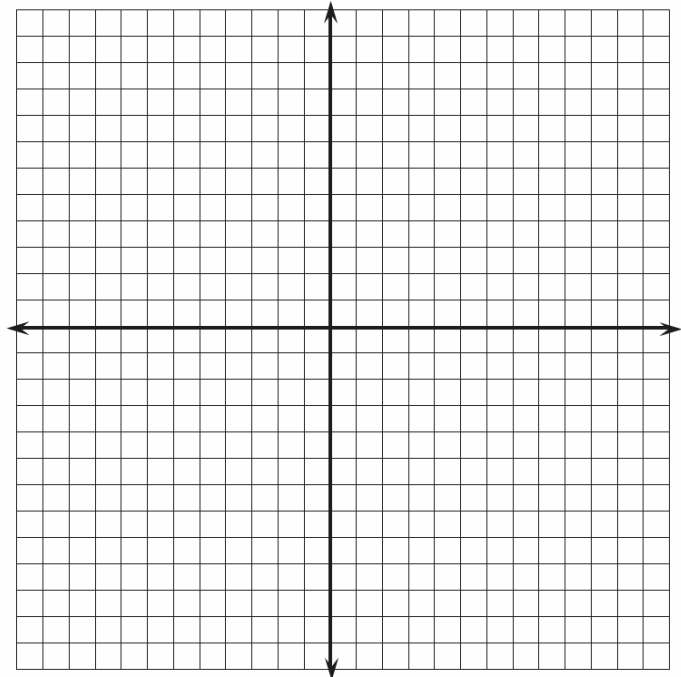
a. $(7, 2)$

b. $(3, -4)$

c. $(1, -5)$

d. $(-3, 8)$

e. $(-2, -1)$



5. Write the coordinates of at least one other point in each of the four quadrants.
- Quadrant I

 - Quadrant II

 - Quadrant III

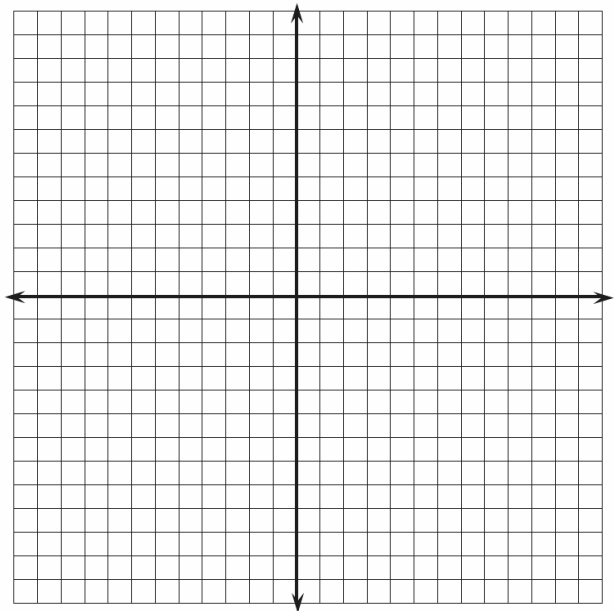
 - Quadrant IV
6. Do you see any similarities in the points within each quadrant? Explain your reasoning.

Lesson Summary

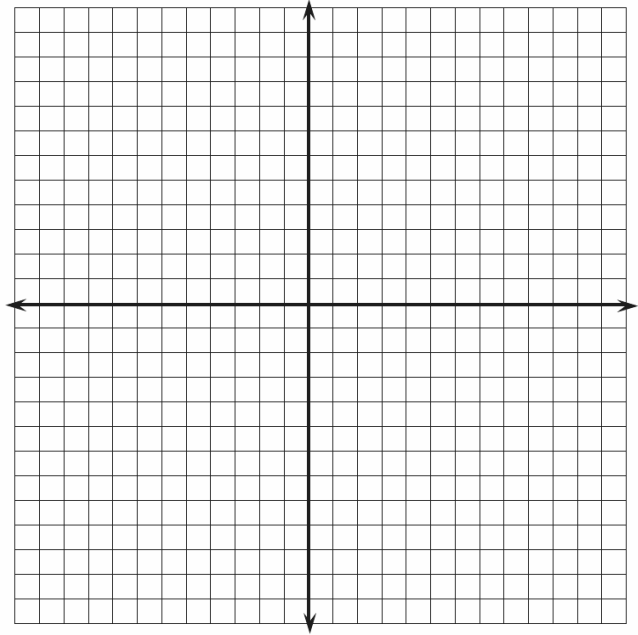
- The x -axis and y -axis of the coordinate plane are number lines that intersect at zero on each number line.
- The axes partition the coordinate plane into four quadrants.
- Points in the coordinate plane lie either on an axis or in one of the four quadrants.

Problem Set

1. Name the quadrant in which each of the points lies. If the point does not lie in a quadrant, specify which axis the point lies on.
 - a. $(-2, 5)$
 - b. $(8, -4)$
 - c. $(-1, -8)$
 - d. $(9.2, 7)$
 - e. $(0, -4)$
2. Jackie claims that points with the same x - and y -coordinates must lie in Quadrant I or Quadrant III. Do you agree or disagree? Explain your answer.
3. Locate and label each set of points on the coordinate plane. Describe similarities of the ordered pairs in each set, and describe the points on the plane.
 - a. $\{(-2, 5), (-2, 2), (-2, 7), (-2, -3), (-2, -0.8)\}$
 - b. $\{(-9, 9), (-4, 4), (-2, 2), (1, -1), (3, -3), (0, 0)\}$
 - c. $\{(-7, -8), (5, -8), (0, -8), (10, -8), (-3, -8)\}$



4. Locate and label at least five points on the coordinate plane that have an x -coordinate of 6.
- What is true of the y -coordinates below the x -axis?
 - What is true of the y -coordinates above the x -axis?
 - What must be true of the y -coordinates on the x -axis?



Lesson 16: Symmetry in the Coordinate Plane

Classwork

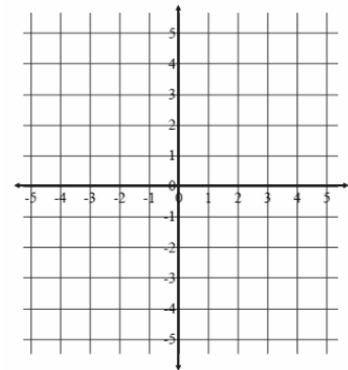
Opening Exercise

Give an example of two opposite numbers, and describe where the numbers lie on the number line. How are opposite numbers similar, and how are they different?

Example 1: Extending Opposite Numbers to the Coordinate Plane

Extending Opposite Numbers to the Coordinates of Points on the Coordinate Plane

Locate and label your points on the coordinate plane to the right. For each given pair of points in the table below, record your observations and conjectures in the appropriate cell. Pay attention to the absolute values of the coordinates and where the points lie in reference to each axis.



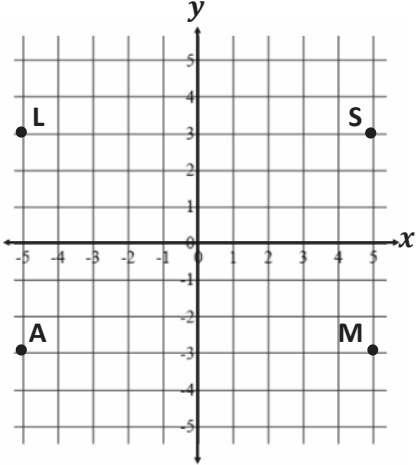
	$(3, 4)$ and $(-3, 4)$	$(3, 4)$ and $(3, -4)$	$(3, 4)$ and $(-3, -4)$
Similarities of Coordinates			
Differences of Coordinates			

Similarities in Location			
Differences in Location			
Relationship Between Coordinates and Location on the Plane			

Exercises

In each column, write the coordinates of the points that are related to the given point by the criteria listed in the first column of the table. Point $S(5, 3)$ has been reflected over the x - and y -axes for you as a guide, and its images are shown on the coordinate plane. Use the coordinate grid to help you locate each point and its corresponding coordinates.

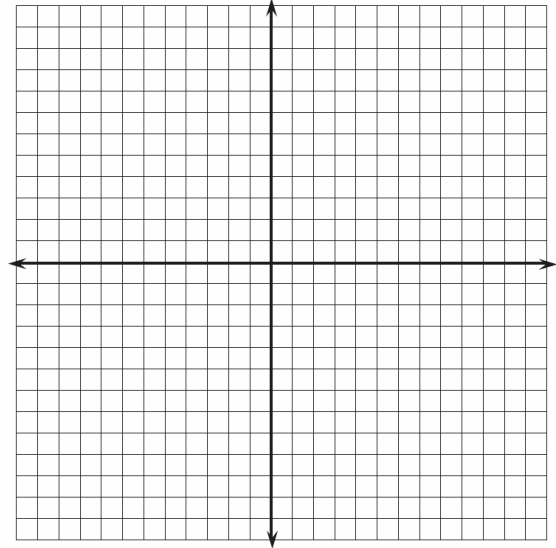
Given Point:	$S(5, 3)$	$(-2, 4)$	$(3, -2)$	$(-1, -5)$
The given point is reflected across the x -axis.				
The given point is reflected across the y -axis.				
The given point is reflected first across the x -axis and then across the y -axis.				
The given point is reflected first across the y -axis and then across the x -axis.				



Problem Set

1. Locate a point in Quadrant IV of the coordinate plane. Label the point A , and write its ordered pair next to it.

- Reflect point A over an axis so that its image is in Quadrant III. Label the image B , and write its ordered pair next to it. Which axis did you reflect over? What is the only difference in the ordered pairs of points A and B ?
- Reflect point B over an axis so that its image is in Quadrant II. Label the image C , and write its ordered pair next to it. Which axis did you reflect over? What is the only difference in the ordered pairs of points B and C ? How does the ordered pair of point C relate to the ordered pair of point A ?
- Reflect point C over an axis so that its image is in Quadrant I. Label the image D , and write its ordered pair next to it. Which axis did you reflect over? How does the ordered pair for point D compare to the ordered pair for point C ? How does the ordered pair for point D compare to points A and B ?



2. Bobbie listened to her teacher's directions and navigated from the point $(-1, 0)$ to $(5, -3)$. She knows that she has the correct answer, but she forgot part of the teacher's directions. Her teacher's directions included the following:

"Move 7 units down, reflect about the ___?___-axis, move up 4 units, and then move right 4 units."

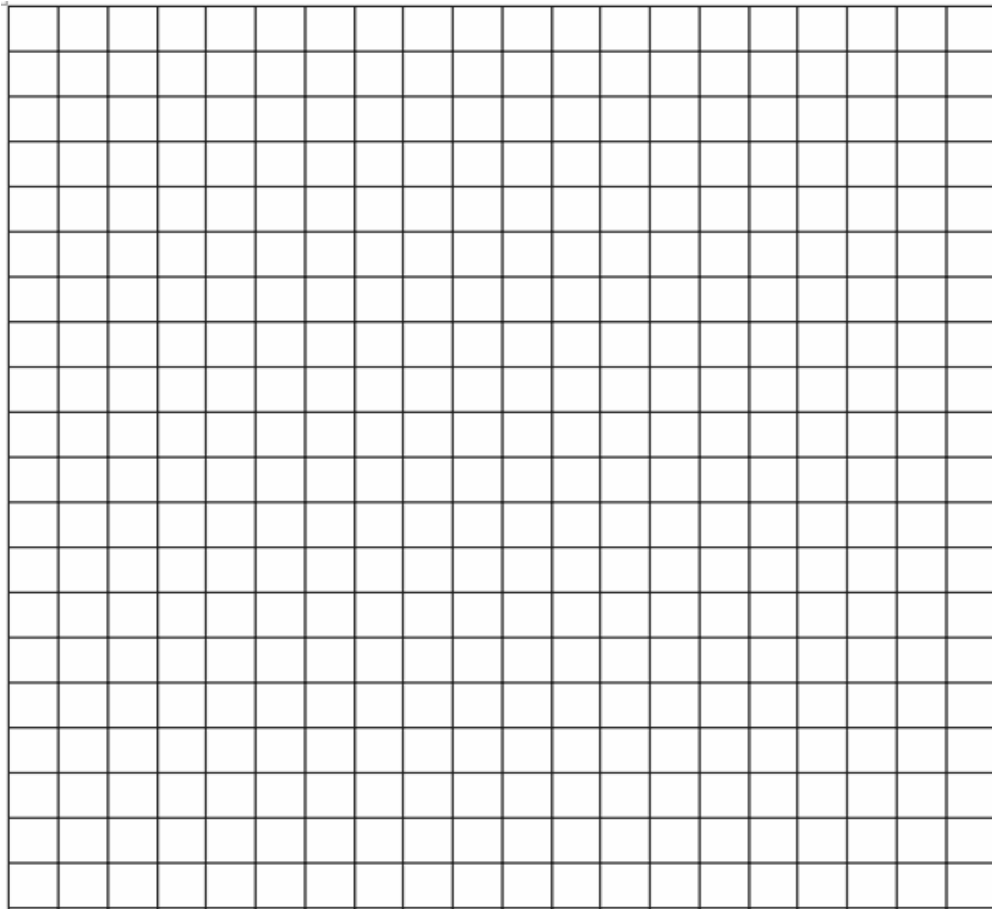
Help Bobbie determine the missing axis in the directions, and explain your answer.

Lesson 17: Drawing the Coordinate Plane and Points on the Plane

Classwork

Opening Exercise

Draw all necessary components of the coordinate plane on the blank 20×20 grid provided below, placing the origin at the center of the grid and letting each grid line represent 1 unit.



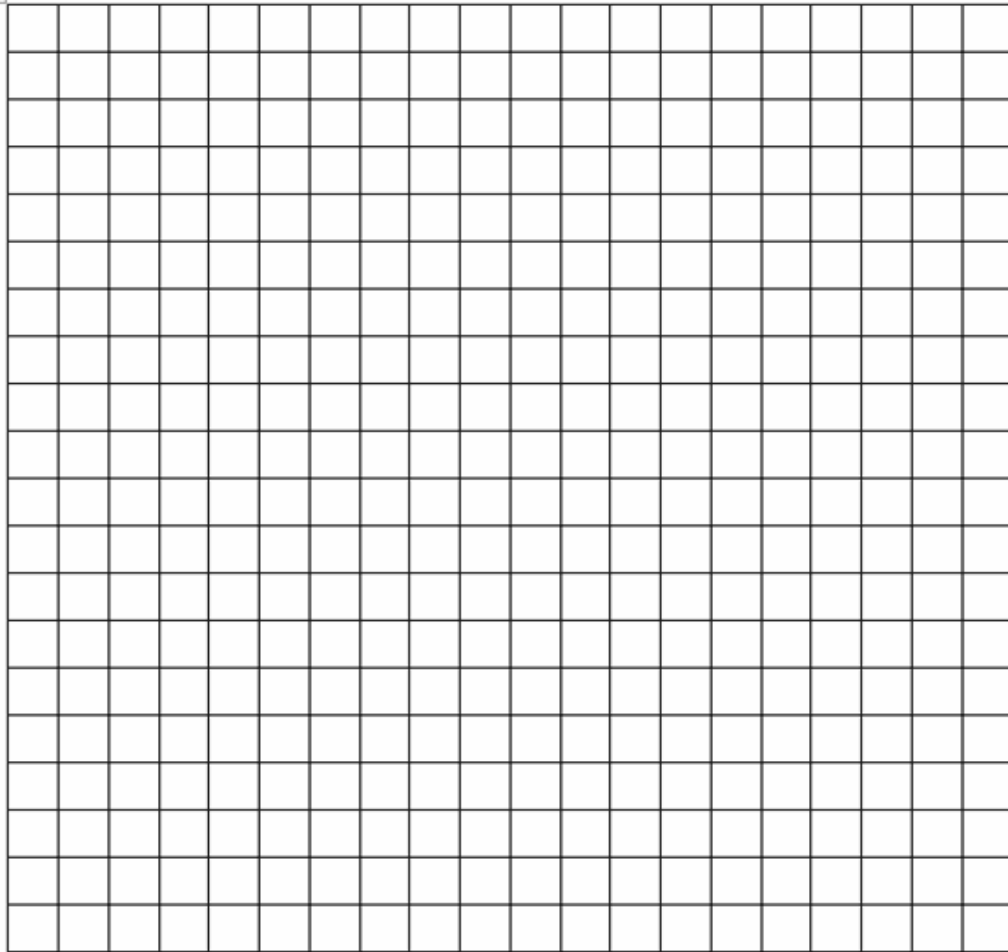
Example 1: Drawing the Coordinate Plane Using a 1: 1 Scale

Locate and label the points $\{(3,2), (8,4), (-3, 8), (-2, -9), (0,6), (-1, -2), (10, -2)\}$ on the grid above.

Example 2: Drawing the Coordinate Plane Using an Increased Number Scale for One Axis

Draw a coordinate plane on the grid below, and then locate and label the following points:

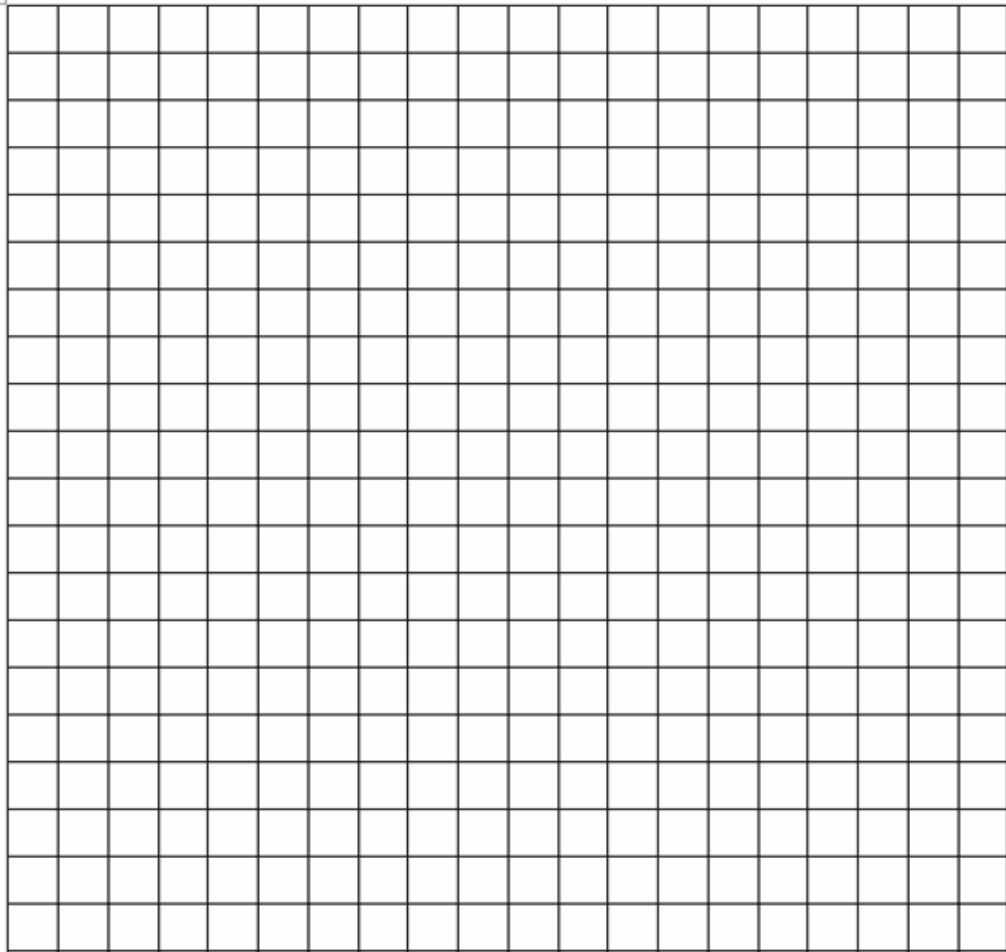
$$\{(-4, 20), (-3, 35), (1, -35), (6, 10), (9, -40)\}.$$



Example 3: Drawing the Coordinate Plane Using a Decreased Number Scale for One Axis

Draw a coordinate plane on the grid below, and then locate and label the following points:

$$\{(0.1, 4), (0.5, 7), (-0.7, -5), (-0.4, 3), (0.8, 1)\}.$$



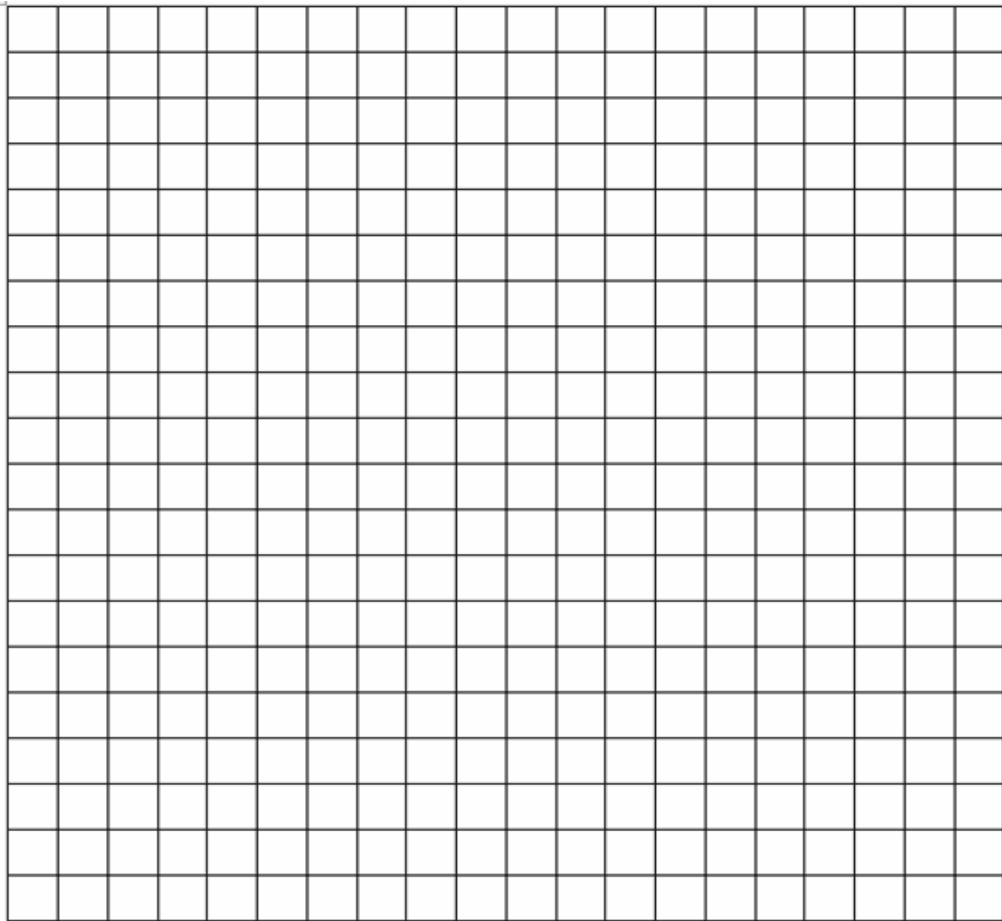
Example 4: Drawing the Coordinate Plane Using a Different Number Scale for Both Axes

Determine a scale for the x -axis that will allow all x -coordinates to be shown on your grid.

Determine a scale for the y -axis that will allow all y -coordinates to be shown on your grid.

Draw and label the coordinate plane, and then locate and label the set of points.

$$\{(-14, 2), (-4, -0.5), (6, -3.5), (14, 2.5), (0, 3.5), (-8, -4)\}$$



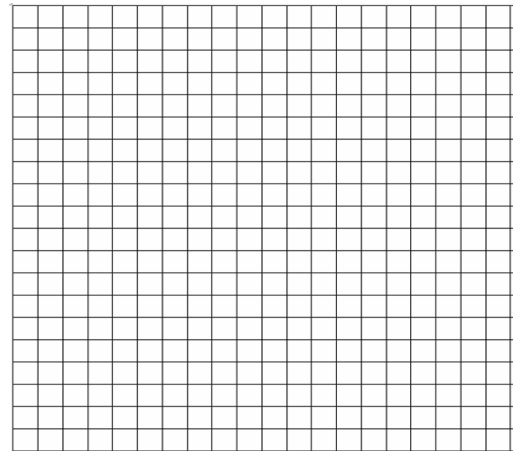
Lesson Summary

- The axes of the coordinate plane must be drawn using a straightedge and labeled x (horizontal axis) and y (vertical axis).
- Before assigning a scale to the axes, it is important to assess the range of values found in a set of points as well as the number of grid lines available. This allows you to determine an appropriate scale so all points can be represented on the coordinate plane that you construct.

Problem Set

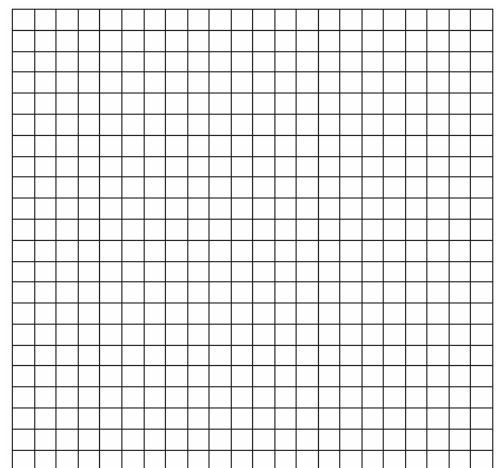
1. Label the coordinate plane, and then locate and label the set of points below.

$$\left\{ \begin{array}{l} (0.3, 0.9), (-0.1, 0.7), (-0.5, -0.1), \\ (-0.9, 0.3), (0, -0.4) \end{array} \right\}$$



2. Label the coordinate plane, and then locate and label the set of points below.

$$\left\{ \begin{array}{l} (90, 9), (-110, -11), (40, 4), \\ (-60, -6), (-80, -8) \end{array} \right\}$$



Extension:

3. Describe the pattern you see in the coordinates in Problem 2 and the pattern you see in the points. Are these patterns consistent for other points too?

Lesson 18: Distance on the Coordinate Plane

Classwork

Opening Exercise

Four friends are touring on motorcycles. They come to an intersection of two roads; the road they are on continues straight, and the other is perpendicular to it. The sign at the intersection shows the distances to several towns. Draw a map/diagram of the roads, and use it and the information on the sign to answer the following questions:

Albertsville ← 8 mi.

Blossville ↑ 3 mi.

Cheyenne ↑ 12 mi.

Dewey Falls → 6 mi.

What is the distance between Albertsville and Dewey Falls?

What is the distance between Blossville and Cheyenne?

On the coordinate plane, what represents the intersection of the two roads?

Example 1: The Distance Between Points on an Axis

Consider the points $(-4, 0)$ and $(5, 0)$.

What do the ordered pairs have in common, and what does that mean about their location in the coordinate plane?

How did we find the distance between two numbers on the number line?

Use the same method to find the distance between $(-4, 0)$ and $(5, 0)$.

Example 2: The Length of a Line Segment on an Axis

Consider the line segment with end points $(0, -6)$ and $(0, -11)$.

What do the ordered pairs of the end points have in common, and what does that mean about the line segment's location in the coordinate plane?

Find the length of the line segment described by finding the distance between its end points $(0, -6)$ and $(0, -11)$.

Example 3: Length of a Horizontal or Vertical Line Segment That Does Not Lie on an Axis

Consider the line segment with end points $(-3, 3)$ and $(-3, -5)$.

What do the end points, which are represented by the ordered pairs, have in common? What does that tell us about the location of the line segment on the coordinate plane?

Find the length of the line segment by finding the distance between its end points.

Exercise

Find the lengths of the line segments whose end points are given below. Explain how you determined that the line segments are horizontal or vertical.

a. $(-3, 4)$ and $(-3, 9)$

b. $(2, -2)$ and $(-8, -2)$

c. $(-6, -6)$ and $(-6, 1)$

d. $(-9, 4)$ and $(-4, 4)$

e. $(0, -11)$ and $(0, 8)$

Lesson Summary

To find the distance between points that lie on the same horizontal line or on the same vertical line, we can use the same strategy that we used to find the distance between points on the number line.

Problem Set

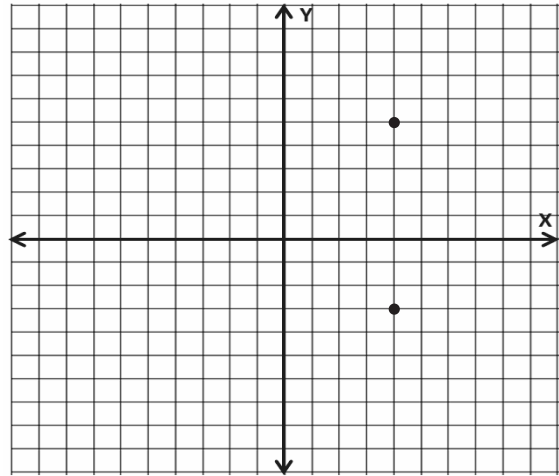
1. Find the length of the line segment with end points $(7, 2)$ and $(-4, 2)$, and explain how you arrived at your solution.
2. Sarah and Jamal were learning partners in math class and were working independently. They each started at the point $(-2, 5)$ and moved 3 units vertically in the plane. Each student arrived at a different end point. How is this possible? Explain and list the two different end points.
3. The length of a line segment is 13 units. One end point of the line segment is $(-3, 7)$. Find four points that could be the other end points of the line segment.

Lesson 19: Problem Solving and the Coordinate Plane

Classwork

Opening Exercise

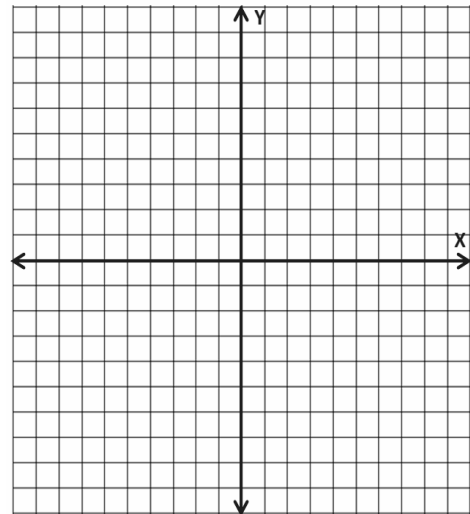
In the coordinate plane, find the distance between the points using absolute value.



Exploratory Challenge

Exercises 1–2: The Length of a Line Segment Is the Distance Between Its End Points

1. Locate and label $(4, 5)$ and $(4, -3)$. Draw the line segment between the end points given on the coordinate plane. How long is the line segment that you drew? Explain.



2. Draw a horizontal line segment starting at $(4, -3)$ that has a length of 9 units. What are the possible coordinates of the other end point of the line segment? (There is more than one answer.)

Which point did you choose to be the other end point of the horizontal line segment? Explain how and why you chose that point. Locate and label the point on the coordinate grid.

Exercise 3: Extending Lengths of Line Segments to Sides of Geometric Figures

3. The two line segments that you have just drawn could be seen as two sides of a rectangle. Given this, the end points of the two line segments would be three of the vertices of this rectangle.
 - a. Find the coordinates of the fourth vertex of the rectangle. Explain how you find the coordinates of the fourth vertex using absolute value.

 - b. How does the fourth vertex that you found relate to each of the consecutive vertices in either direction? Explain.

 - c. Draw the remaining sides of the rectangle.

Exercises 4–6: Using Lengths of Sides of Geometric Figures to Solve Problems

4. Using the vertices that you have found and the lengths of the line segments between them, find the perimeter of the rectangle.

5. Find the area of the rectangle.

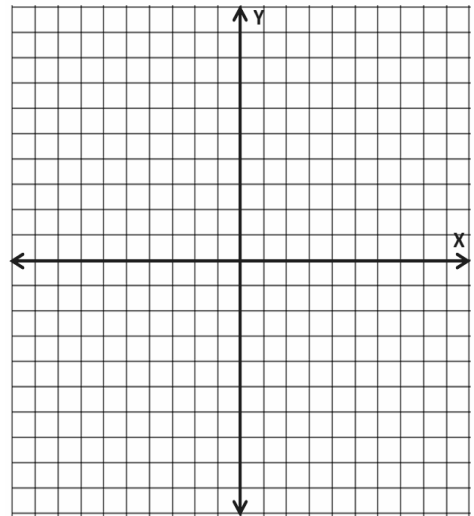
6. Draw a diagonal line segment through the rectangle with opposite vertices for end points. What geometric figures are formed by this line segment? What are the areas of each of these figures? Explain.

Extension (If time allows): Line the edge of a piece of paper up to the diagonal in the rectangle. Mark the length of the diagonal on the edge of the paper. Align your marks horizontally or vertically on the grid, and estimate the length of the diagonal to the nearest integer. Use that estimation to now estimate the perimeter of the triangles.

Exercise 7

7. Construct a rectangle on the coordinate plane that satisfies each of the criteria listed below. Identify the coordinate of each of its vertices.
- Each of the vertices lies in a different quadrant.
 - Its sides are either vertical or horizontal.
 - The perimeter of the rectangle is 28 units.

Using absolute value, show how the lengths of the sides of your rectangle provide a perimeter of 28 units.



Lesson Summary

- The length of a line segment on the coordinate plane can be determined by finding the distance between its end points.
- You can find the perimeter and area of figures such as rectangles and right triangles by finding the lengths of the line segments that make up their sides and then using the appropriate formula.

Problem Set

1. One end point of a line segment is $(-3, -6)$. The length of the line segment is 7 units. Find four points that could serve as the other end point of the given line segment.
2. Two of the vertices of a rectangle are $(1, -6)$ and $(-8, -6)$. If the rectangle has a perimeter of 26 units, what are the coordinates of its other two vertices?
3. A rectangle has a perimeter of 28 units, an area of 48 square units, and sides that are either horizontal or vertical. If one vertex is the point $(-5, -7)$ and the origin is in the interior of the rectangle, find the vertex of the rectangle that is opposite $(-5, -7)$.